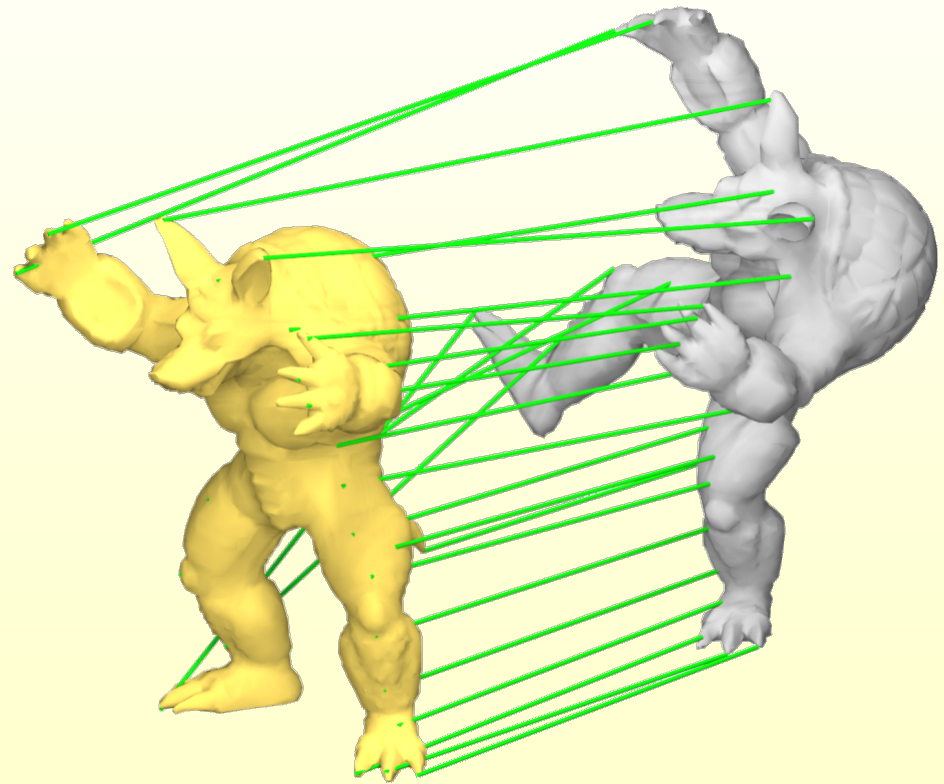


# CS233: Geometric and Topological Data Analysis

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Shape search, non-rigid  
alignment, one point  
isometric matching, Möbius  
voting, blended intrinsic maps

14 May 2018



Slides ack: Yaron Lipman, Peter Huang, Vova Kim,  
Maks Ovsjanikov, Michael Bronstein

Last Time:  
3D Alignments, Shape  
Descriptors and Features  
(Local, Global)

# The (3D) Registration Problem

## Given:

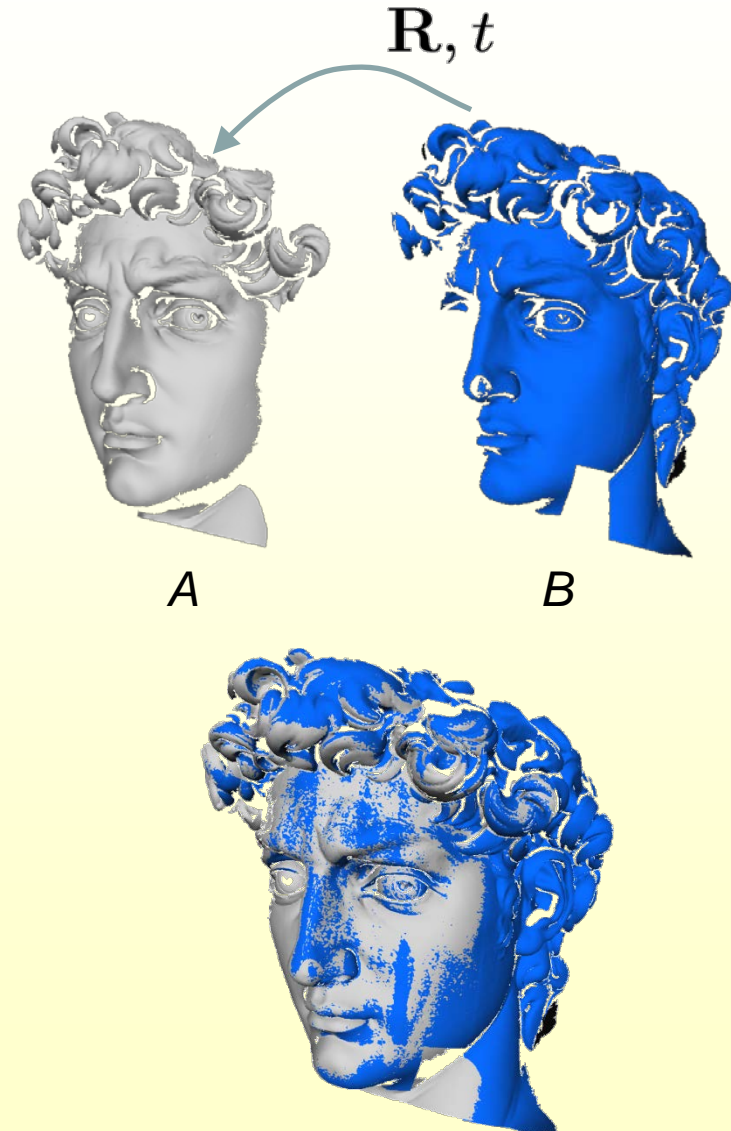
Two shapes  $A$  and  $B$  which partially overlap

## Goal:

Using only **rigid transforms**, register  $B$  against  $A$  by minimizing a measure of **distance** between  $A$  and  $B$

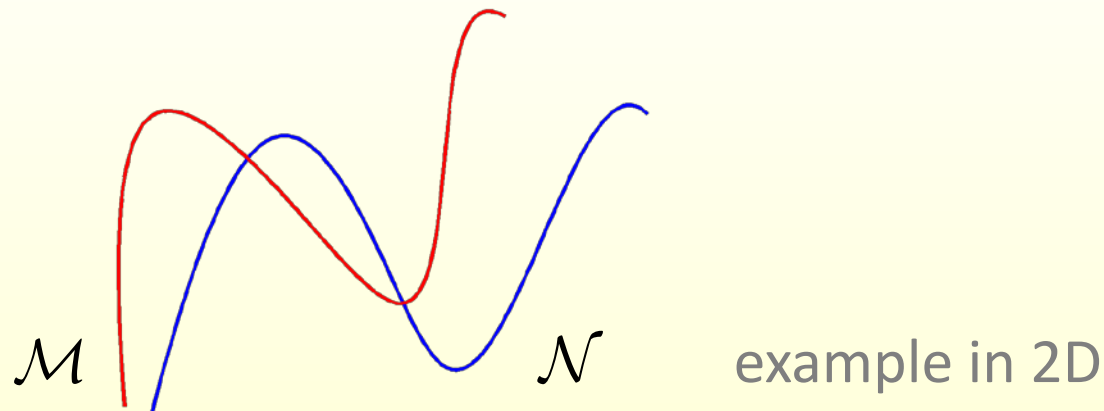
## Assume:

$A$  and  $B$  are positioned close to each other



# Iterative Closest Point (ICP)

- Classical approach: iterate between finding correspondences and finding the transformation:



Given a pair of shapes,  $\mathcal{M}$  and  $\mathcal{N}$ , iterate:

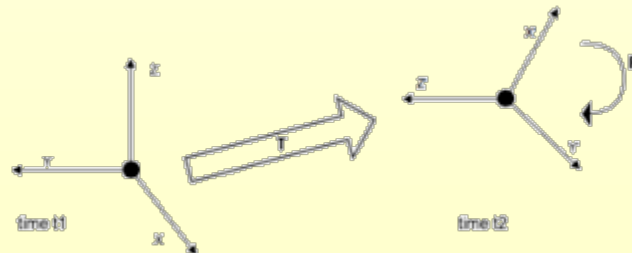
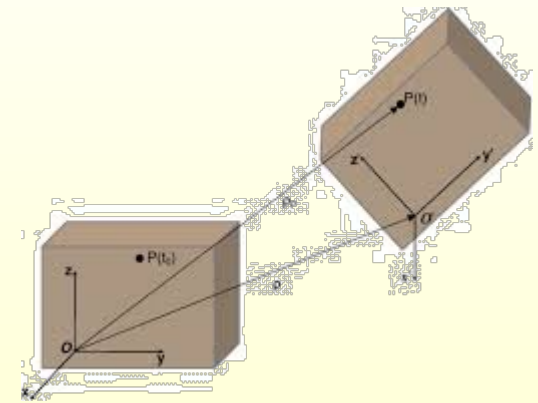
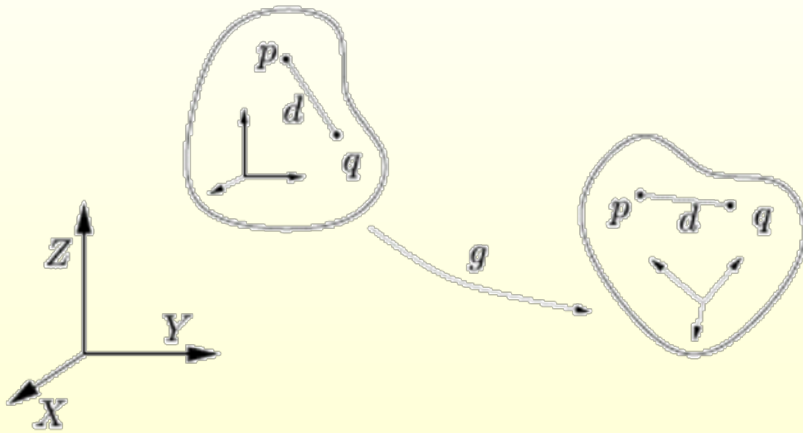
1. For each  $x_i \in \mathcal{M}$  find **nearest** neighbor  $y_i \in \mathcal{N}$
2. Find optimal transformation  $\mathbf{R}, t$  minimizing:

$$\arg \min_{R, t} \sum_i \|Rx_i + t - y_i\|_2^2$$

Classic problem,  
solvable by SVD

# Dimensionality of Aligning Transform is Crucial

- ◆ Rigid motions in 3D have six degrees of freedom.

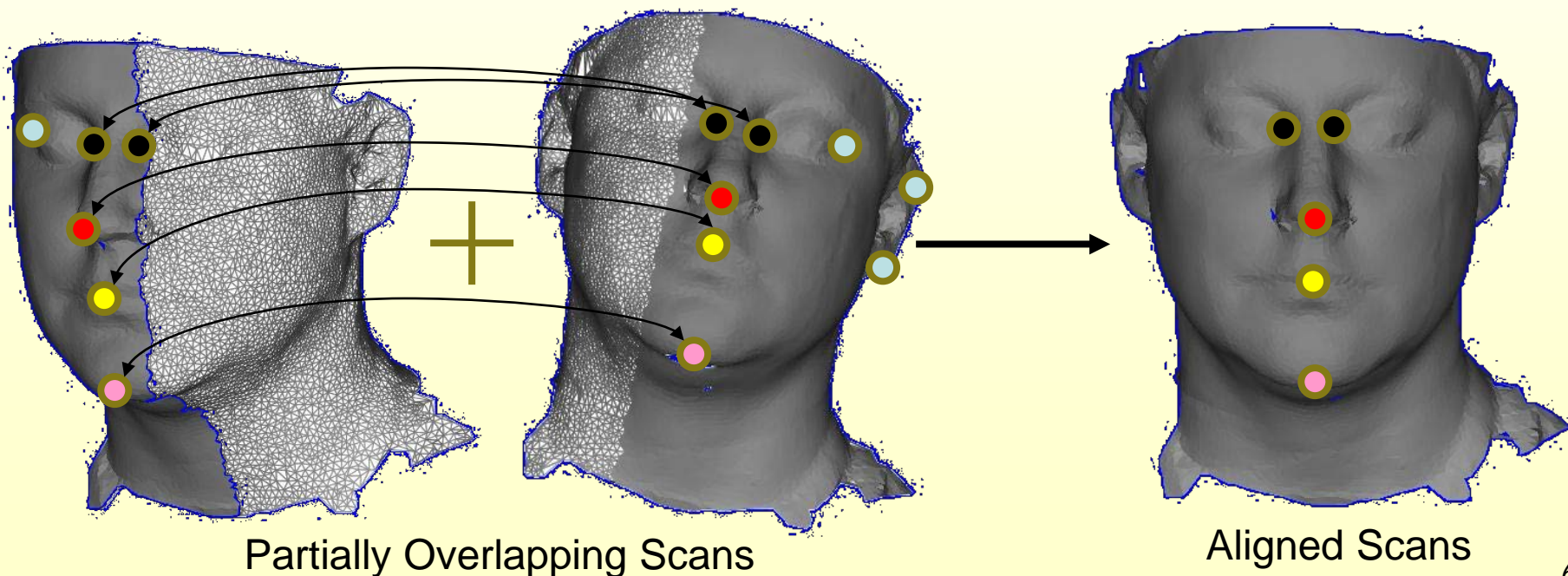


Low dimensional

# With Sparse Correspondences

1. (Find feature points on the two scans)
2. Establish correspondences
3. Compute the aligning transformation

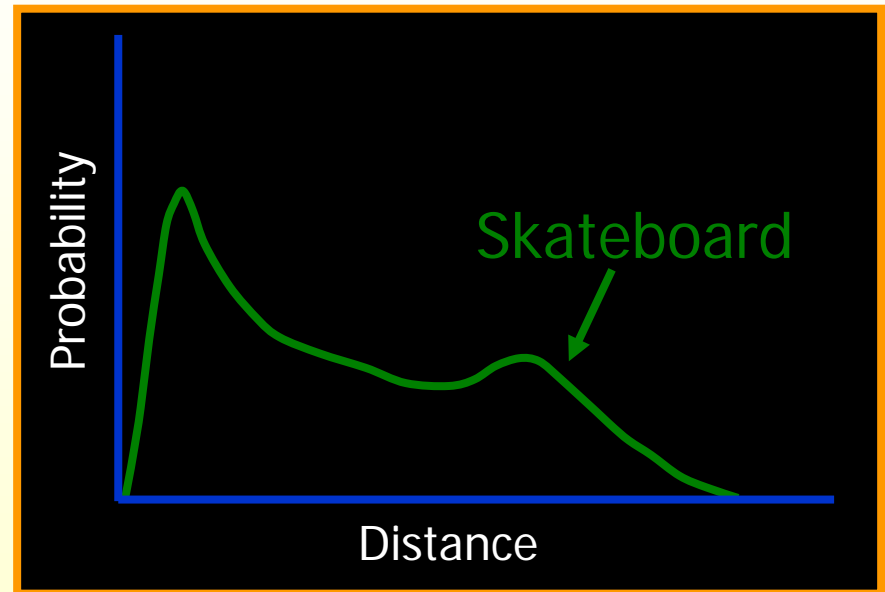
Preserve features  
Various regularizers



# Global: D2 Shape Distributions

## ◆ Properties

- ◆ Concise to store?
- ◆ Quick to compute?
- ◆ Invariant to transforms?
- ◆ Efficient to match?
- ◆ Insensitive to noise?
- ◆ Insensitive to topology?
- ◆ Robust to degeneracies?
- ◆ Invariant to deformations?
- ◆ Discriminating?



