

# Color I: trichromatic theory

CS 178, Spring 2014

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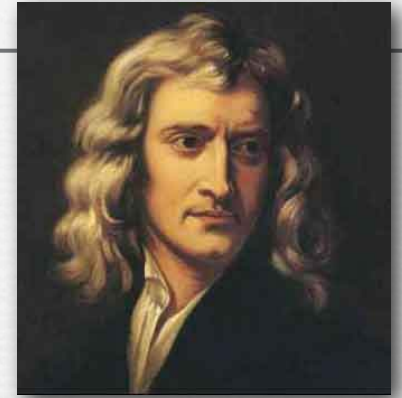
Marc Levoy  
Computer Science Department  
Stanford University

# Outline

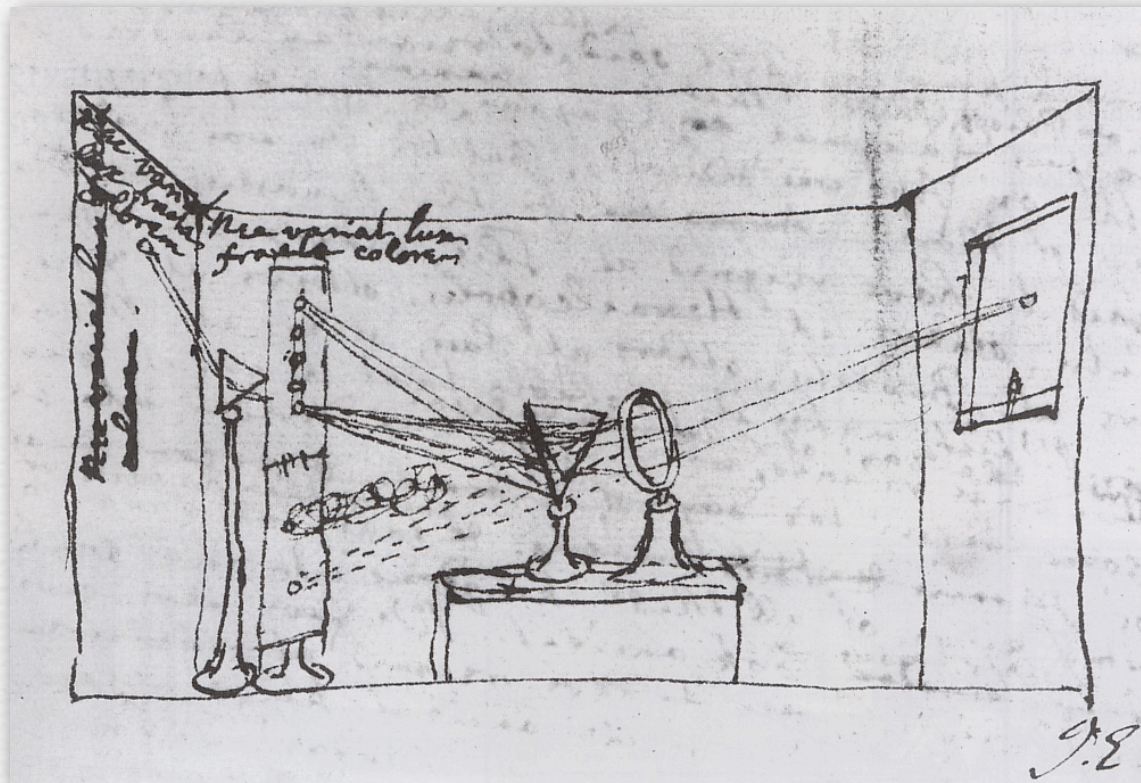
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- ◆ spectral power distributions
  - ◆ color response in animals and humans
  - ◆ 3D colorspace of the human visual system
    - and color filter arrays in cameras
  - ◆ reproducing colors using three primaries
    - including computer screens
  - ◆ additive versus subtractive color mixing
- 
- ◆ cylindrical color systems used by artists (and Photoshop)
  - ◆ chromaticity diagrams
    - color temperature and white balancing
    - standardized color spaces and gamut mapping

# Newton's Experimentum Crucis



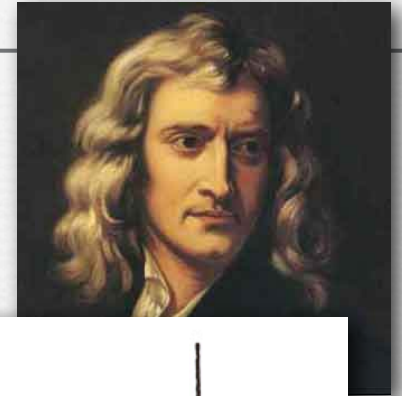
Isaac Newton  
(1643-1727)



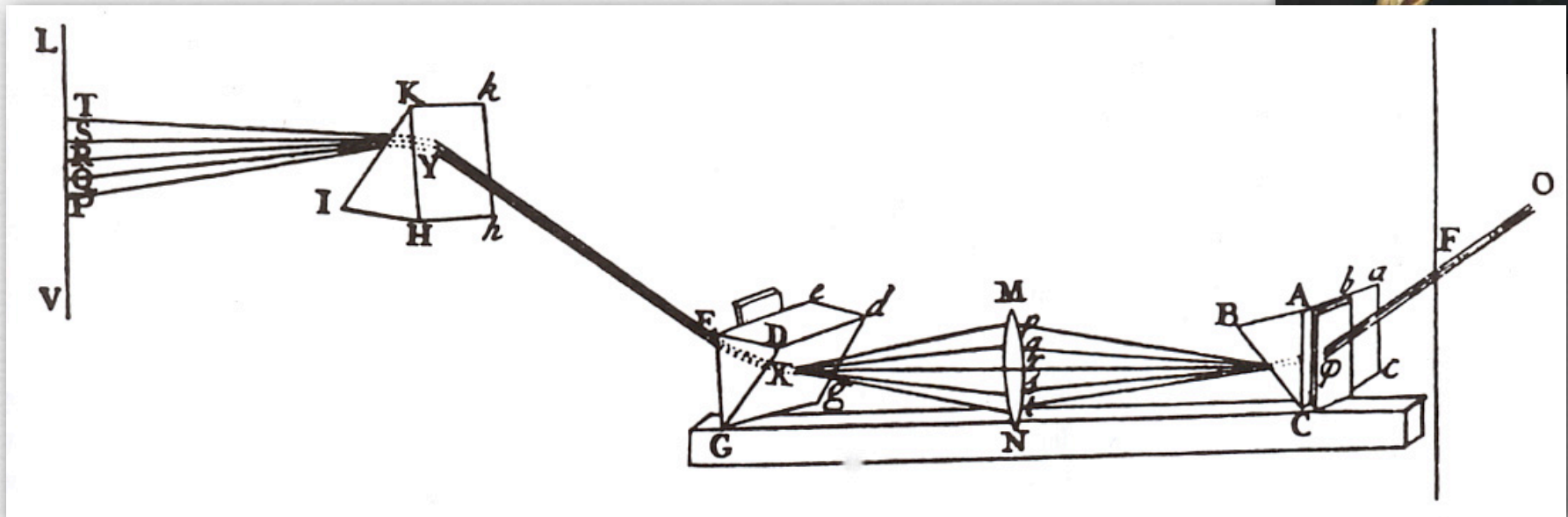
(Robin)

- ◆ sunlight can be divided into colors using a prism
- ◆ these colors cannot be further divided using a 2nd prism
- ◆ experiment performed 1665, drawing made in 1672

# Newton's Experimentum Crucis

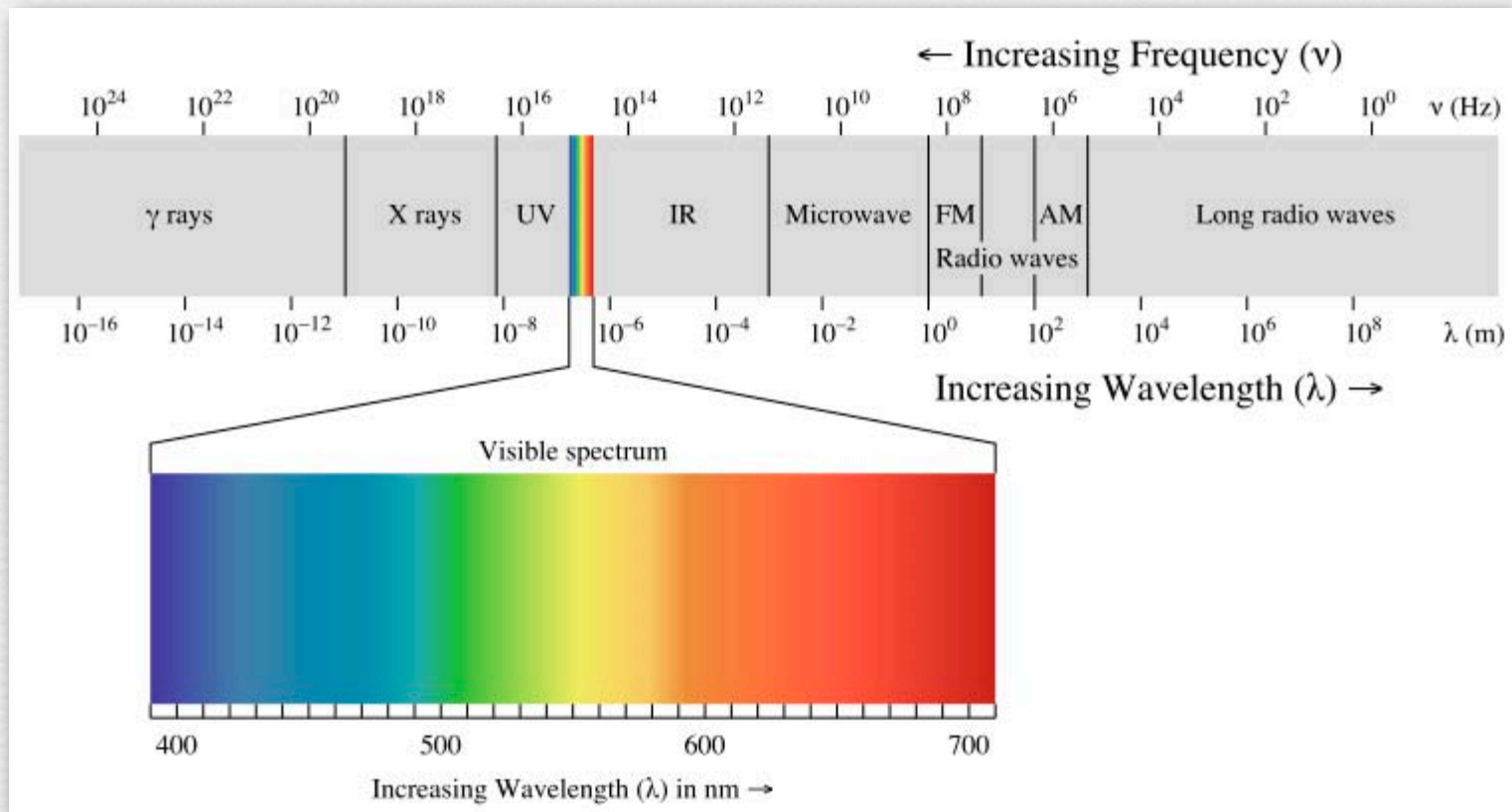


(Robin)



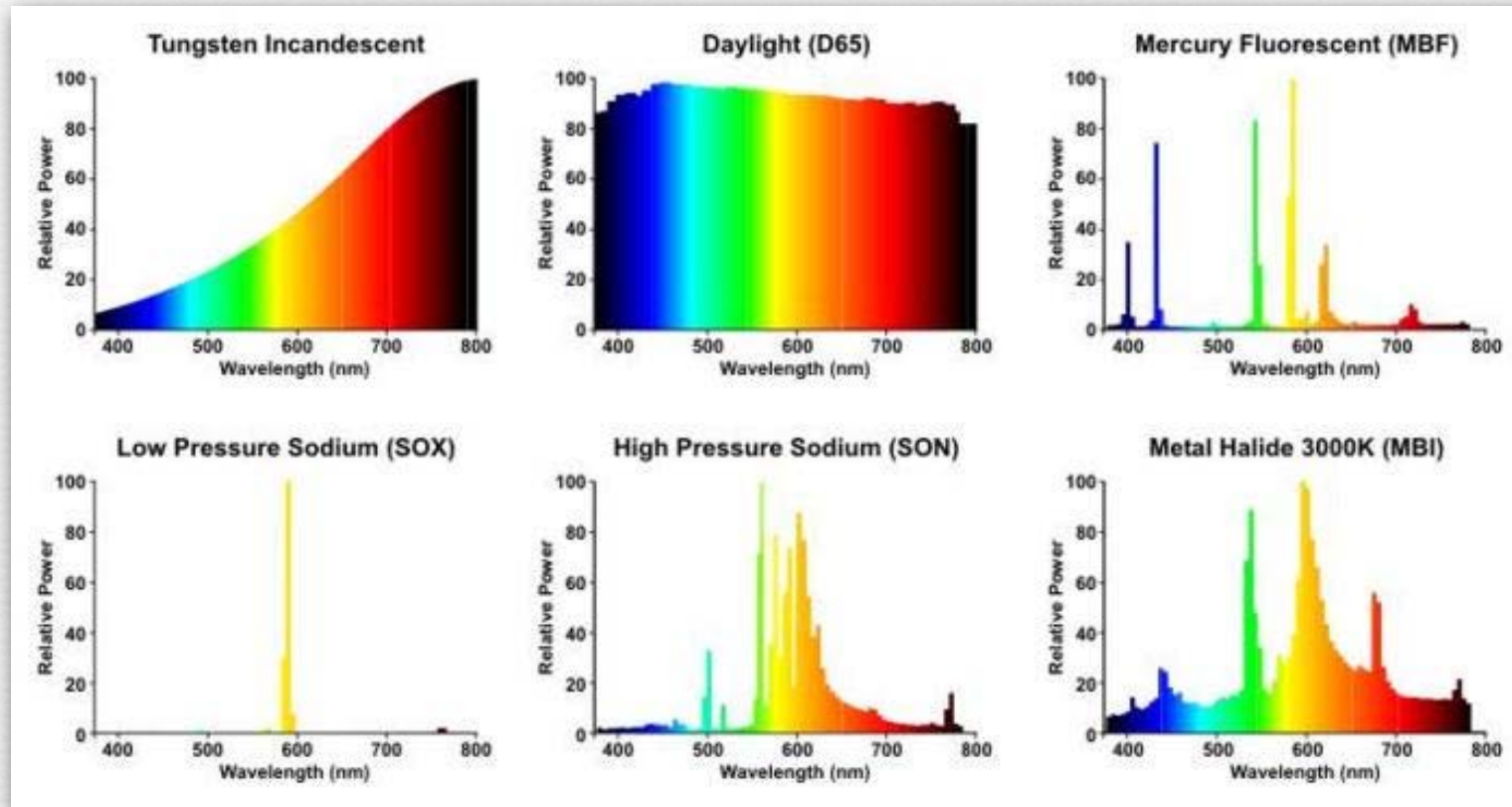
- ♦ alternatively, the divided colors can be recombined using a lens and 2nd prism into a new beam that has exactly the same properties as the original

# The visible light spectrum



- ◆ wavelengths between 400nm and 700 nm ( $0.4\mu - 0.7\mu$ )
- ◆ exactly the colors in a rainbow

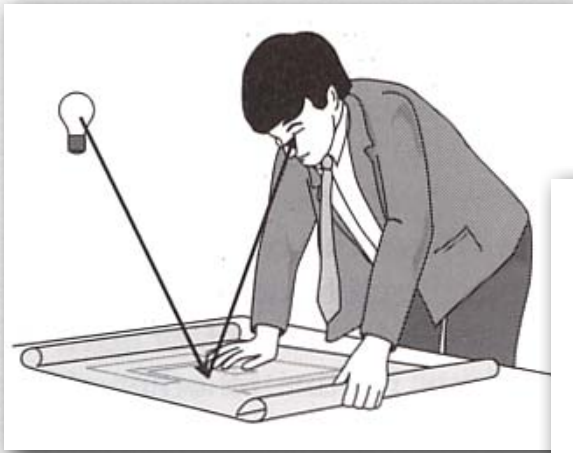
# Spectral power distribution (SPD)



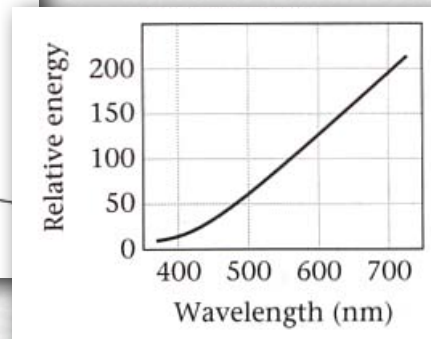
(LampTech)

- ◆ units of power are watts (joules per second)
- ◆ shown here are spectra of common illumination sources
- ◆ plots above are relative amounts (%) of each wavelength