512 bytes (64 values)

0.5 seconds ( $10^6$  samples)

# Local: Spin Images

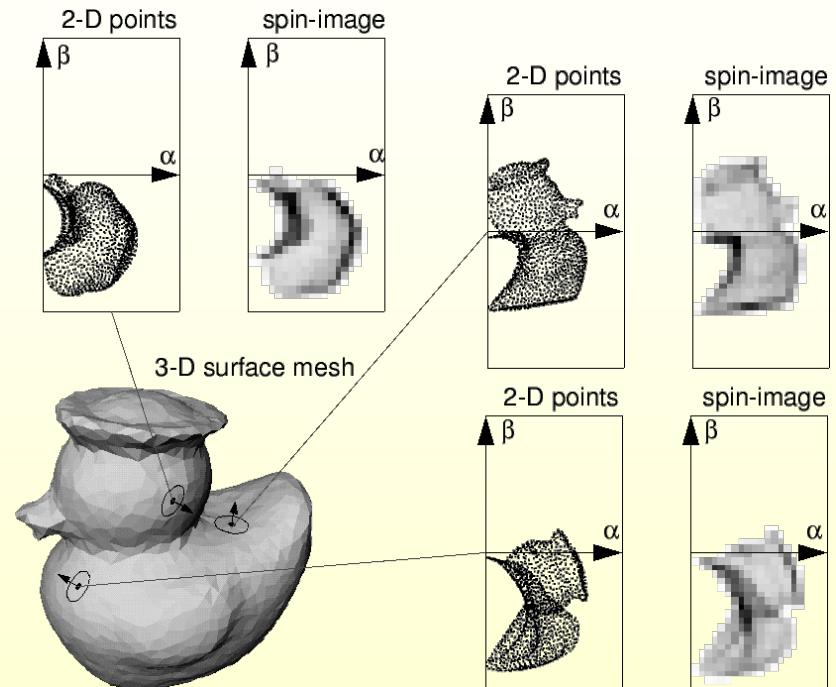
Creates an image associated with a neighborhood of a point.

Compare points by comparing their *spin images* (2D).

Given a point and a normal, every other point is indexed by two parameters:

$\beta$  distance to tangent plane

$\alpha$  distance to normal line



*Using Spin Images for Efficient Object Recognition in Cluttered 3D Scenes*  
Johnson et al, PAMI 99

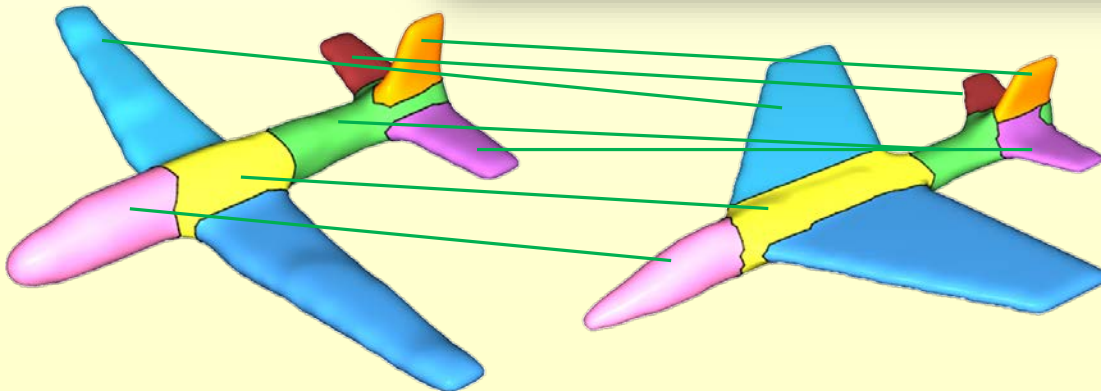
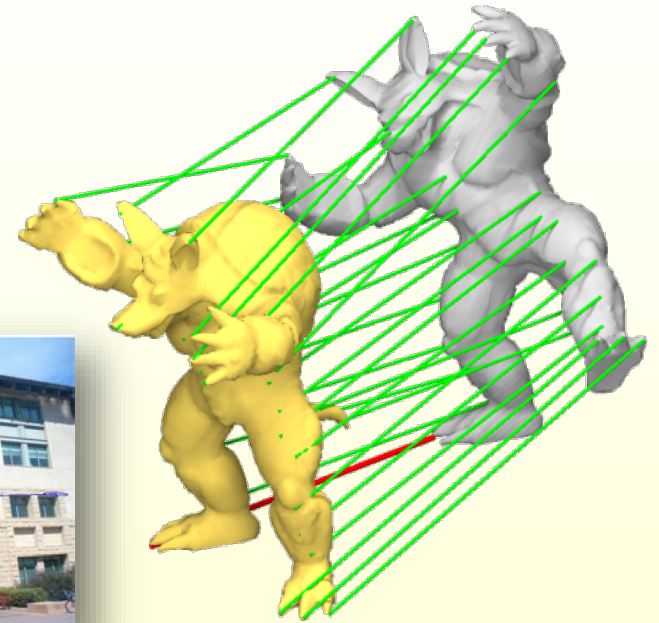
# Maps Between Geometric Data

# Mapping Between Data Sets

- ◆ Multiscale mappings

- ◆ Point/pixel level

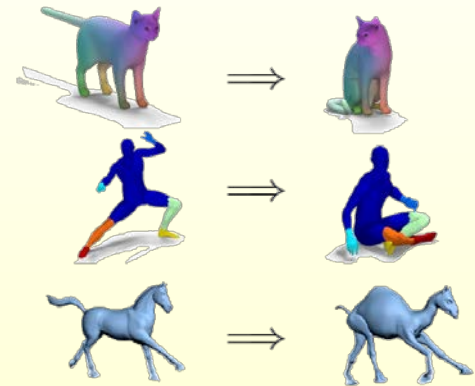
- ◆ part level



Maps capture what is the same or similar across two data sets

# Why Do We Care About Maps and Alignments?

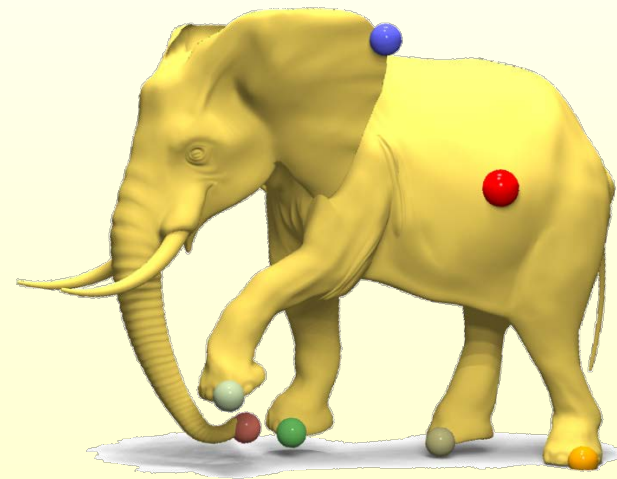
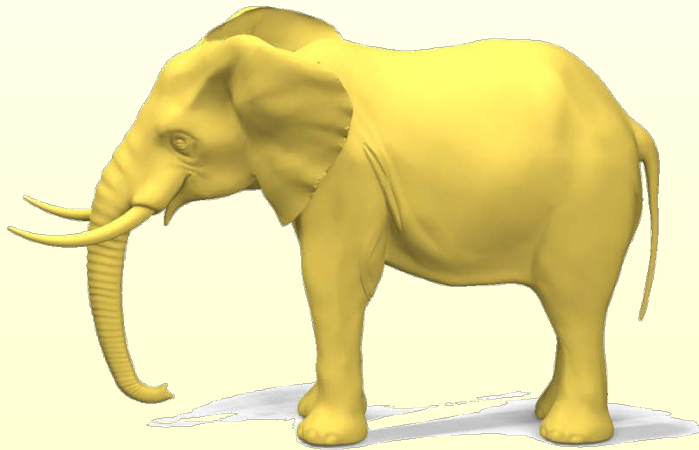
- ◆ To stitch data together
- ◆ To transfer information
- ◆ To compute distances and similarities
- ◆ To perform joint analysis



# Shape Matching

## Problem:

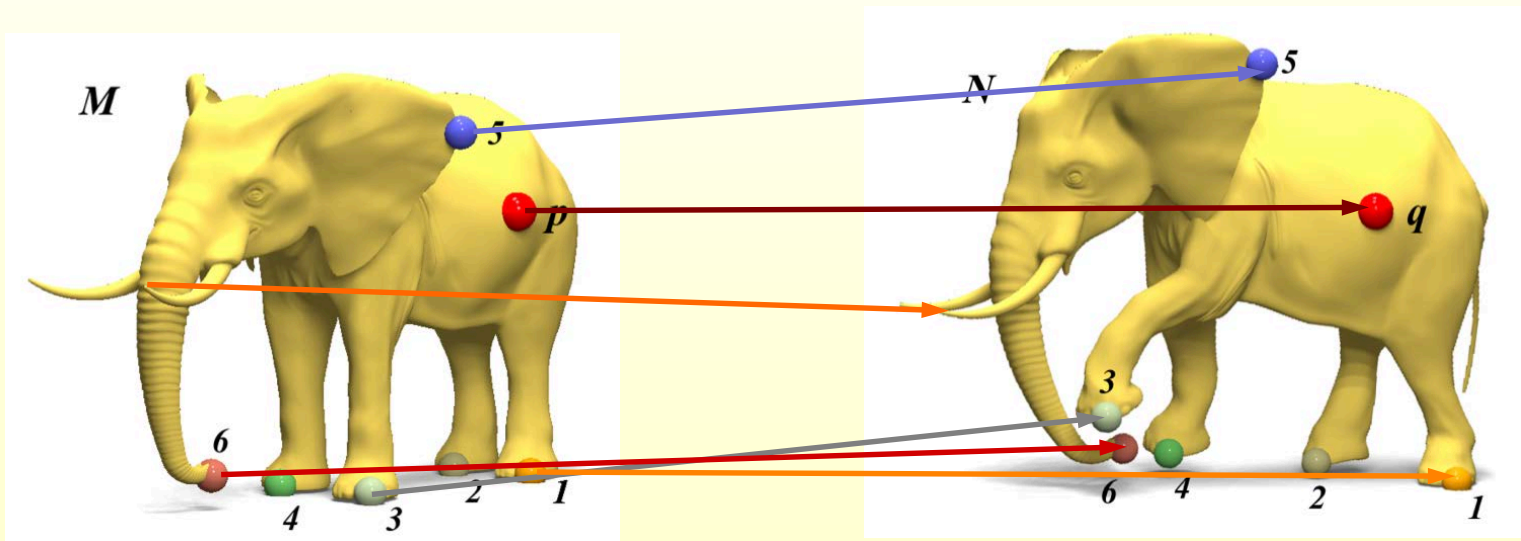
Given a pair of shapes, find corresponding points.



# Shape Matching

## Problem:

Given a pair of shapes, find corresponding points.

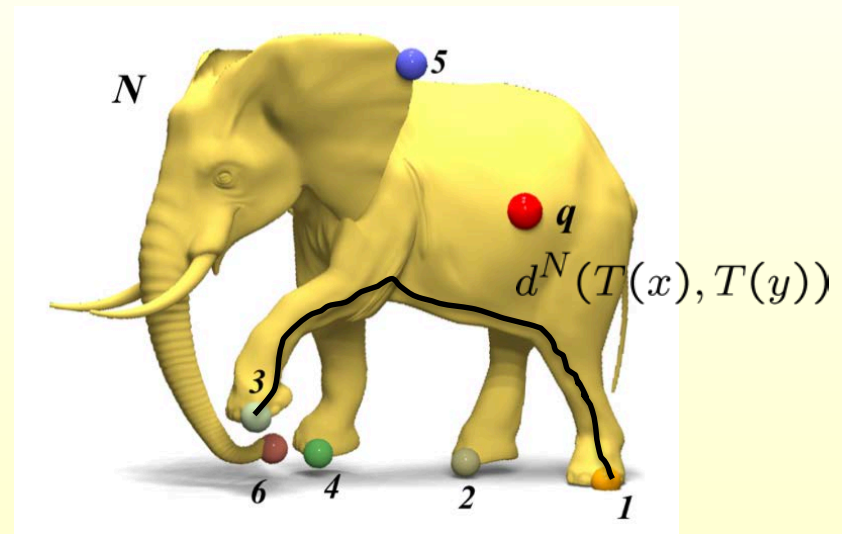
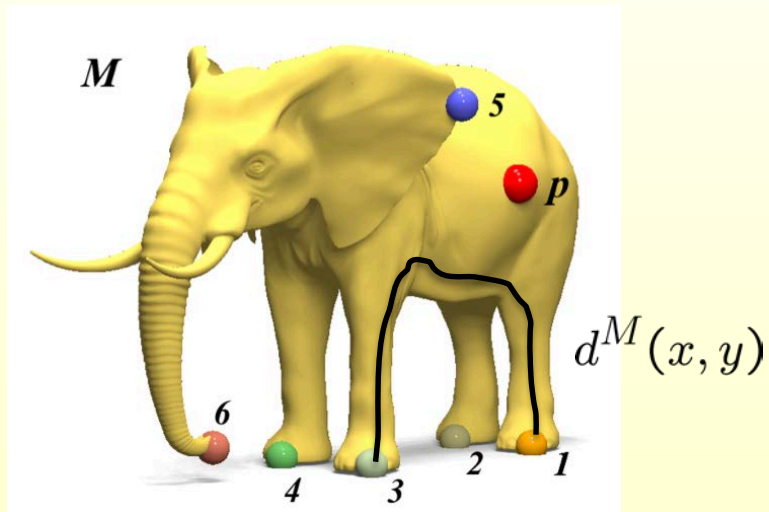


Unary and binary terms in correspondence problems

# Non-Rigid Shape Matching

## Problem:

Given a pair of shapes, find corresponding points.



Find a correspondence that preserves intrinsic (geodesic) distances on the shapes – 2<sup>nd</sup> order info.

# Gromov-Hausdorff: Aligning Metric Spaces

- ◆ Gromov-Hausdorff distance: a second order optimization over mappings or correspondences

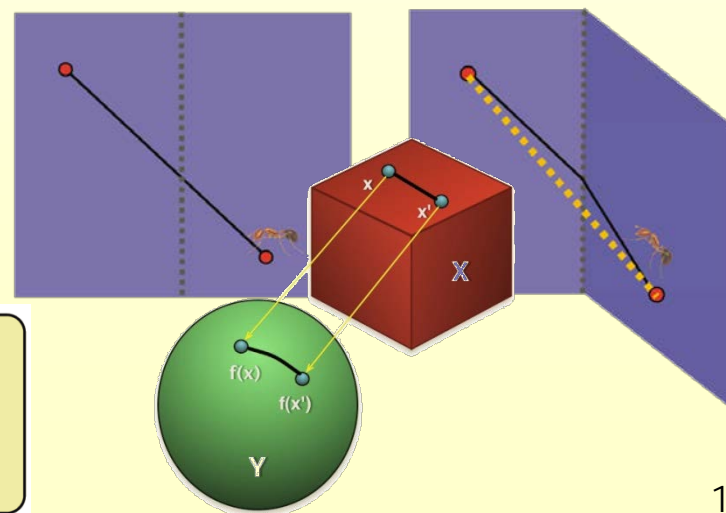
$$\Gamma_{X,Y}(x, y, x', y') := |d_X(x, x') - d_Y(y, y')|$$

intrinsic distance  
distortion

$$d_{\mathcal{GH}}(X, Y) = \frac{1}{2} \inf_R \|\Gamma_{X,Y}\|_{L^\infty(R \times R)}$$

evaluated via **intrinsic** distances

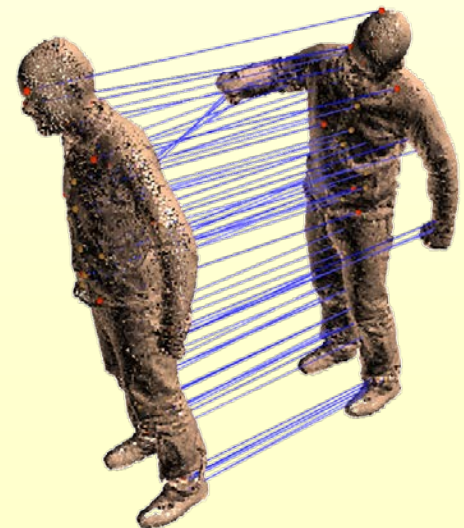
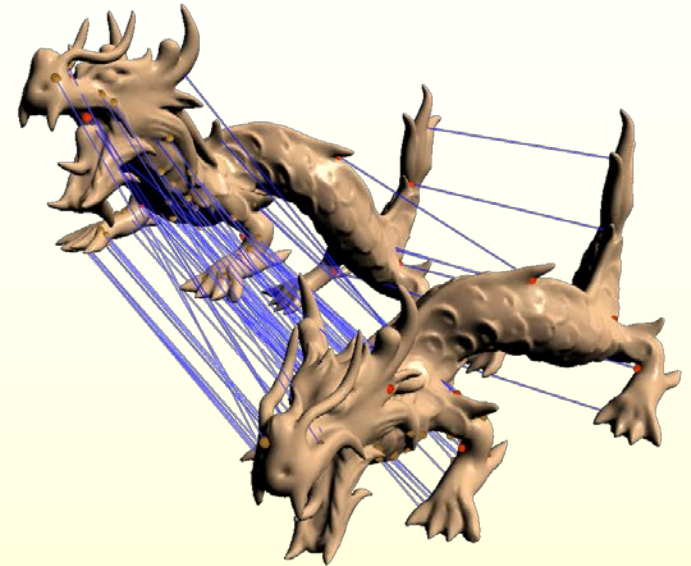
This is a minimization over all correspondences!



$$\text{dist}(\text{cat}, \text{cat}) = 0 ?$$

# Near Isometries Preserve Local Structure

- ◆ Optimal alignment can be defined in terms of certain intrinsic but hard-to-compute shape distances, such as Gromov-Hausdorff
- ◆ We are mostly interested in the near isometric case
- ◆ Can we define certain **descriptors** of point neighborhoods which can guide us at establishing good mappings?
- ◆ Note that 1<sup>st</sup> order matching (linear assignment) is polynomial, but 2<sup>nd</sup> order matching (quadratic assignment) is NP-complete



# Intrinsic vs. Extrinsic Correspondences

# Extrinsic vs. Intrinsic Alignment

- ◆ Coordinate root mean squared distance

$$\text{cRMS}^2(\mathbf{P}, \mathbf{Q}) = \min_{\mathbf{R}, \mathbf{t}} \frac{1}{n} \sum_{i=1}^n \|\mathbf{R}\mathbf{p}_i + \mathbf{t} - \mathbf{q}_i\|^2$$

estimate transform

- ◆ Distance root mean squared distance

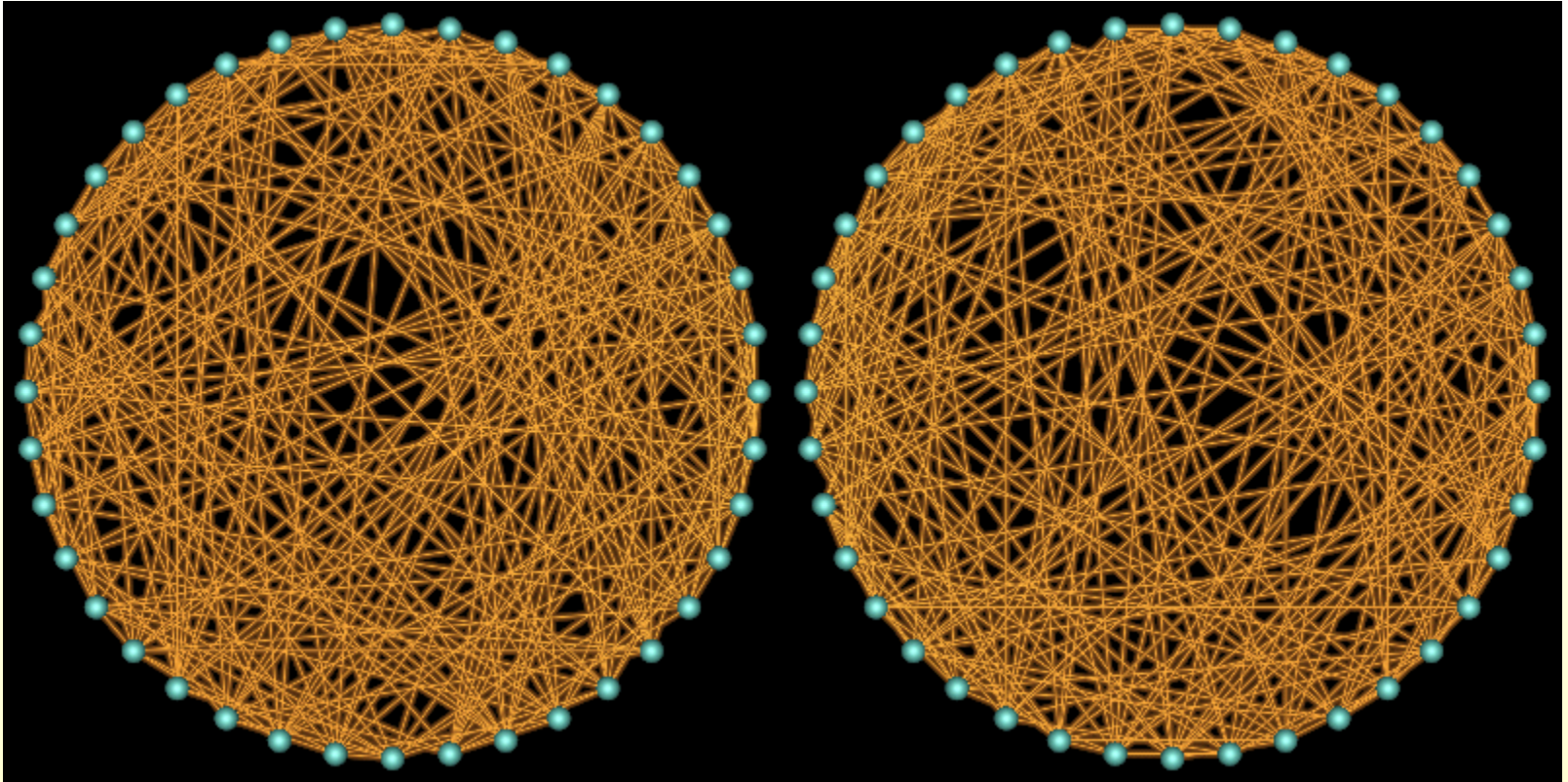
$$\text{dRMS}^2(\mathbf{P}, \mathbf{Q}) = \frac{1}{n^2} \min_{\sigma} \sum_{i=1}^n \sum_{j=1}^n (\|\mathbf{p}_i - \mathbf{p}_j\| - \|\mathbf{q}_{\sigma(i)} - \mathbf{q}_{\sigma(j)}\|)^2$$

estimate correspondences

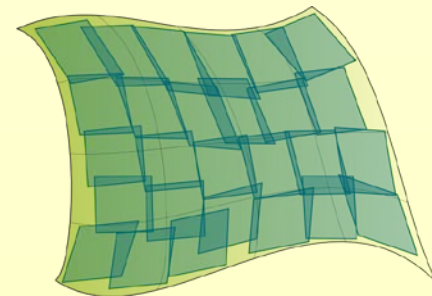
metric space, intrinsic alignment

Gromov-Hausdorff distances

# Graph Isomorphism



Intrinsic alignment of **manifolds**



# Why Intrinsic?



$\mathcal{M}_1$

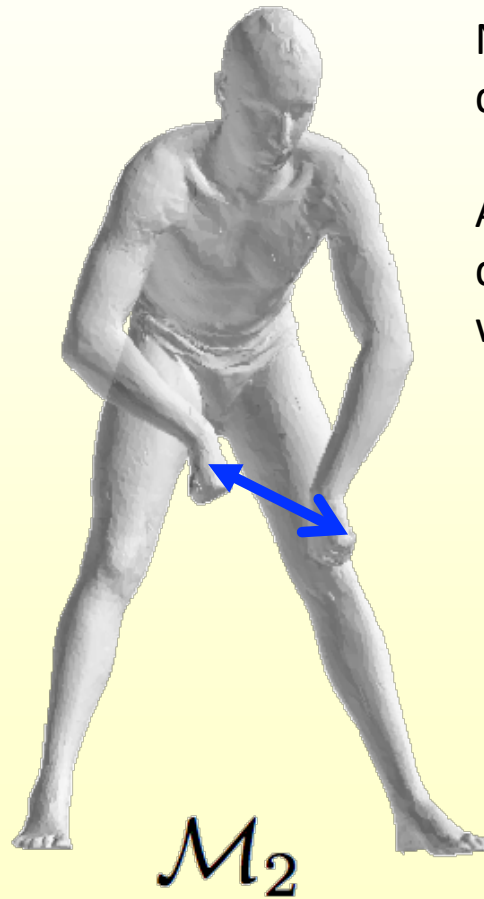
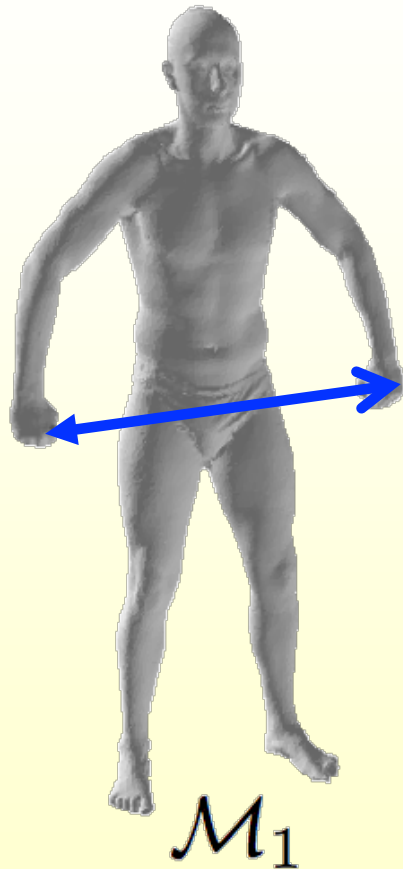


$\mathcal{M}_2$

Many shapes have natural deformations and articulations that do not change the nature of the shape.

But they change its embedding 3D space.

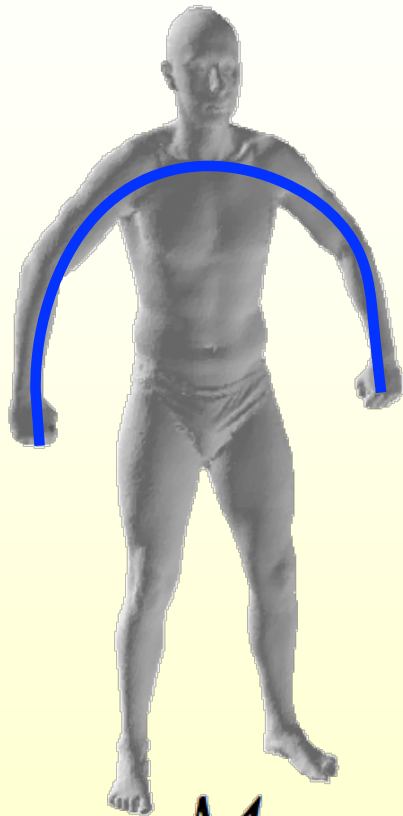
# Why Intrinsic?