# Interaction of light with matter

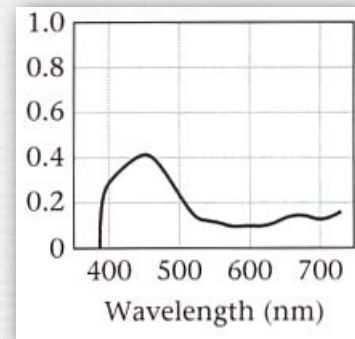


light is reflected  
by an object



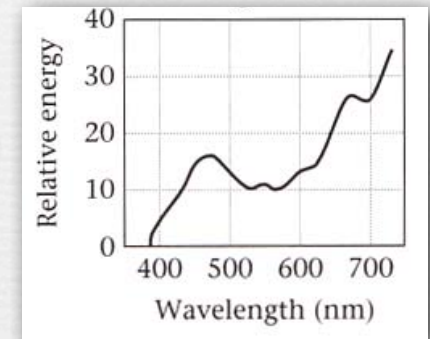
illumination

×



reflectance

=

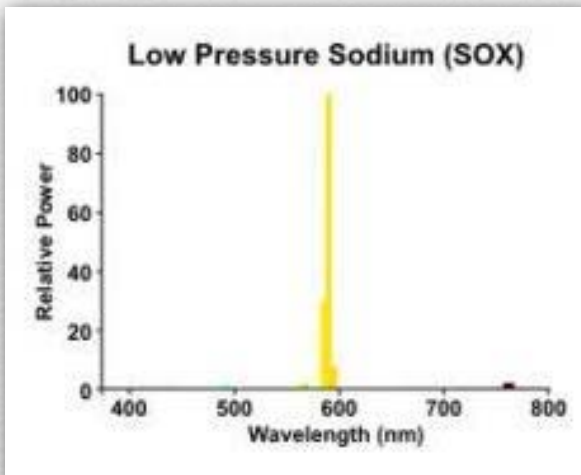


stimulus that  
enters your eye

- ◆ illumination is multiplied wavelength-by-wavelength by reflectance of object at that wavelength
  - cause is absorption by the material
  - so the spectrum you see depends on the illumination
- ◆ transmittance operates the same way

# Example

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×

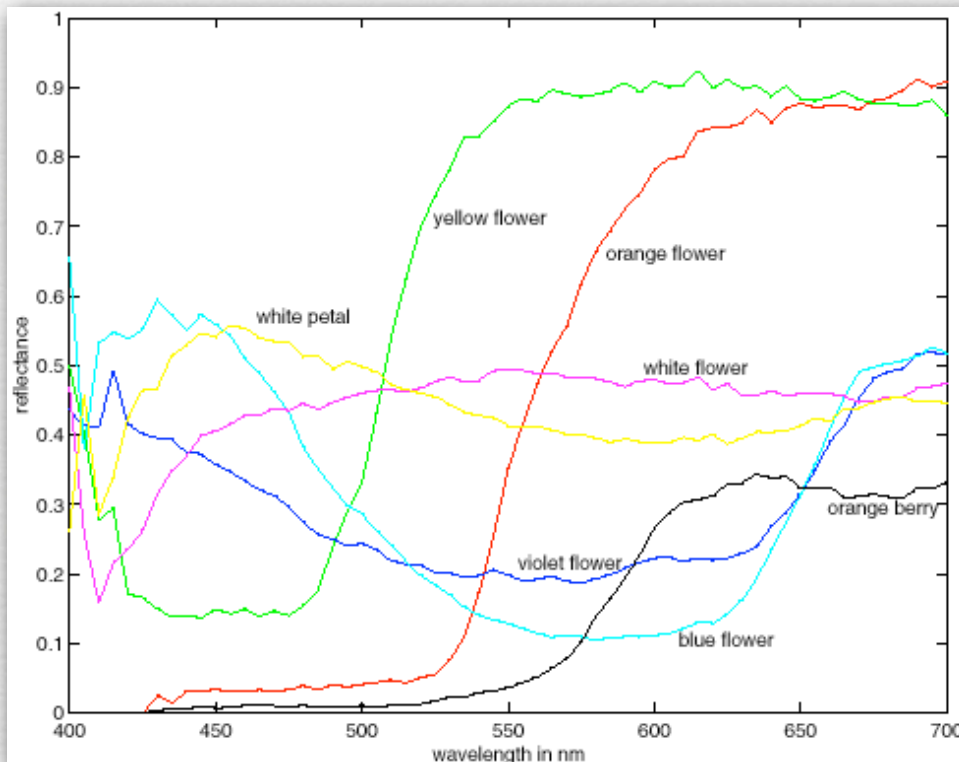


= nearly  
black

my old van



# Examples of reflectance spectra




- two reflectance spectra that match (i.e. are metamers) under one illuminant may not match under another
- clothes that match in the store may not match outdoors

## Questions?

- ◆ two different spectra may appear alike to us
  - white petal and white flower (above left)
  - these are called *metamers*
- ◆ Newton observed this, but could not explain it

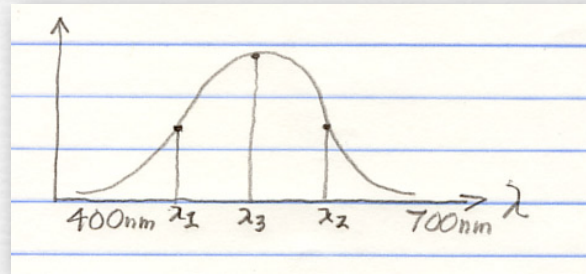
# Outline

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- ◆ spectral power distributions
-  ◆ color response in animals and humans
- ◆ 3D colorspace of the human visual system
  - and color filter arrays in cameras
- ◆ reproducing colors using three primaries
- ◆ additive versus subtractive color mixing
- ◆ cylindrical color systems used by artists (and Photoshop)
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  - color temperature and white balancing
  - standardized color spaces and gamut mapping

# Monochromats (contents of whiteboard)

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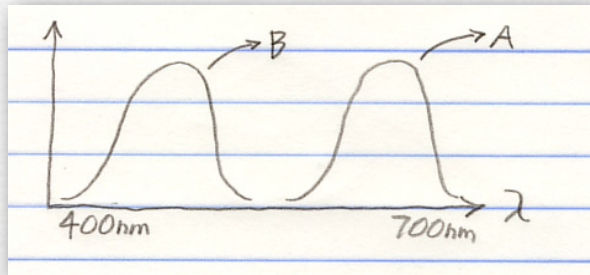


1

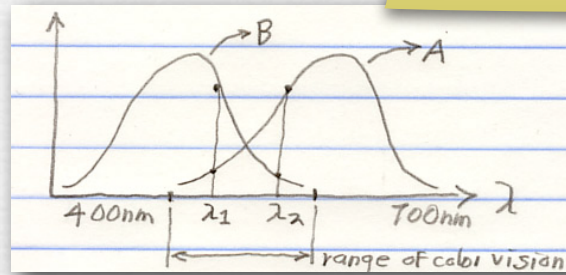
1. organisms having only one kind of retinal receptor cannot distinguish changes in intensity from changes in wavelength, hence they have no *color discrimination*
  - for example a unit amount of  $\lambda_1$  versus  $\lambda_2$  above
  - or a unit amount of  $\lambda_1$  versus half as much of  $\lambda_3$  (assuming the response to  $\lambda_3$  is twice the response to  $\lambda_1$ )
  - example: horseshoe crab

# Dichromats (contents of whiteboard)

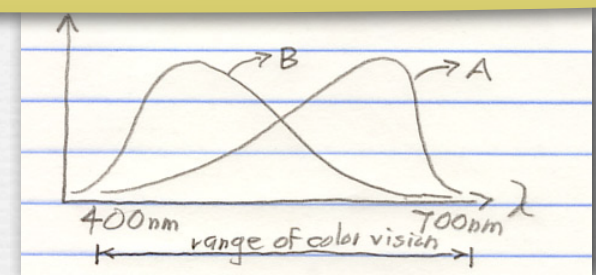
As a student pointed out after class, my definition that color vision consists of not being able to discriminate changes in wavelength from changes in brightness applies only if the stimulus is a single wavelength. If the stimulus contains multiple wavelengths, the dichromat in case 2 below can discriminate many such mixtures. In this situation their color vision should be analyzed in terms of the shape of the spectral locus and the gamut of perceivable colors, as described in later slides.



2



3

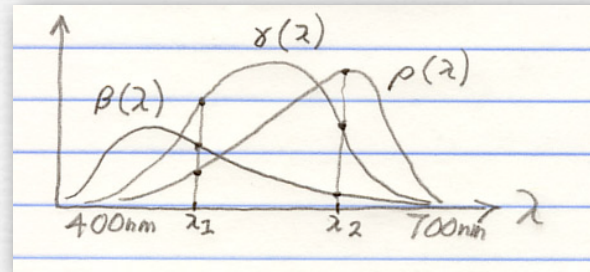


4

2. this organism can discriminate a response in the range of wavelengths covered by A versus by B, but cannot discriminate within those ranges
3. this organism has color discrimination over the range of wavelengths shown
  - for each wavelength within this range, the ratio of responses of receptors A and B is unique; hence the organism can identify which wavelength (e.g.  $\lambda_1$  or  $\lambda_2$ ) it's looking at
4. this organism has a larger range of color vision
  - example: dog, horse