Normal distances can change drastically under such deformations

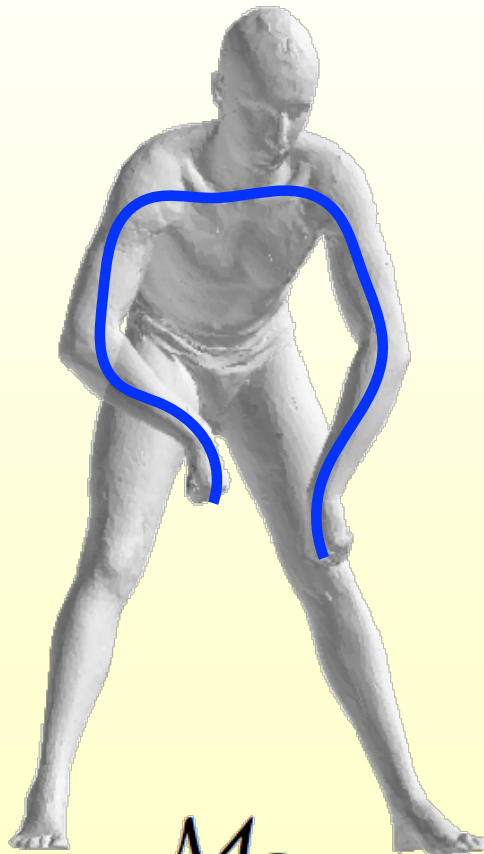
A descriptor based on Euclidean distance histograms, like D2, would fail

# Geodesic / Intrinsic Distances



$\mathcal{M}_1$

geodesic = intrinsic



$\mathcal{M}_2$

isometry = length-preserving transform

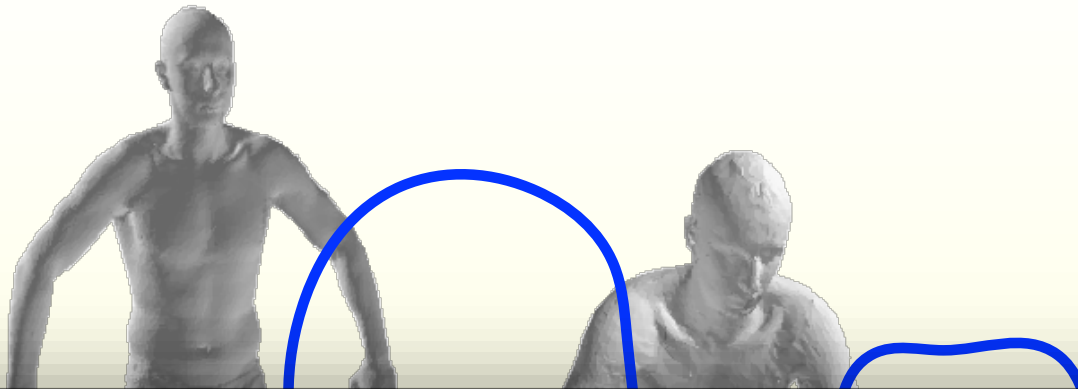
Near isometric deformations are common for both organic and man-made shapes

Intrinsic distances are invariant to isometric deformations

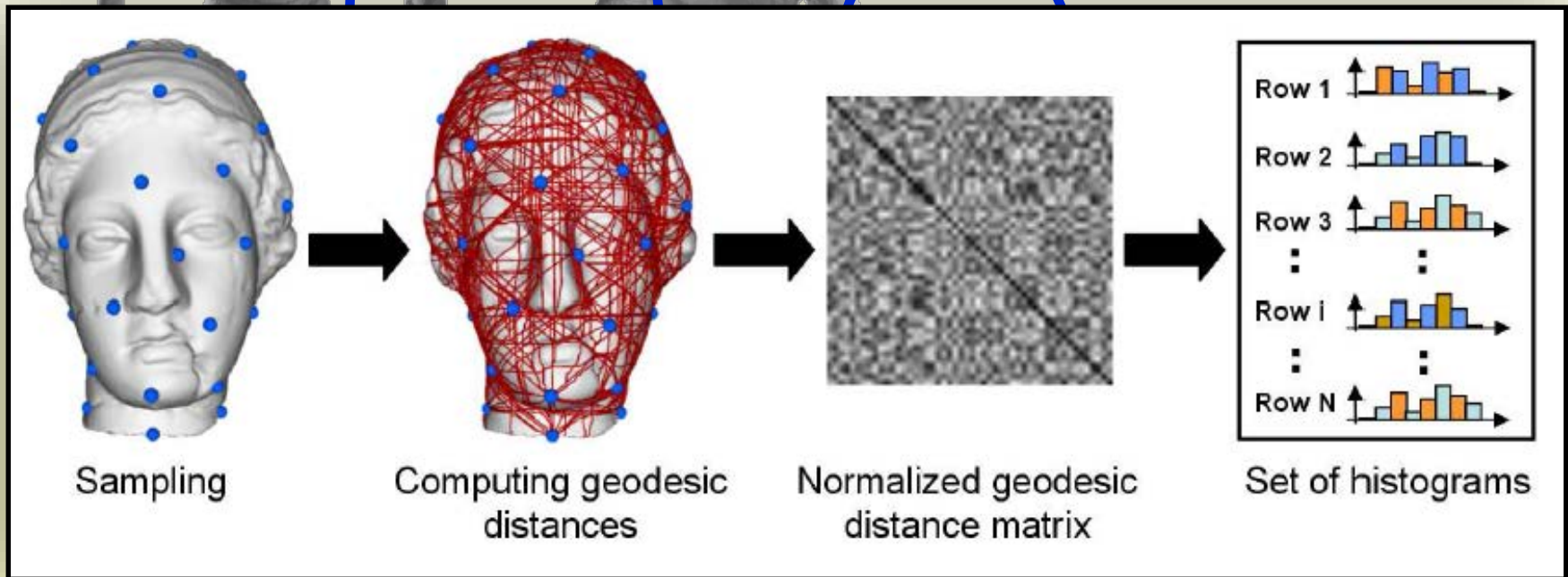


No stretching, shrinking, or tearing

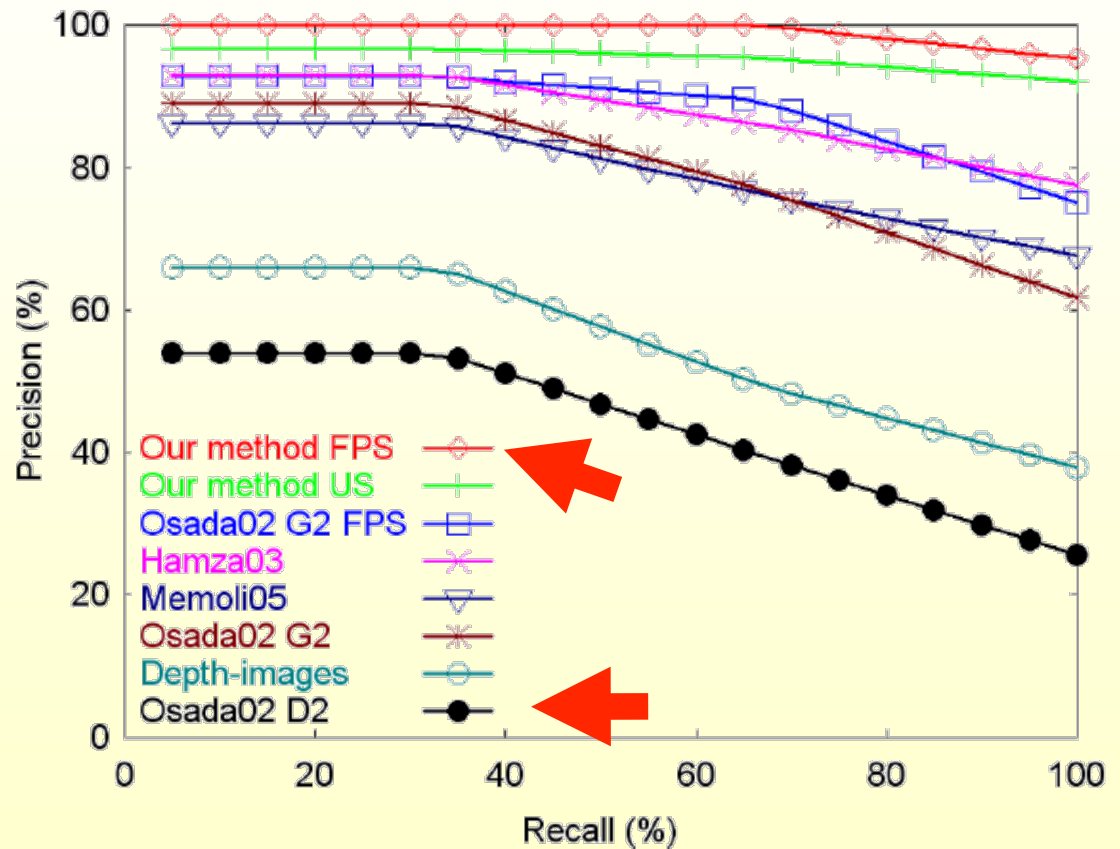
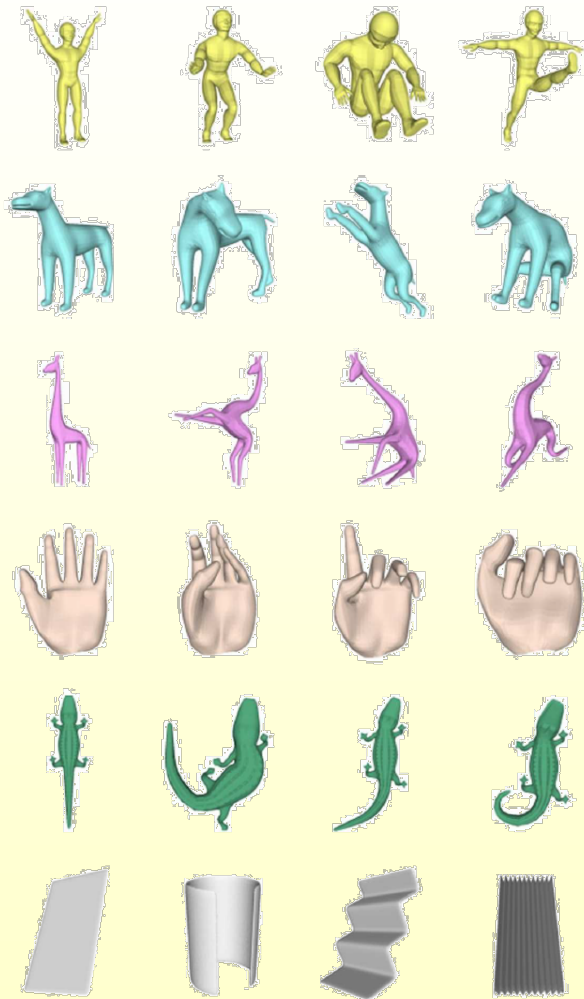
# Geodesic / Intrinsic Distances



We can use geodesic distance histograms

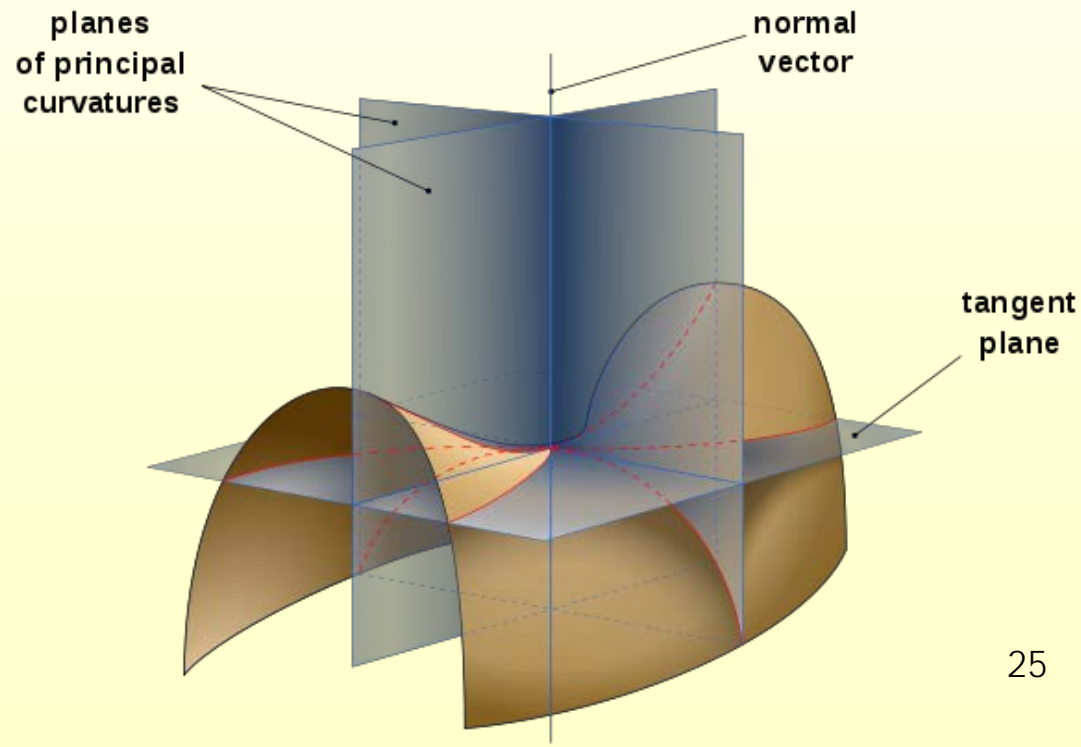


# Geodesic / Intrinsic Distances

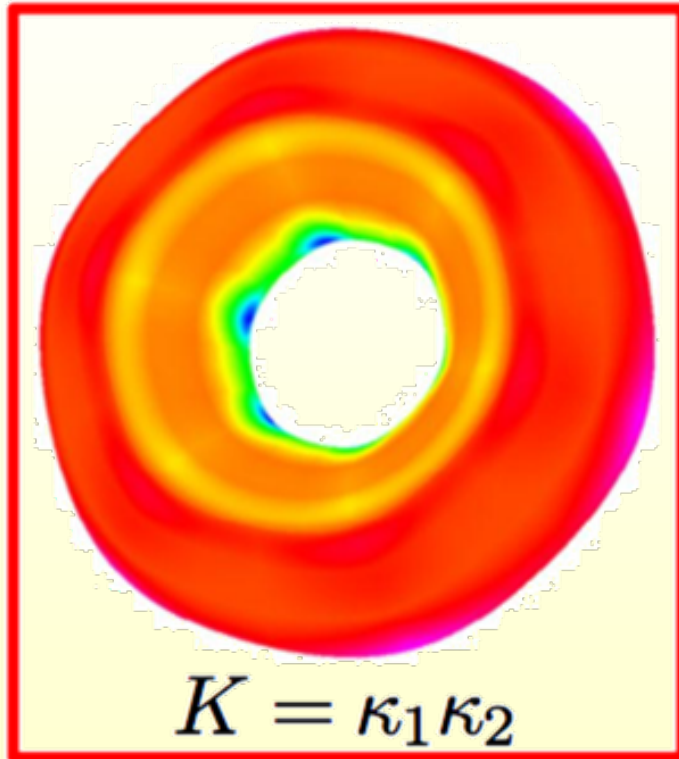


# What About Local Intrinsic Descriptors?

- ◆ Isometrically invariant features
  - ◆ Curvature
  - ◆ Geodesic Distance
  - ◆ Histogram of Geodesic Distances (similar to D2)
  - ◆ Global Point Signature
  - ◆ Heat Kernel Signature
  - ◆ Wave Kernel Signature



# Gaussian Curvature



**Theorema Egregium**  
("Remarkable Theorem"):  
**Gaussian curvature**  
**is intrinsic.**

# Gaussian Curvature

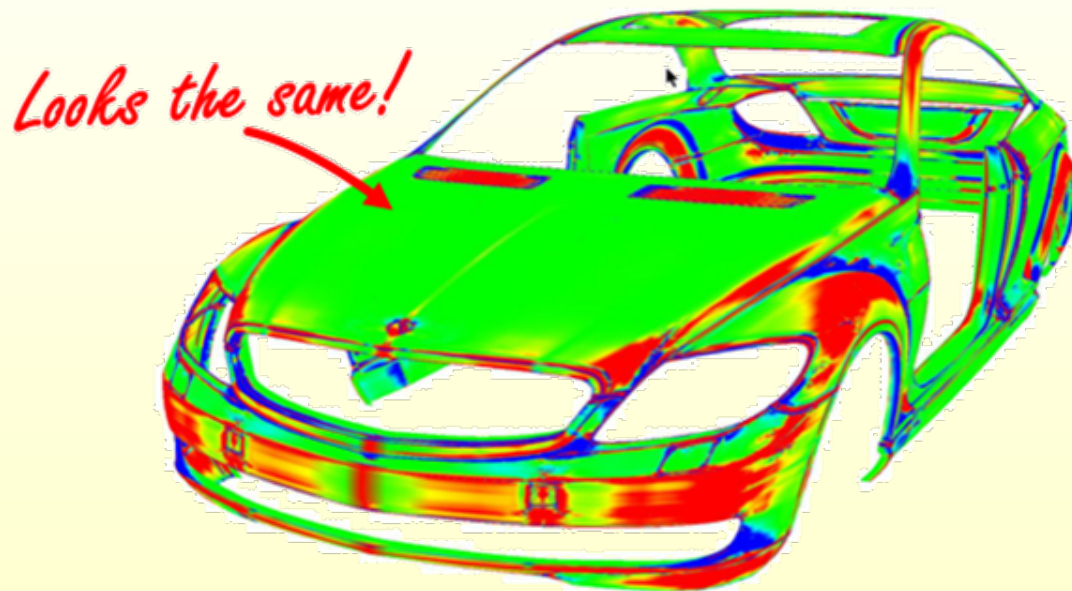
Problems



$$K = \kappa_1 \kappa_2$$

# Gaussian Curvature

## Problems



<http://www.integrityware.com/images/MercedesGaussianCurvature.jpg>

$$K = \kappa_1 \kappa_2$$

# Spectral Intrinsic Signatures

# Heat Diffusion on Manifolds

- Heat diffusion on a Riemannian manifold:

If  $u(x, t)$  is the amount of heat at point  $x$  at time  $t$ ,  
then

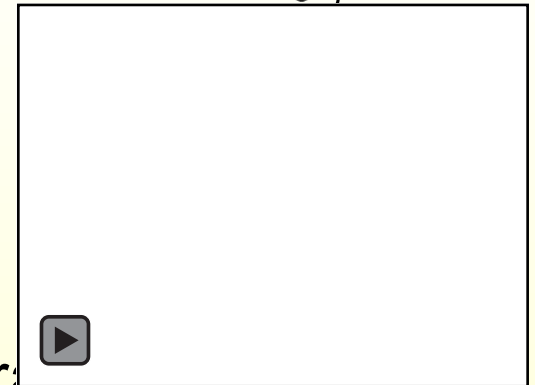
$$\frac{\partial u}{\partial t} = \Delta u$$

$\Delta$  : Laplace-Beltrami Operator (div grad)

- Given an initial distribution  $f(x)$ . After time  $t$ :

$$f(x, t) = e^{-t\Delta} f$$

$H_t$  heat operator



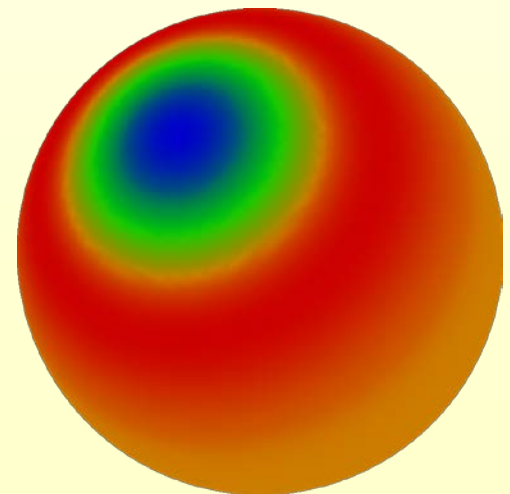
# Laplace-Beltrami Operator

- Analog of Fourier transform on the sphere, but now on a general 2D manifold
- LB is an operators that can be applied to functions on manifolds to yield other functions

$$\Delta : C^\infty(M) \rightarrow C^\infty(M), \Delta f = \operatorname{div} \nabla f$$

$$\operatorname{div} \nabla f$$

$$\frac{\partial f}{\partial t} = \Delta f = \operatorname{div} \nabla f.$$



# LB Eigen-decomposition

- ◆ The Laplace-Beltrami operator  $\Delta$  has an eigendecomposition

$$\Delta\phi_i = \lambda_i\phi_i$$

Invariant to isometric deformations!



$$\lambda_0 = 0$$

$$\lambda_1 = 2.6$$

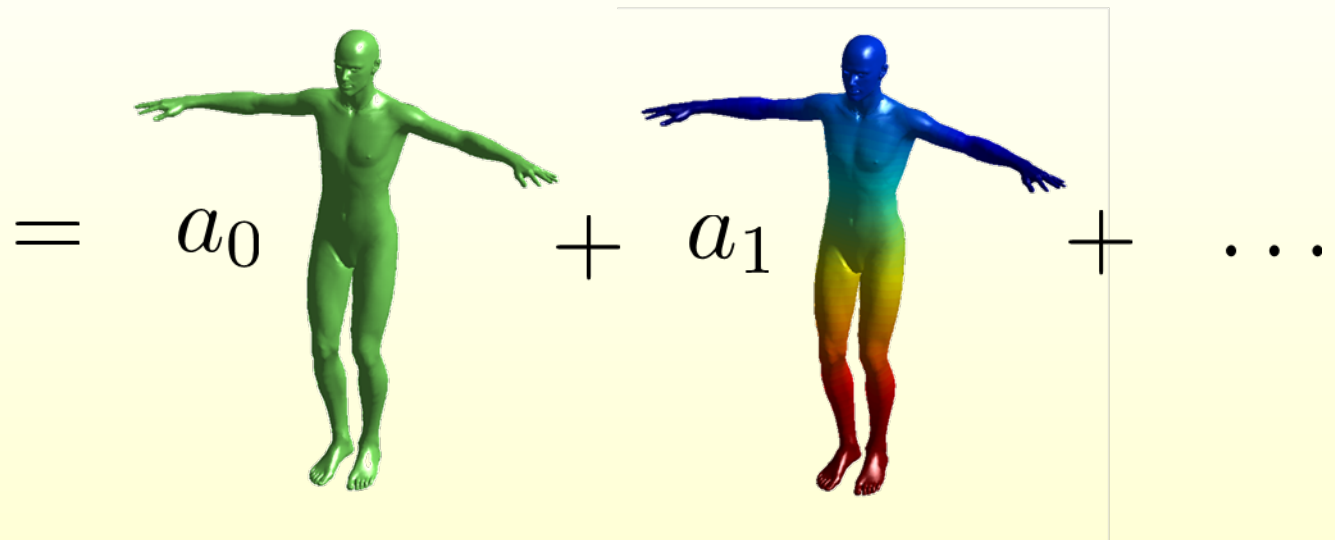
$$\lambda_2 = 3.4$$

$$\lambda_3 = 5.1$$

$$\lambda_4 = 7.6$$

# Multiscale Basis for a Function Space

$$f : M \rightarrow \mathbb{R}$$



$$f = \sum_{i=0}^{\infty} a_i \phi_i \quad a_i = \int_M f(x) \phi_i(x) d\mu$$

# Heat Kernel Signature

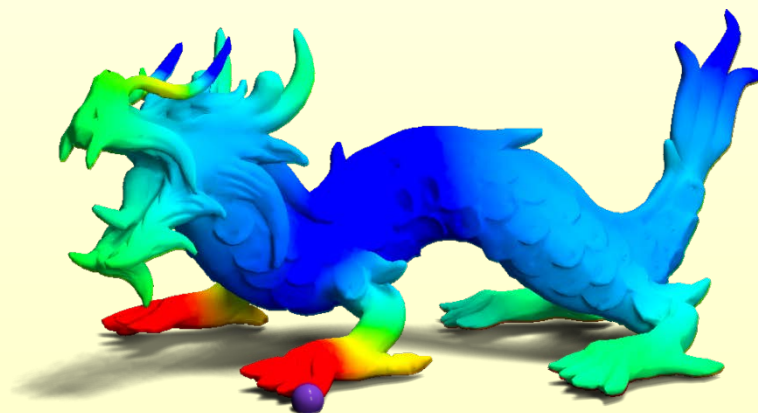
- Heat kernel is related to the eigenvalues and eigenfunctions of the LB-operator:

$$\text{HKS}(x, t) = \sum_{i=0}^{\infty} e^{-\lambda_i t} \phi_i^2(x)$$

- Invariant to rotations within the eigenspace



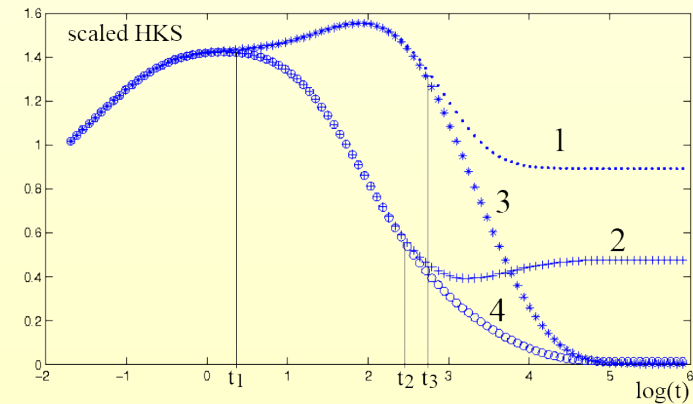
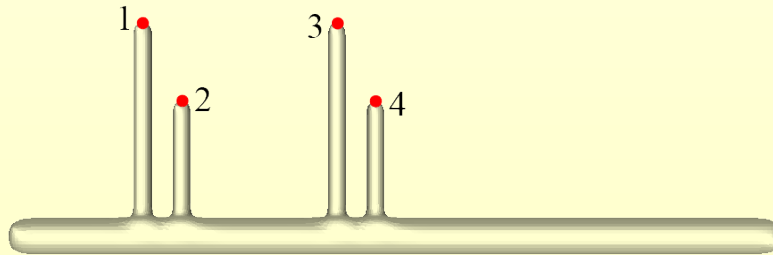
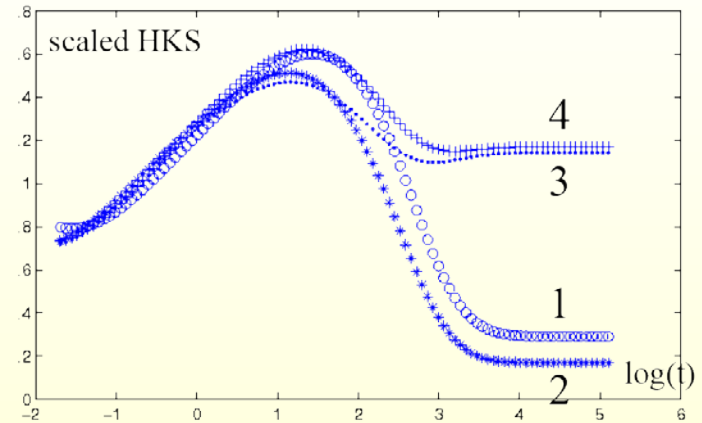
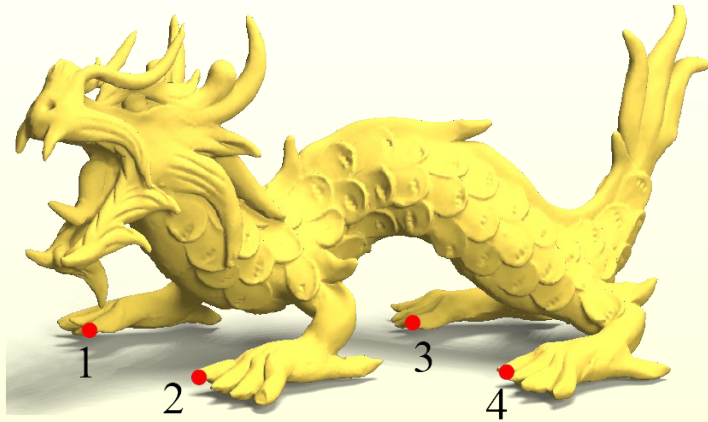
Medium scale



Full scale

# Multiscale Matching

- Comparing points through their HKS signatures:



# Heat Kernel Signature

$$k_t(x, x) = \sum_{n=0}^{\infty} e^{-\lambda_n t} \phi_n(x)^2$$

## ◆ Pros

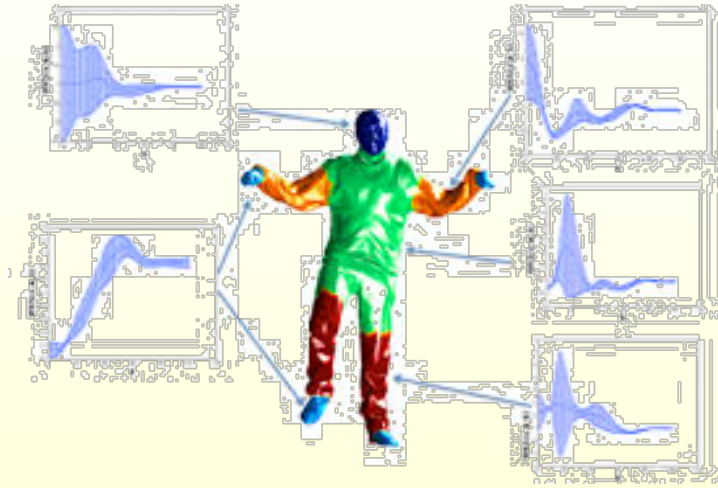
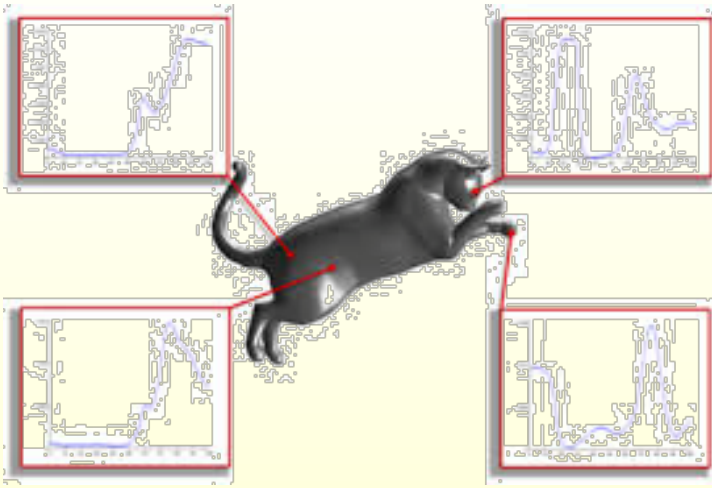
- ◆ Isometry-invariant
- ◆ Multi-scale (depends on range of t)
- ◆ Eigenfunction sign flips not a problem
- ◆ Change in eigenfunction positions is not a problem

## ◆ Cons

- ◆ Theoretical invariance only for isometric deformations

A Few Related Items:  
Wave-Kernel Signature,  
Isometry-Invariant Shape  
Search

# The Wave Kernel Signature (WKS)



Based on solutions of the quantum Schrödinger equation

$$\frac{\partial \psi}{\partial t}(x, t) = i\Delta \psi(x, t)$$


# Wave Kernel Signature

$$WKS(E, x) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |\psi_E(x, t)|^2 dt = \sum_{n=0}^{\infty} \phi_n(x)^2 f_E(\lambda_k)^2$$

**Initial energy  
distribution**



**Average probability  
over time that  
particle is at x.**



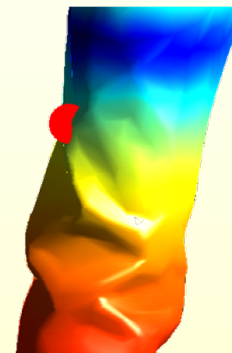
# Wave Kernel Signature

$$WKS(E, x) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |\psi_E(x, t)|^2 dt = \sum_{n=0}^{\infty} \phi_n(x)^2 f_E(\lambda_k)^2$$

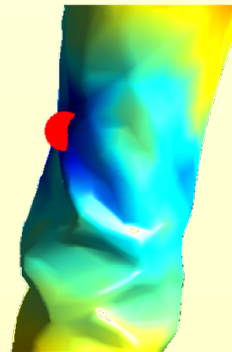
HKS



Note the shoulder



WKS



reference shape

new shape

zoom on the new shape

# Wave Kernel Signature

$$WKS(E, x) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T |\psi_E(x, t)|^2 dt = \sum_{n=0}^{\infty} \phi_n(x)^2 f_E(\lambda_k)^2$$

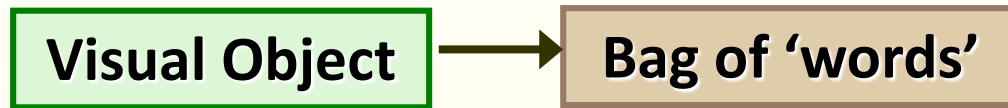
## Pros

- Isometry-invariant
- More accurate in higher frequencies

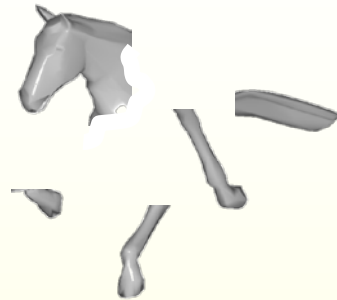
## Cons

- Similar to HKS

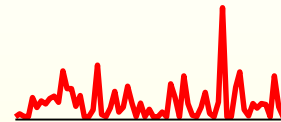
# Bag-of-Words Models (BoW) for Search



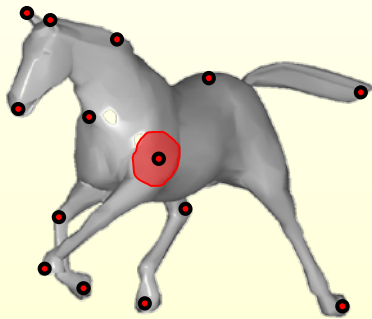
# Spatial Words



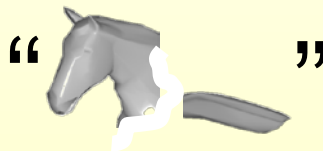
**Geometric words**



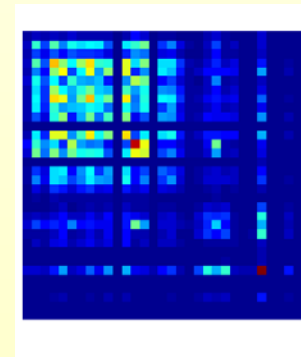
**Bag of geometric words**



**Feature descriptor**

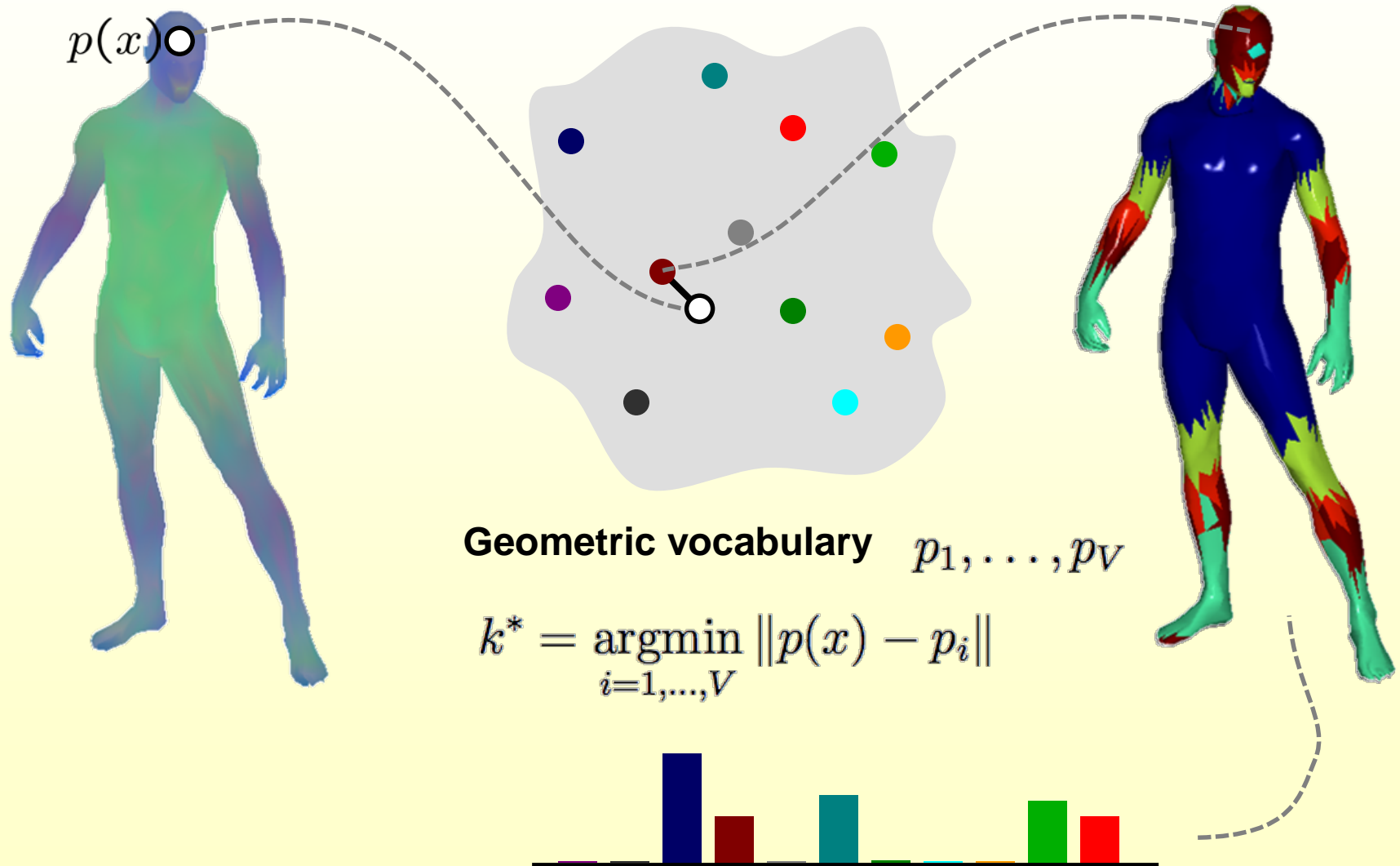


**Geometric expressions**

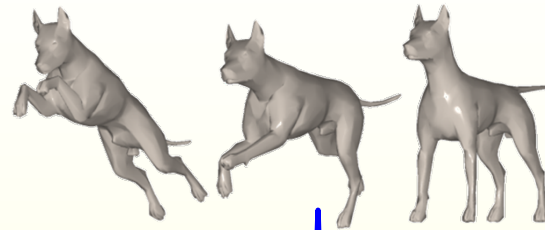


**Spatially-sensitive bag of words**

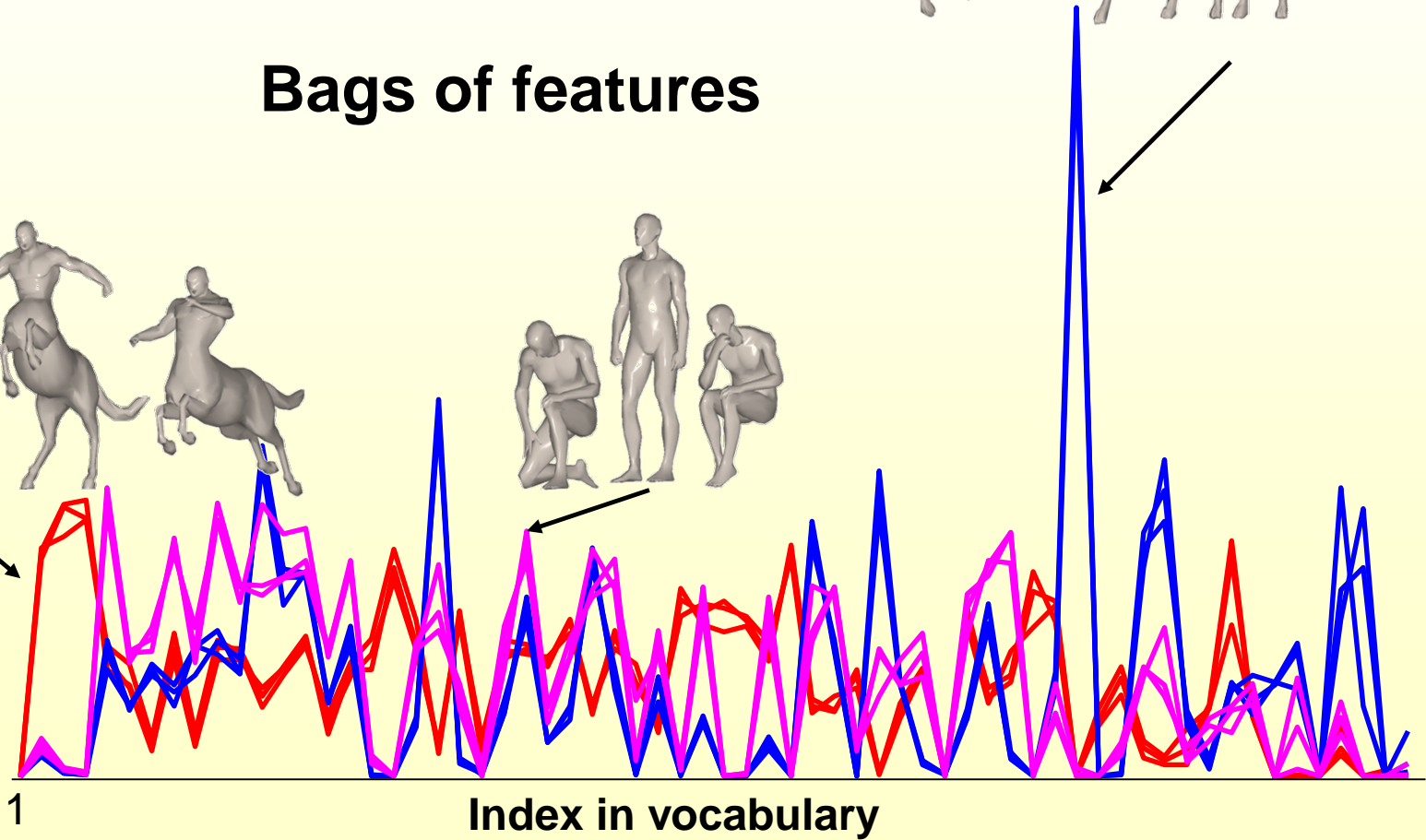
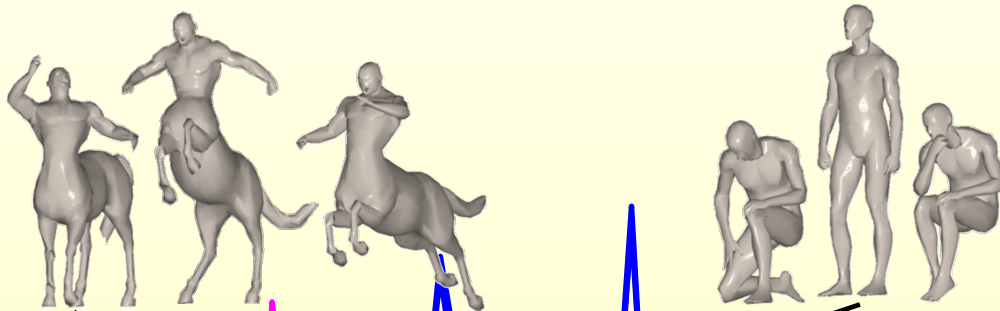
# ShapeGoogle: HKS/WKS-Based BoW Shape Search



# Shape Signatures



**Bags of features**



# HKS/WKS-Based Isometric Matching

# Key HKS / WKS Property

- The set of all HKSs on a shape almost always defines it up to isometry
- **The Informative Theorem:** If  $X$  and  $Y$  are two compact manifolds, such that  $\Delta_X$  and  $\Delta_Y$  have only non-repeating eigenvalues, then a homeomorphism  $T : X \rightarrow Y$  is an isometry **if and only if**, for all  $x$

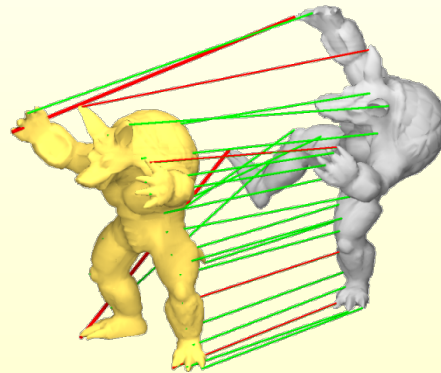
$$\text{HKS}(x) = \text{HKS}(T(x))$$

- The set of all HKSs fully characterizes the intrinsic metric structure of the manifold
- Same is true for WKS

# How Many Points Define an Isometry?

- Isometric matching of shapes

Given two possibly deformed shapes and  $k$  point correspondences, we can extend them to a unique map that preserves all pairwise distances.