# Trichromats (contents of whiteboard)

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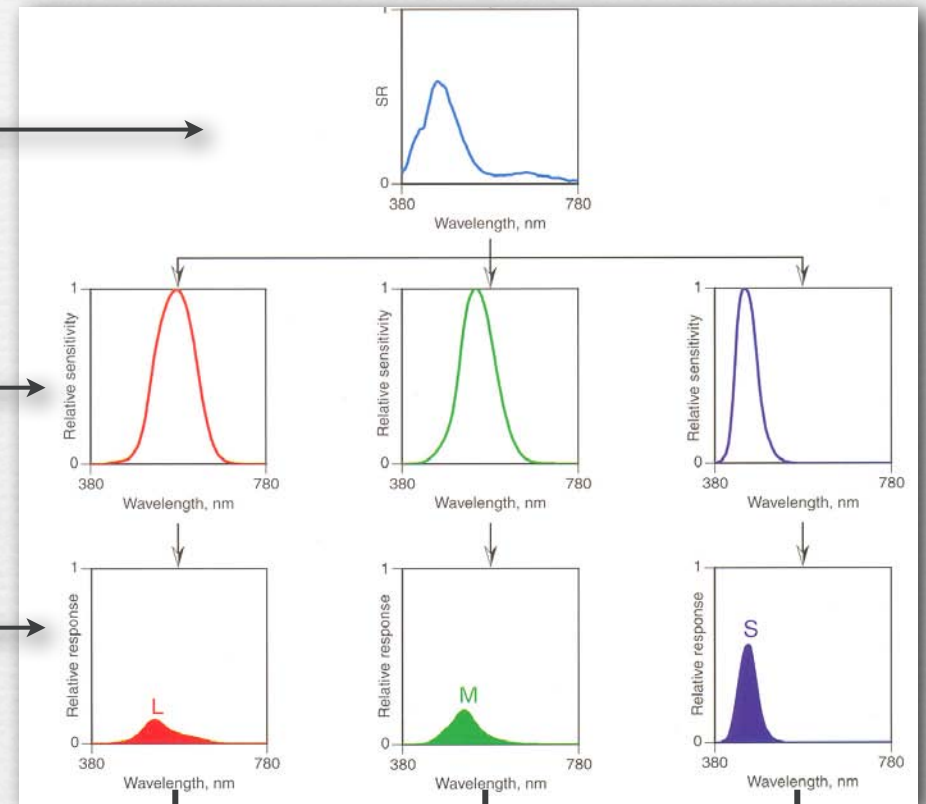
5

5. humans can discriminate wavelengths from 400nm to 700nm
- we can also discriminate mixtures of wavelengths that dichromats cannot; this will become clearer later
- ♦ at the retinal level, our response to light is linear
- if the response to a unit stimulus at  $\lambda_1$  is  $(\rho_1, \gamma_1, \beta_1)$ , and to a unit stimulus at  $\lambda_2$  is  $(\rho_2, \gamma_2, \beta_2)$ , then the response to a superposition of stimuli  $\lambda_1$  and  $\lambda_2$  is  $(\rho_1 + \rho_2, \gamma_1 + \gamma_2, \beta_1 + \beta_2)$
  - the response to  $n$  units of a stimulus at  $\lambda_1$  is  $(n \rho_1, n \gamma_1, n \beta_1)$
  - a system that obeys *superposition* (a) and *scaling* (b) is *linear*

# Human response to an arbitrary stimulus

(Berns)

spectrum of stimulus arriving  
in one small area on retina  
 $\times$   
spectral sensitivity of each  
type of cone (L,M,S)  
 $=$   
multiply wavelength-by-  
wavelength to get response spectra  
 $\int$   
integrate over wavelengths to get  
total response for that type of cone



♦ output is three numbers ( $\rho$ ,  $\gamma$ ,  $\beta$ ) per area on retina

# Human response to an arbitrary stimulus

(Berns)

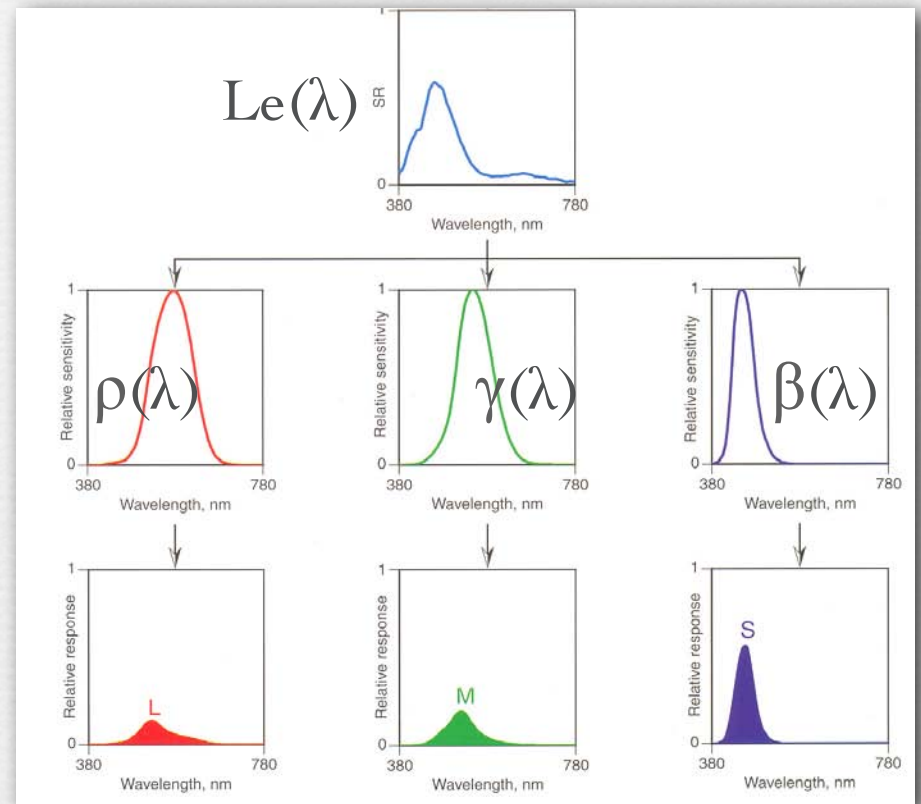
- ♦ stated algebraically, given a stimulus spectrum  $L_e(\lambda)$ , the human response to it ( $\rho, \gamma, \beta$ ) are the integrals over all visible wavelengths of our responses

$$L_e(\lambda) \rho(\lambda),$$

$$L_e(\lambda) \gamma(\lambda),$$

$$L_e(\lambda) \beta(\lambda)$$

to each constituent wavelength  $\lambda$ , i.e.




$$(\rho, \gamma, \beta) = \left( \int_{400 \text{ nm}}^{700 \text{ nm}} L_e(\lambda) \rho(\lambda) d\lambda, \int_{400 \text{ nm}}^{700 \text{ nm}} L_e(\lambda) \gamma(\lambda) d\lambda, \int_{400 \text{ nm}}^{700 \text{ nm}} L_e(\lambda) \beta(\lambda) d\lambda \right)$$

## Questions?

# Outline

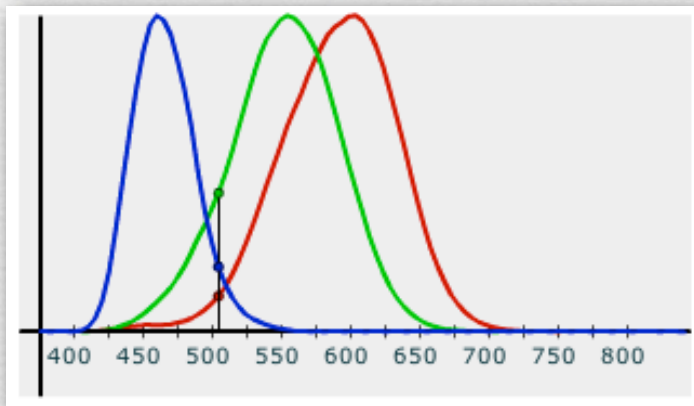
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# Human 3D colorspace

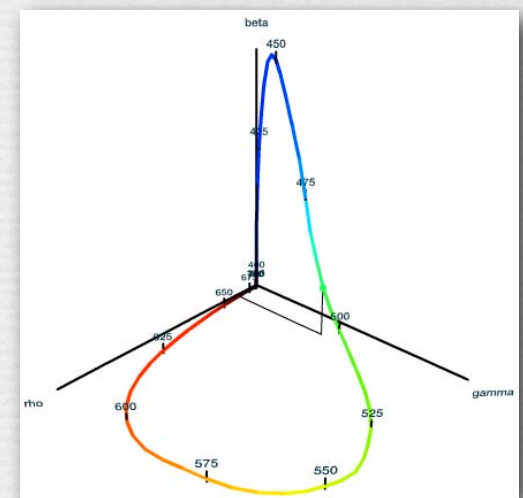
- ◆ the three types of cones in our retina (Long, Medium, Short wavelength) define the axes of a three-dimensional space
- ◆ our response to any stimulus spectrum can be summarized by three numbers ( $\rho$ ,  $\gamma$ ,  $\beta$ ) and plotted as a point in this space
- ◆ our responses to all visible single-wavelength spectra (a.k.a. pure wavelengths  $\lambda$ , i.e. positions along the rainbow), if connected together, form a curve in this space, called the *locus of spectral colors*; the sequence of ( $\rho$ ,  $\gamma$ ,  $\beta$ ) numbers form the *tristimulus sensitivity functions*  $\rho(\lambda)$ ,  $\gamma(\lambda)$ , and  $\beta(\lambda)$



sensitivity functions

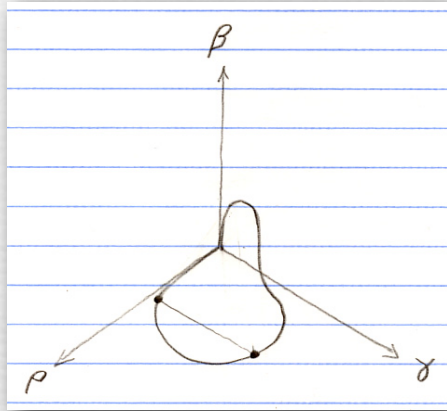
**(FLASH DEMO)**

<http://graphics.stanford.edu/courses/cs178/applets/locus.html>

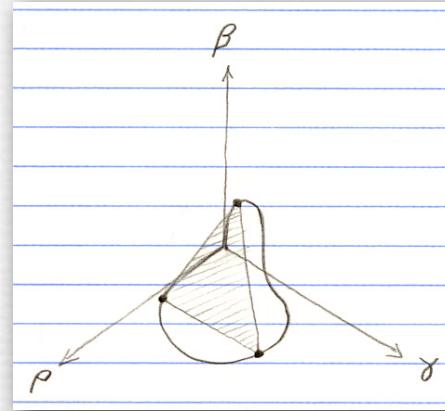


spectral locus

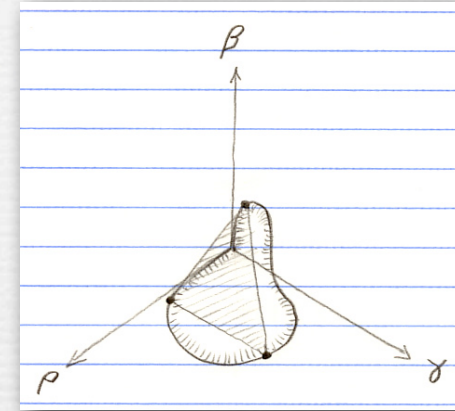
# Properties of human 3D colorspace (1 of 2) (contents of whiteboard)



1



2

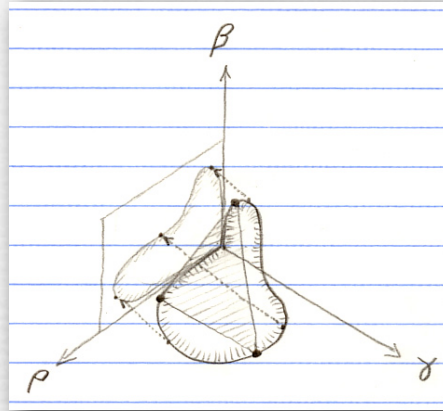


3

1. our response to any mixture ( $\sum = 1$ ) of two pure wavelengths falls on a line connecting the responses to each wavelength
2. our response to any mixture ( $\sum = 1$ ) of three pure wavelengths falls on a triangle connecting the responses to each wavelength; our response to any mixture or scaling ( $\sum \leq 1$ ) of three pure wavelengths falls in a tetrahedron defined by this triangle and the origin
3. our responses to all possible mixtures or scalings ( $\sum \leq 1$ ) of all visible wavelengths forms an irregular volume called the *gamut of perceivable colors*, equal to the convex hull of the spectral locus

# Properties of human 3D colorspace (2 of 2) (contents of whiteboard)

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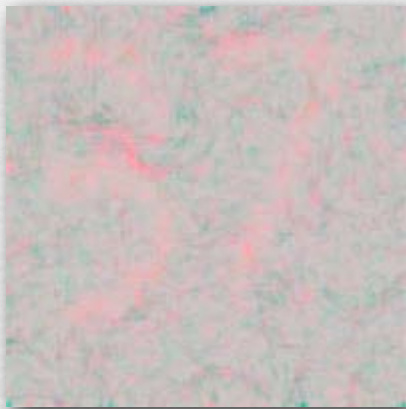
4

4. to a deuteranope - a color-blind person who is missing their medium-wavelength receptor, i.e. their gamma receptor - this diagram is squashed into the rectangle shown above on the rho-beta plane
- as a result, spectra whose  $(\rho, \gamma, \beta)$  responses lie along the dotted lines cannot be distinguished; they will appear as the same color, i.e. as metamers
  - by a similar argument, many spectra distinguishable to pentachromats (e.g. Mallard ducks), whose colorspace is 5-dimensional, are indistinguishable to trichromats (humans), whose colorspace is only 3-dimensional

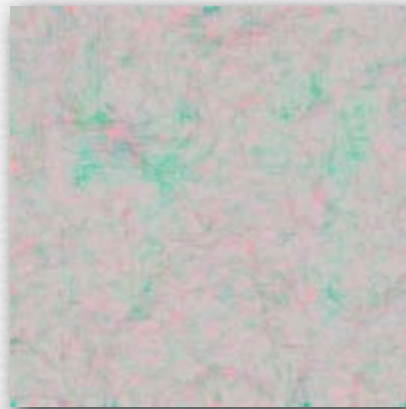
# Color blindness

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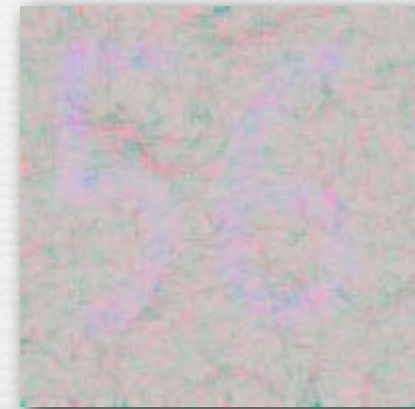
37?



49?



56?

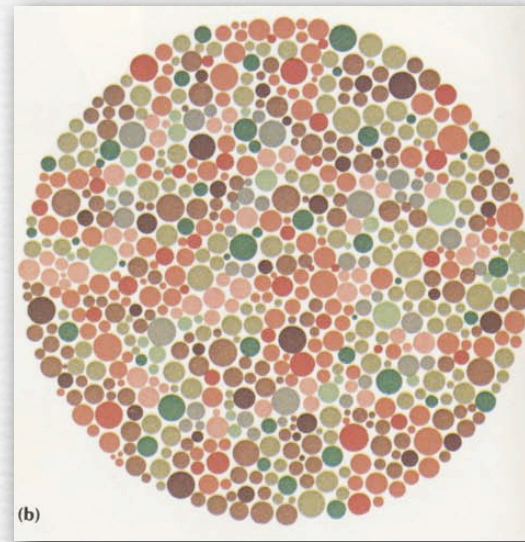
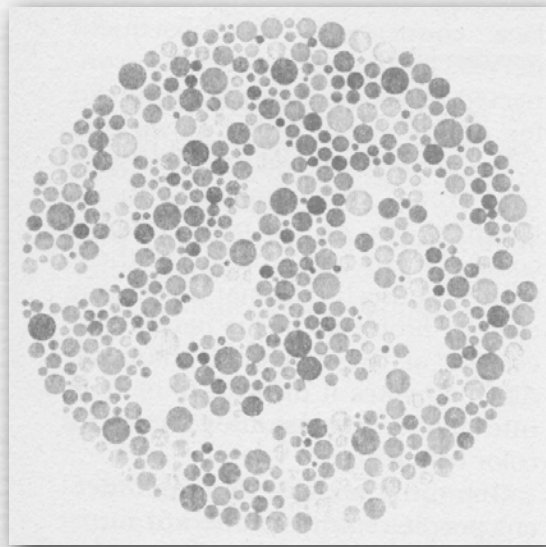


(wikipedia)

- ◆ protanopia (1% of males)
- ◆ deuteranopia (1% of males)
- ◆ tritanopia (< 1% of both genders)
- ◆ protanomaly (1% of males)
- ◆ deuteranomaly (6% of males)
- ◆ tritanomaly (< 1% of both genders)

# The advantage of being color blind

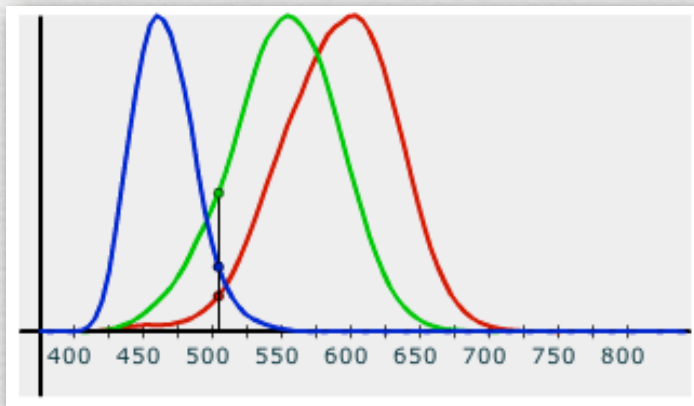
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- ◆ the maze (at left) is recreated (at right) using subtle intensity differences, but overridden by stronger red-green color differences
- ◆ a deuteranope can easily see the maze at right