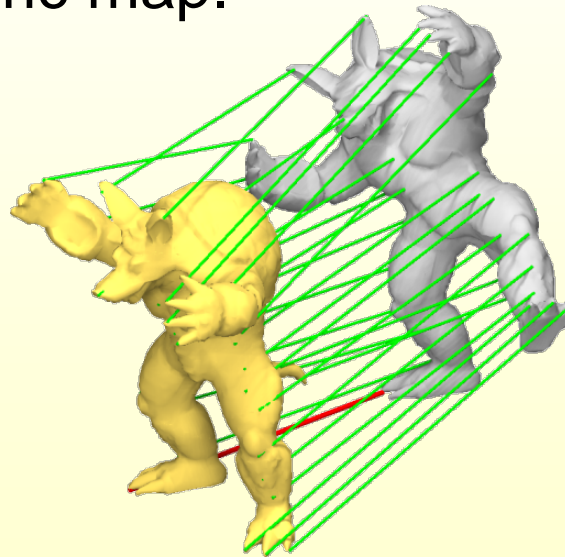
Find  $f : M \rightarrow N$  such that:  
 $d^M(x, y) = d^N(f(x), f(y))$   
for all  $x, y \in M$ .

- Question:

What is the minimum  $k$  that makes this possible?

# Surprising Result

- Under mild genericity conditions:  
**One** correspondence is sufficient to define a unique isometric map.



- Moreover, both the correspondence and its extension can be found efficiently in practice.

# Method Outline

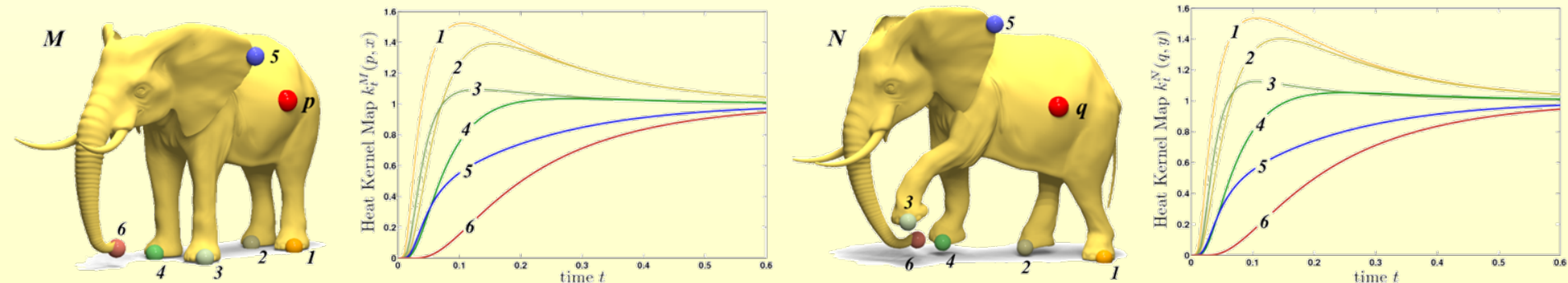
- Given a correspondence:

$$p \in M \longrightarrow q \in N$$

Consider the heat kernel:  $k_t^M(p, x)$  for any  $x \in M$ .  
Amount of heat transferred from  $p$  to  $x$  in time  $t$ .

- If  $f : M \rightarrow N$  is an isometry, heat kernel is preserved:

$$k_t^M(p, x) = k_t^N(q, f(x)) \quad \forall x \in M, t > 0$$



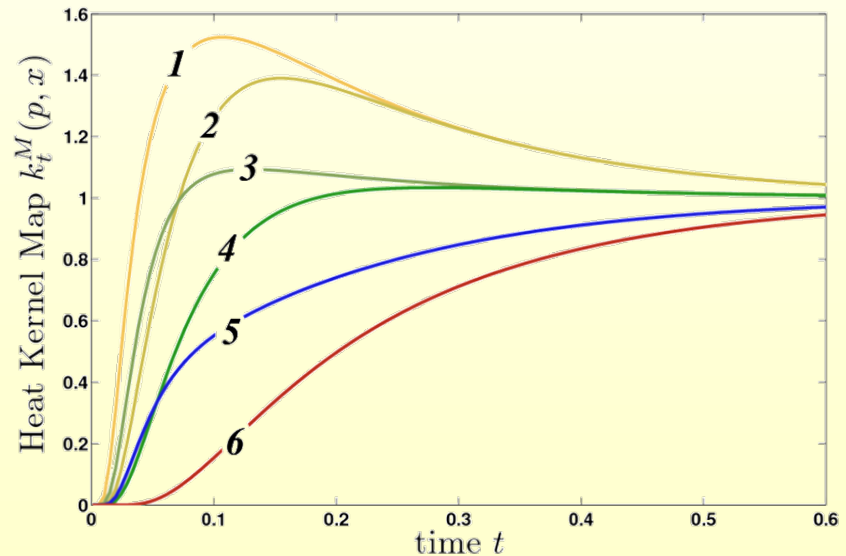
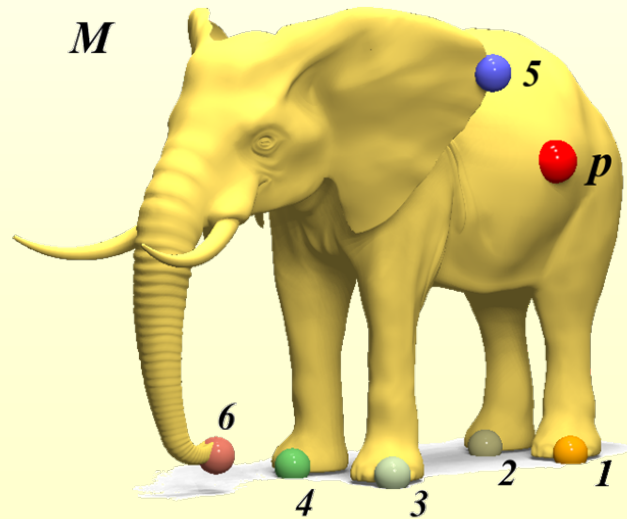
# Method Overview

- Given a compact manifold  $M$  and a point  $p \in M$  :

Heat Kernel Map:

$$x \rightarrow k_t^M(p, x) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$$

Associates with each point a function from  $\mathbb{R}^+$  to  $\mathbb{R}^+$ .



# Method Overview

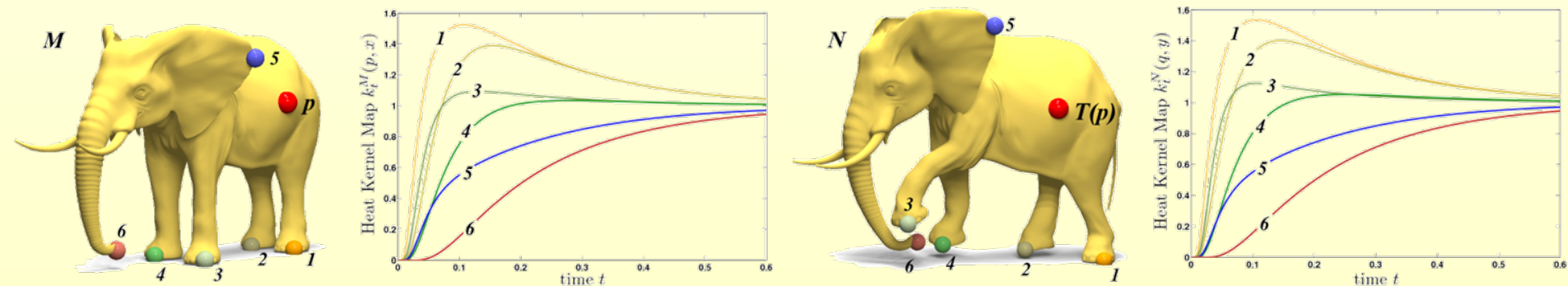
- Given a compact manifold  $M$  and a point  $p \in M$  :

Heat Kernel Map (HKM):

$$x \rightarrow k_t^M(p, x) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$$

- If  $T : M \rightarrow N$  is an isometry, HKM is preserved:

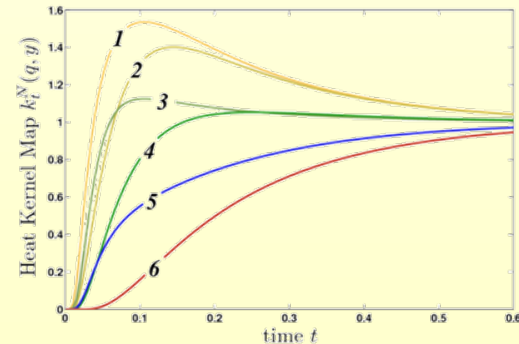
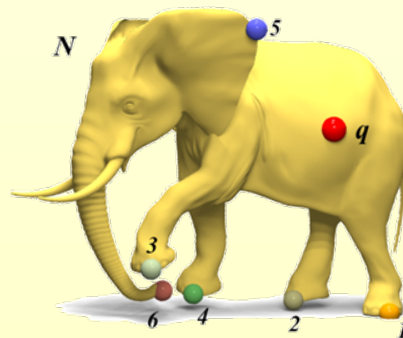
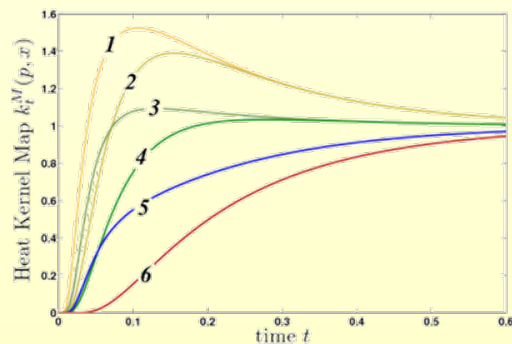
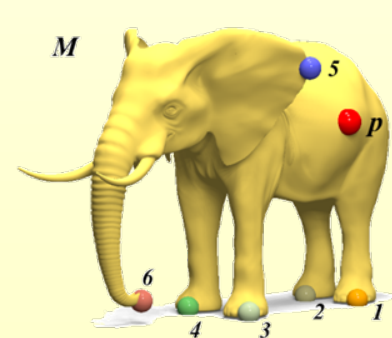
$$k_t^M(p, x) = k_t^N(T(p), T(x)) \quad \forall x \in M, t > 0$$



# Method Outline

- We show that if  $M$  and  $p$  are “generic” then the HKM

$$x \in M \rightarrow k_t^M(p, x) \quad \text{is injective.}$$



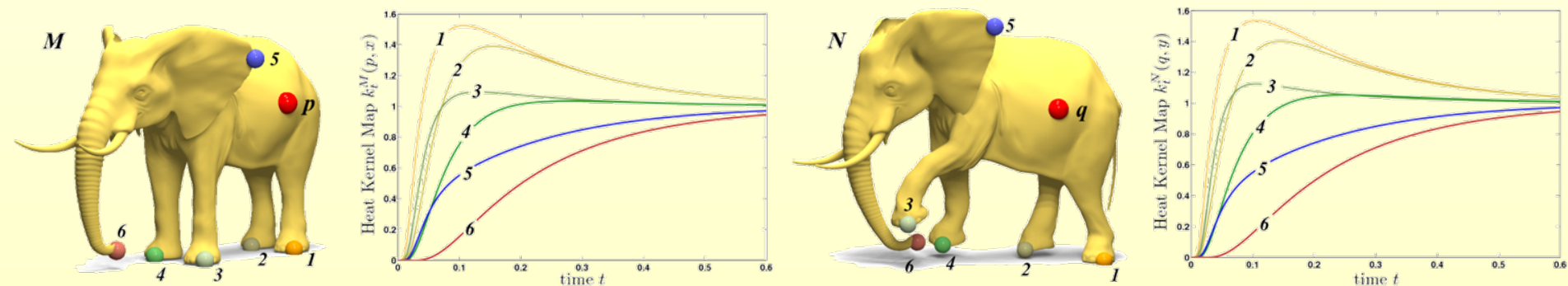
# Method Outline

- We show that if  $M$  and  $p$  are “generic” then the HKM:

$$x \in M \rightarrow k_t^M(p, x) \quad \text{is injective.}$$

$\Rightarrow$  Given two isometric shapes  $M$  and  $N$ , and a single correspondence  $T(p) = q$ , then for any  $x$ ,  $T(x)$  is the *unique* point, s.t.:  $\text{HKM}(x) = \text{HKM}(T(x))$ . I.e.