# Human 3D colorspace

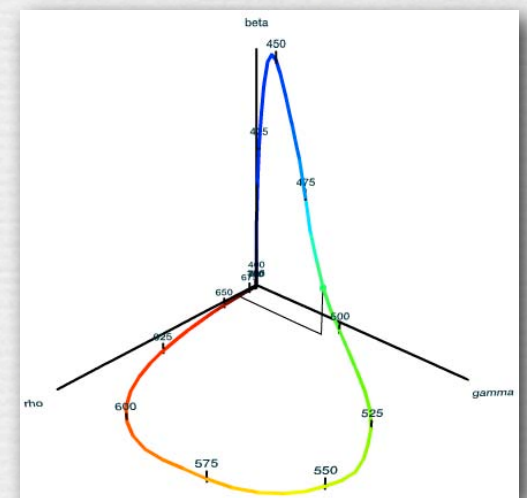
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sensitivity functions

**(FLASH DEMO)**

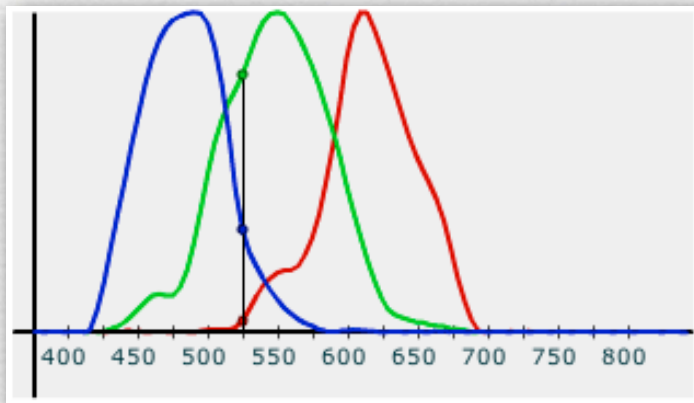
<http://graphics.stanford.edu/courses/cs178/applets/locus.html>



spectral locus

# Canon 30D color filters

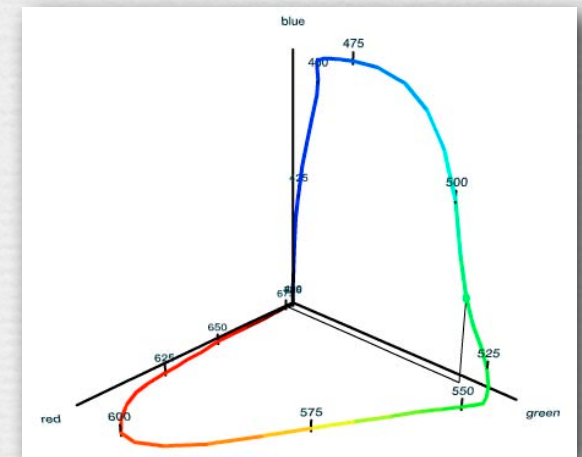
- ♦ you want the camera's R, G, and B color filters to have the same spectral sensitivities as our L, M, and S cones
  - you don't want objects in the real world to be metamers to one system and not the other
  - otherwise, colored textures the camera sees might be invisible to a person (bad), or textures you see might be invisible to a camera (also bad)



filter transmissivity

**(FLASH DEMO)**

<http://graphics.stanford.edu/courses/cs178/applets/locus.html>




spectral locus

**Questions?**

# Outline

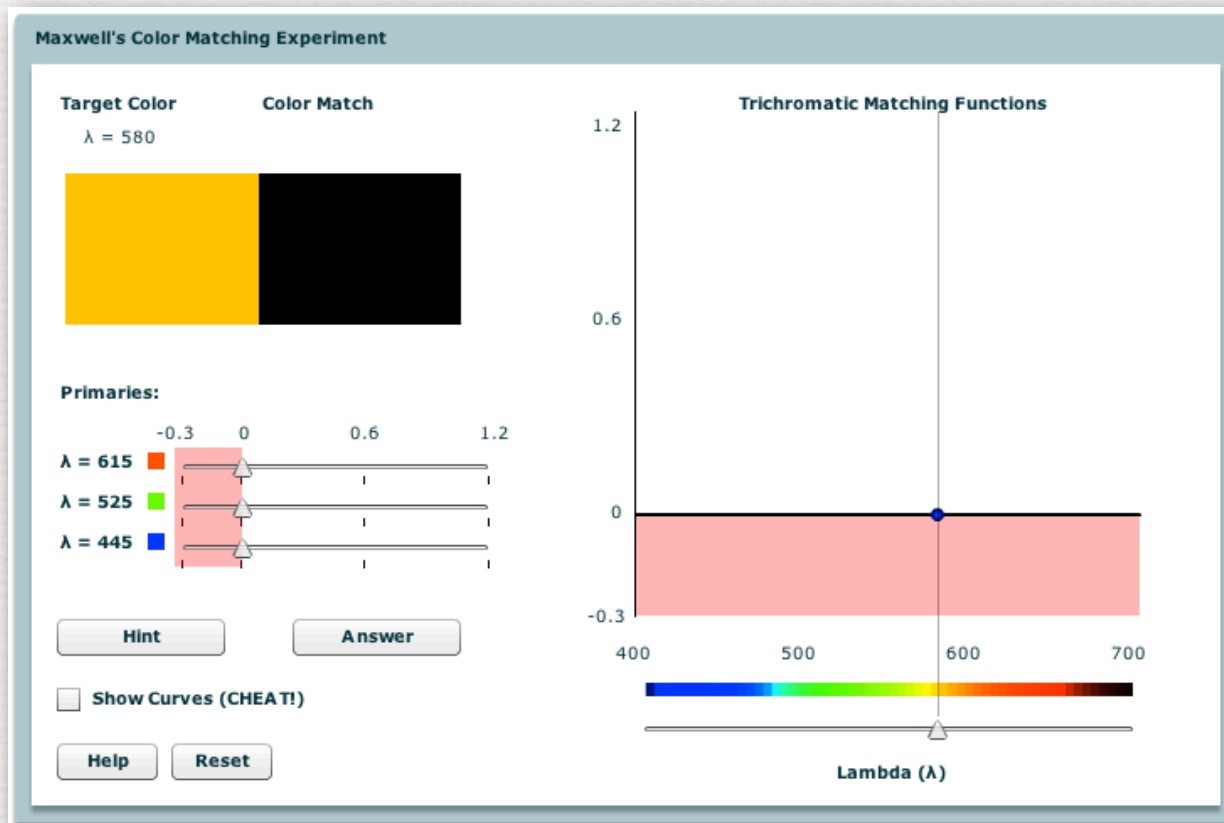
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# Maxwell's color matching experiment

- ◆ Maxwell actually used a slightly different procedure
  - see <http://www.handprint.com/HP/WCL/color6.html> for details
  - the procedure below is used in modern versions of the experiment

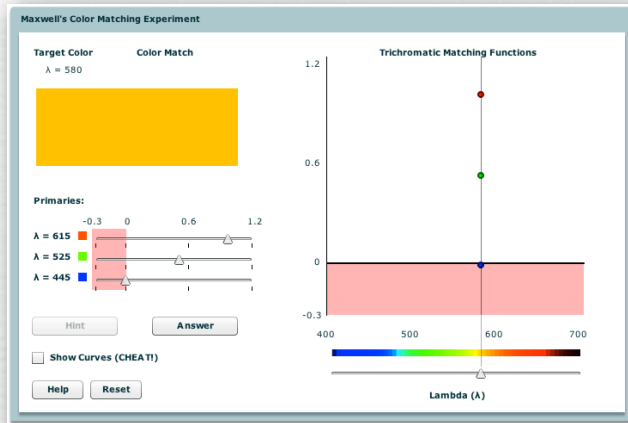


**(FLASH DEMO)**

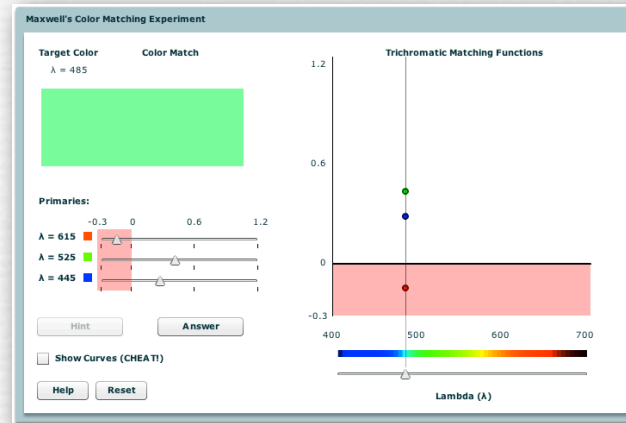
<http://graphics.stanford.edu/courses/cs178/applets/colormatching.html>

# Maxwell's color matching experiment

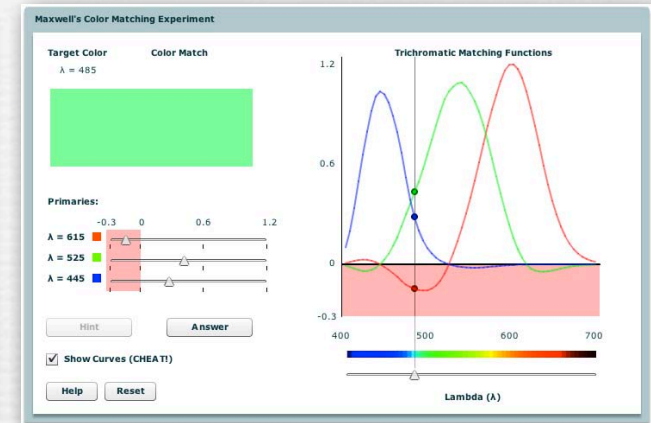
(summary of live demo)



1



2



3

1. given a stimulus wavelength, the amount of each primary required to match it is given by three numbers  $(r, g, b)$
2. some stimuli cannot be matched unless first desaturated by adding some of one primary to it before matching; the amount added is denoted by negative values of  $r$ ,  $g$ , or  $b$
3. the sequence of  $(\bar{r}, \bar{g}, \bar{b})$  values, some negative, required to match the locus of spectral colors across all  $\lambda$ , form the *trichromatic matching functions*  $\bar{r}(\lambda)$ ,  $\bar{g}(\lambda)$ , and  $\bar{b}(\lambda)$  for a particular set of 3 primaries

# Human response to an arbitrary stimulus (contents of whiteboard)

spectrum of stimulus

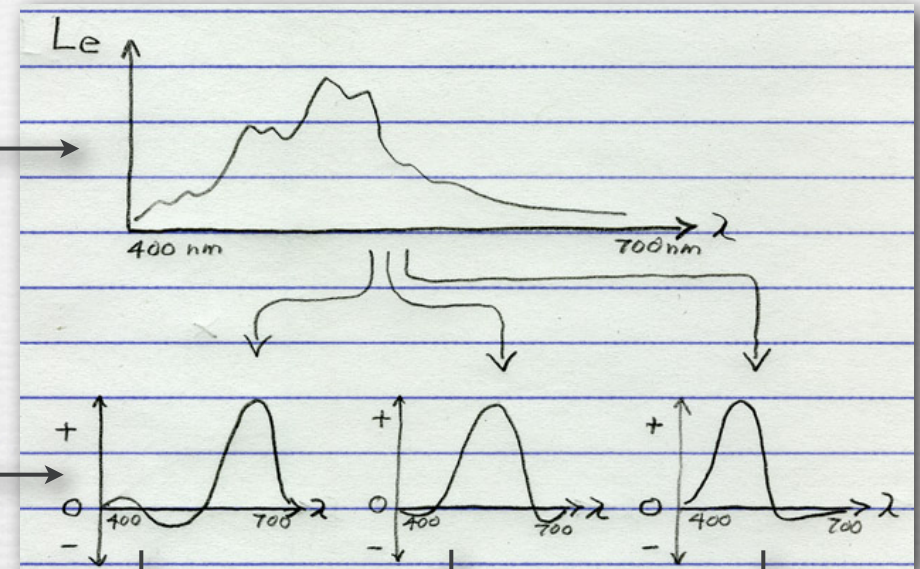
$\times$

multiply wavelength-by-wavelength  
by the matching functions  
 $\bar{r}(\lambda)$ ,  $\bar{g}(\lambda)$ , and  $\bar{b}(\lambda)$

for a particular set of 3 primaries

$\int$

then integrate over wavelengths to  
get the amount of that primary  
required to reproduce that spectrum



R

G

B

# Young-Helmholtz trichromatic theory



Thomas Young  
(1773-1829)



James Clerk Maxwell  
(c. 1860)



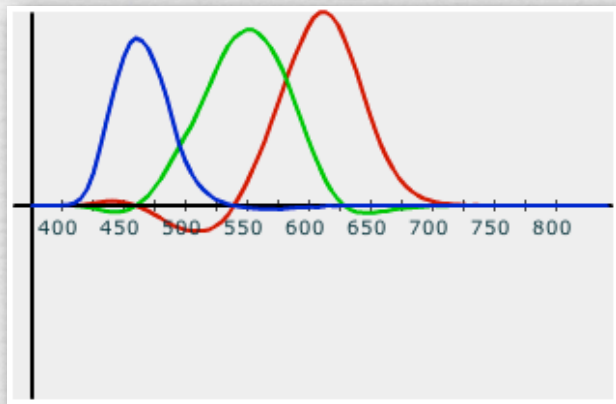
Hermann von Helmholtz  
(1821-1894)

- ◆ spectra can be visually matched using mixtures of *primary colors*; such matches are called *metamers*
- ◆ due to the linearity of human retinal response, given a stimulus spectrum  $L_e(\lambda)$ , the amounts of each primary R, G, B required to match it, for any particular choice of 3 primaries, are the integrals over all visible wavelengths of the amounts  $r(\lambda)$ ,  $g(\lambda)$ , and  $b(\lambda)$  required to match each constituent wavelength  $\lambda$ , *i.e.*

$$(R, G, B) = \left( \int_{400nm}^{700nm} L_e(\lambda) \bar{r}(\lambda) d\lambda, \int_{400nm}^{700nm} L_e(\lambda) \bar{g}(\lambda) d\lambda, \int_{400nm}^{700nm} L_e(\lambda) \bar{b}(\lambda) d\lambda \right)$$

# 3D interpretation of color matching

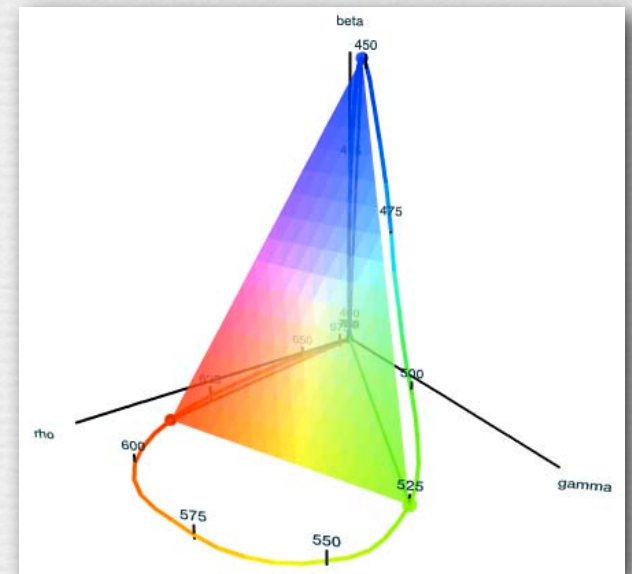
- ◆ our response to varying amounts of a primary forms a vector in  $(\rho, \gamma, \beta)$  space, rooted at the origin
- ◆ to provide a normal range of color vision, three primaries are required, and their vectors must not lie on a plane
- ◆ our responses to all possible mixtures and scales ( $\sum \leq 1$ ) of three primaries form a tetrahedron called the *gamut of reproducible colors* for these primaries



RGB matching functions

**(FLASH DEMO)**

<http://graphics.stanford.edu/courses/cs178/applets/locus.html>

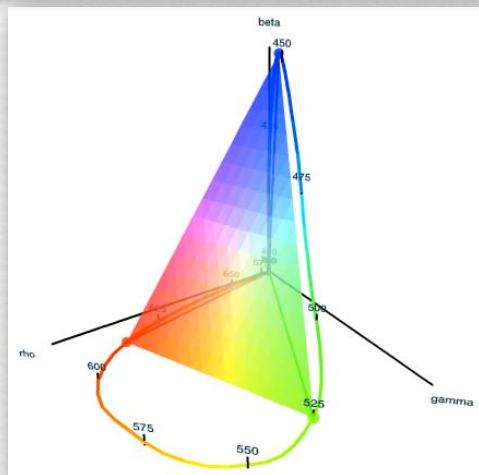
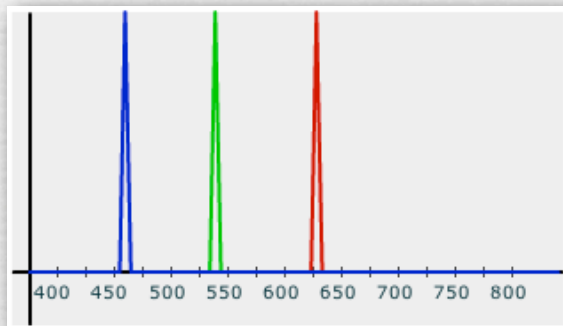


gamut of reproducible colors

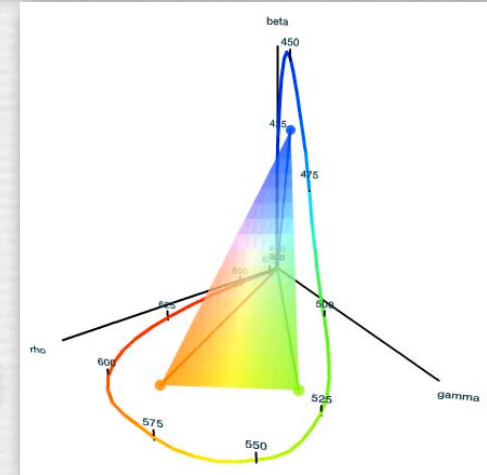
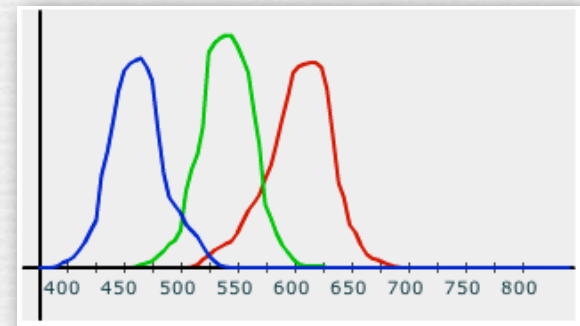
# 3D interpretation of color matching

- ◆ the spectrum of each of the three primaries can be a pure wavelength (1) or a mixture of wavelengths (2)
- ◆ impure primaries have a smaller gamut in  $(\rho, \gamma, \beta)$  space
- ◆ additional primaries can be added to increase the gamut

1



2




**(FLASH DEMO)**

<http://graphics.stanford.edu/courses/cs178/applets/locus.html>

**Questions?**

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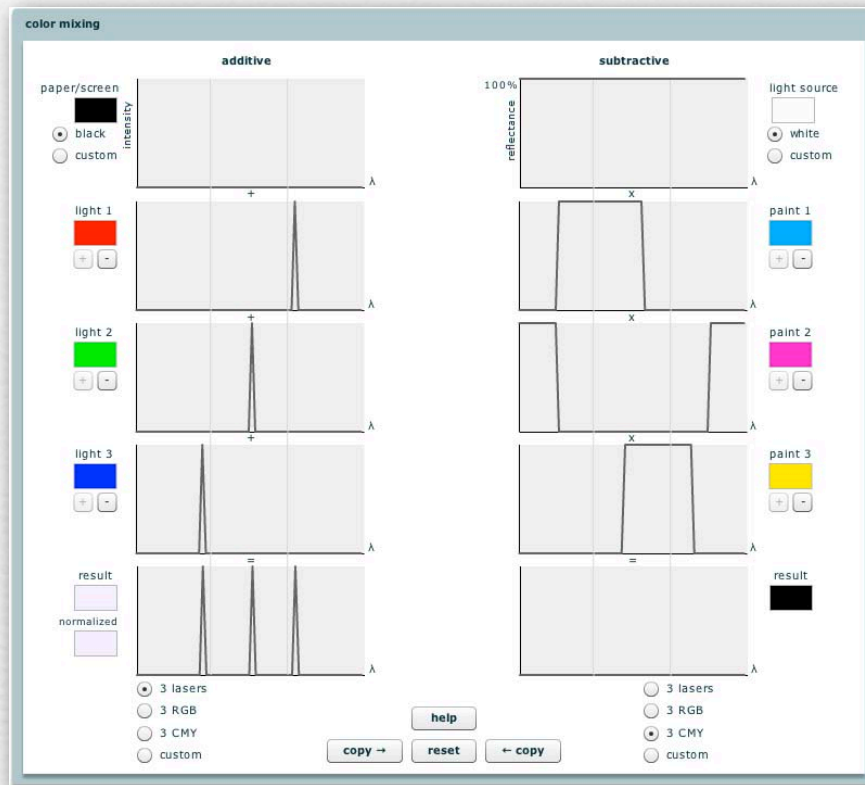
# Additive versus subtractive mixing

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- ◆ demo using color guns and filters



# Additive versus subtractive mixing



**(FLASH DEMO)**

<http://graphics.stanford.edu/courses/cs178/applets/colormixing.html>

- ◆ superimposed colored lights or small adjacent dots combine *additively* - by adding their spectra wavelength-by-wavelength
- ◆ layered dyes or sequenced color filters combine *subtractively* - by multiplying their transmittance spectra wavelength-by-wavelength

# Additive versus subtractive mixing



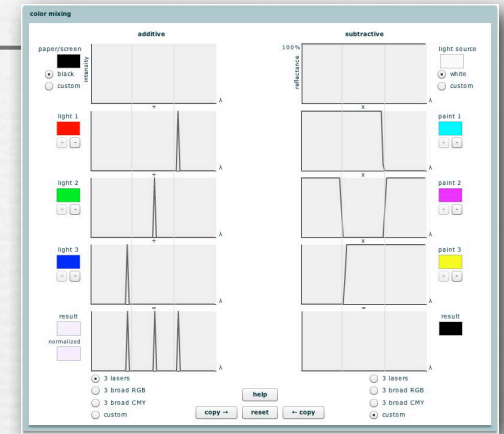
**(FLASH DEMO)**

<http://graphics.stanford.edu/courses/cs178/applets/colormixing.html>

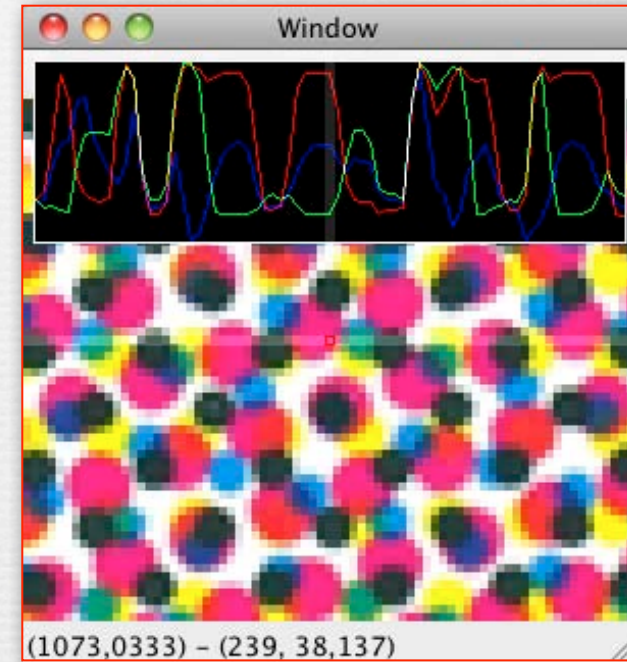
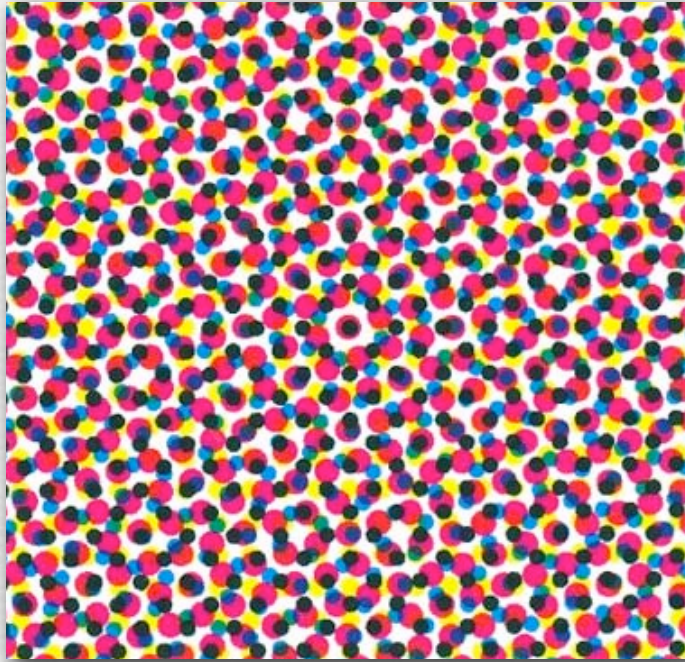
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# Additive versus subtractive mixing

- ◆ narrow spectra, widely spaced in wavelength, are best for primaries to be combined additively
- ◆ wide spectra that overlap are best for primaries to be combined subtractively, but product of all three must be black
- ◆ the particular spectra chosen are flexible; additive primaries need not be R,G,B, nor subtractive primaries C,M,Y
- ◆ additional primaries may be added to either system, resulting in a larger gamut of reproducible colors; adding black to a subtractive system (called CMYK) ensures a deep black
- ◆ note: additive mixing can be interpreted as interpolation between points in rho-gamma-beta space, but subtractive mixing cannot, because the two spectra must be multiplied together, not added



# Color printing



- ◆ patches of the 3 subtractive primaries (C,M,Y) overlap partially on the page, making patches of 8 meta-primaries (Wh,C,M,Y,MY,CY,CM,CMY), which combine additively in the eye when viewed from a distance
  - $M \times Y = R$ ,  $C \times Y = G$ ,  $C \times M = B$
  - these effects are modeled by the *Neugebauer equations*