$$k_t^M(p, x) = k_t^N(q, T(x)) \quad \forall t > 0$$

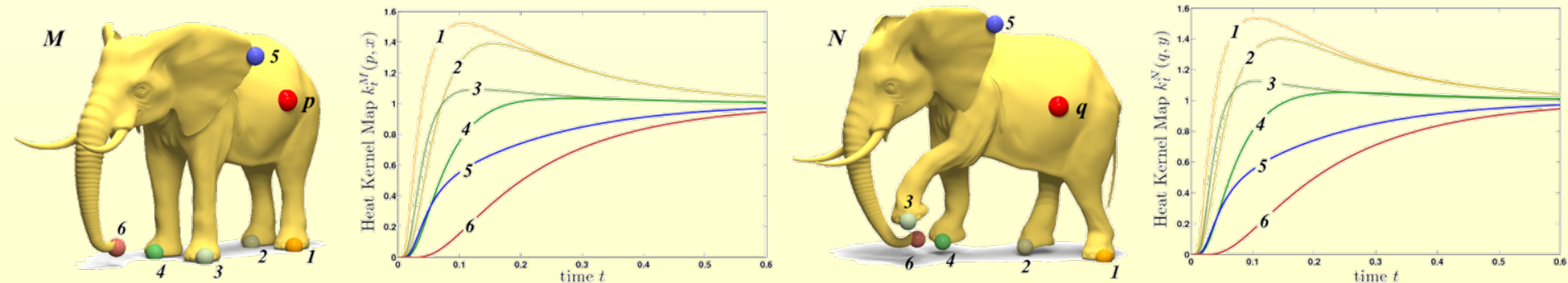


# Method Outline

- We show that if  $M$  and  $p$  are “generic” then the HKM:

$$x \in M \rightarrow k_t^M(p, x) \quad \text{is injective.}$$

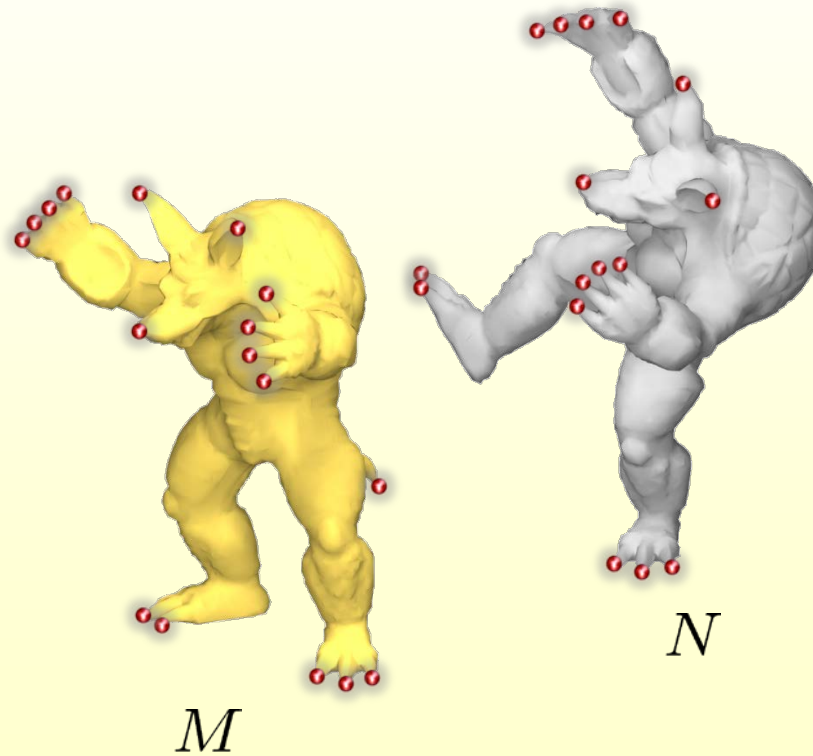
- Conversely, given two generic shapes  $M$  and  $N$  and a correspondence  $T(p) = q$  s.t.  $\forall x \in M \exists T(x)$  with  $\text{HKM}(x) = \text{HKM}(T(x))$  then  $T$  is an isometry.



No assumption of continuity!

# Isometric Matching

1. Detect a set of **feature points** (local maxima of HKS)

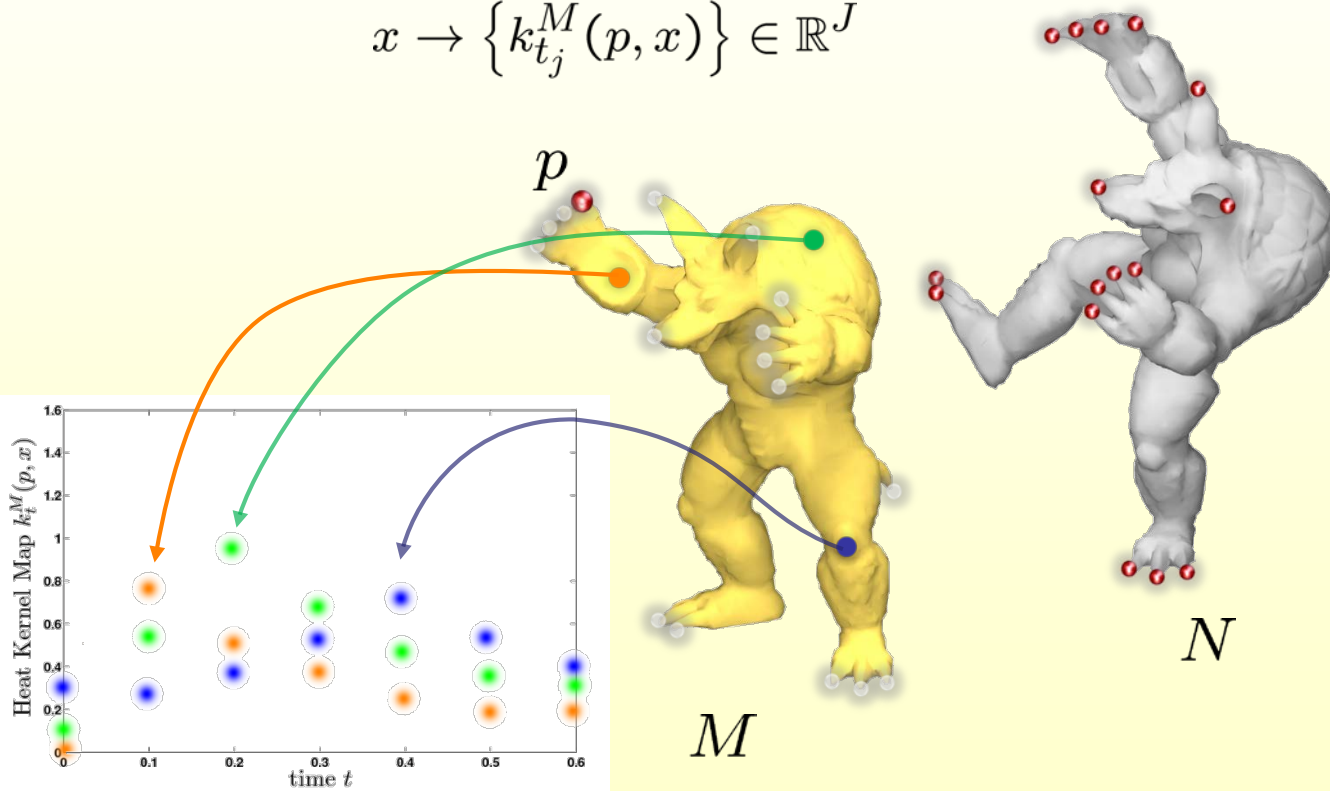


# Isometric Matching

2. Fix a feature point  $p$  on the source shape  $M$

For every point  $x \in M$  discretize the Heat Kernel Map

$$x \rightarrow \{k_{t_j}^M(p, x)\} \in \mathbb{R}^J$$



# Isometric Matching

For every feature point  $q_i \in N$  score the potential correspondence  $p \longrightarrow q_i$  to pick the best  $q_i$

$$E(p, q_i) = \sum_{x \in M} \min_{y \in N} \sup_j t_j |k_{t_j}^M(p, x) - k_{t_j}^N(q_i, y)|$$

If  $E(p, q_i) = 0$  then the map

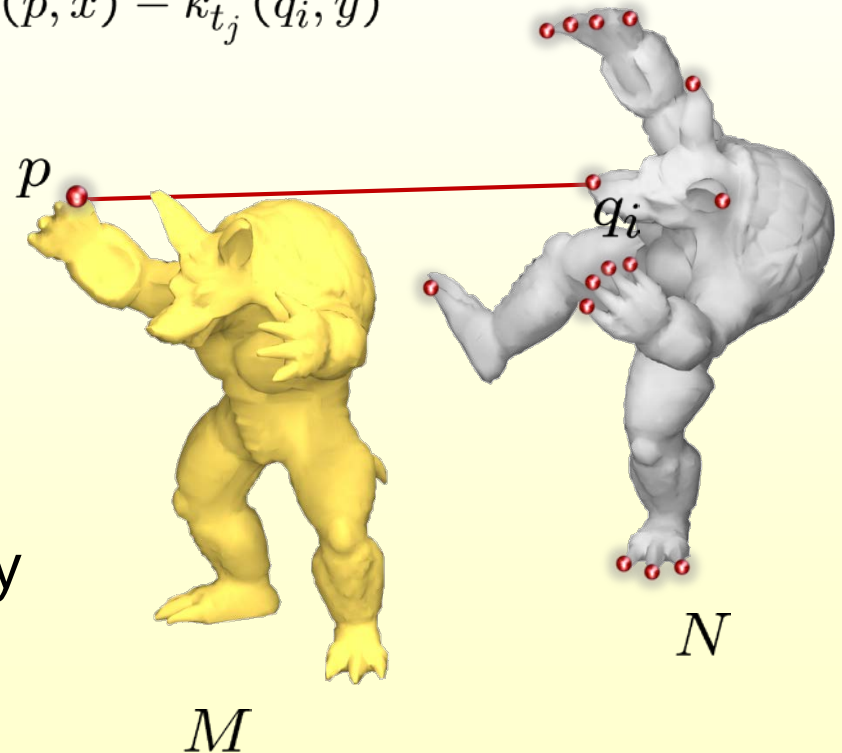
$$x \rightarrow T(x)$$

$$\text{s.t. } k_t^M(p, x) = k_t^N(q_i, T(x)) \quad \forall t$$

is an isometry.

Now extend the map  $p \longrightarrow q_i$  by

$$x \rightarrow \operatorname{argmin}_{y \in N} \left\| \Phi_p^M(x) - \Phi_{q_i}^N(y) \right\|$$



Again: no continuity assumption!

# Isometric Matching – Extensions

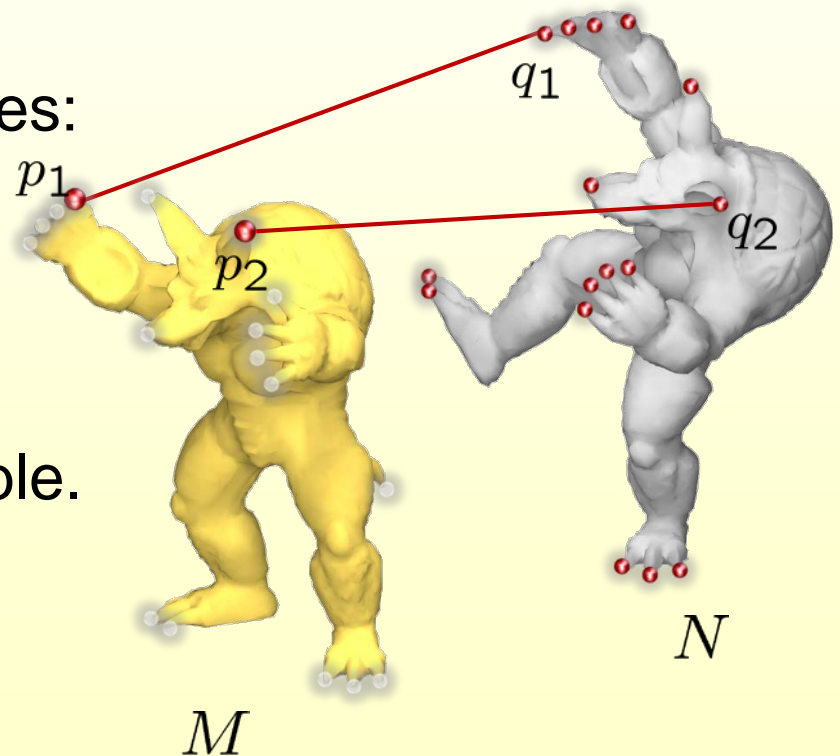
1. Augment the Heat Kernel Map with the HKS:

$$x \rightarrow \{k_{t_j}^M(p, x), k_{t_j}^M(x, x)\} \in \mathbb{R}^{2J}$$

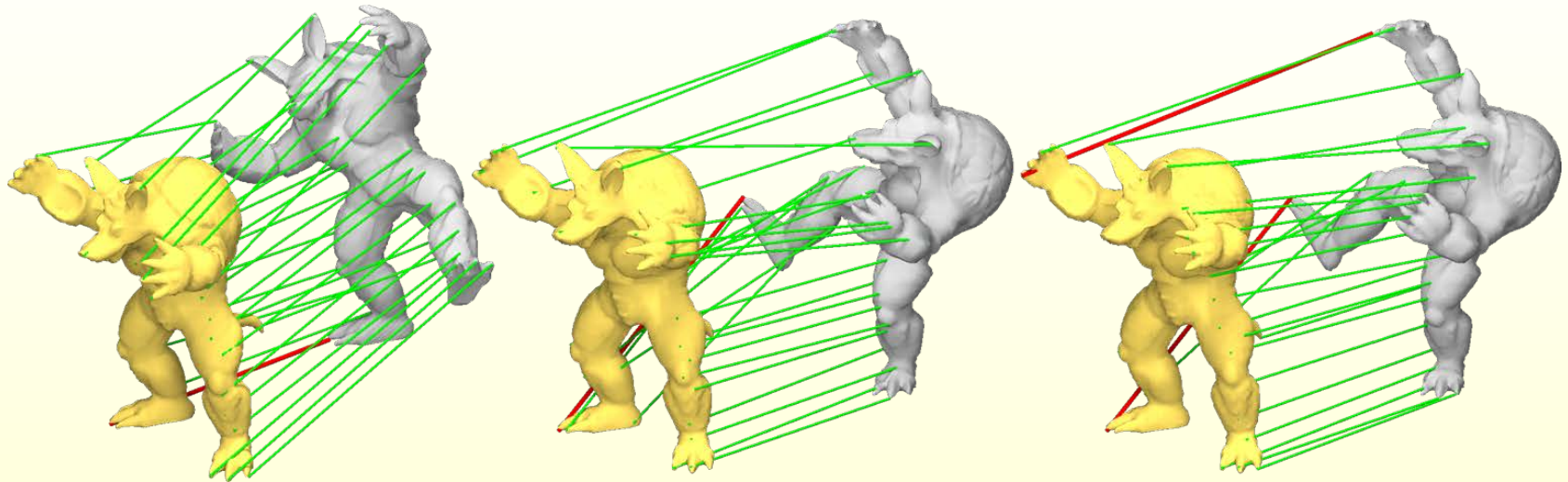
2. Use several correspondences:

$$x \rightarrow \{k_{t_j}^M(p_1, x), k_{t_j}^M(p_2, x)\} \in \mathbb{R}^{2J}$$

All quantities are commensurable.  
Can use the same metric!

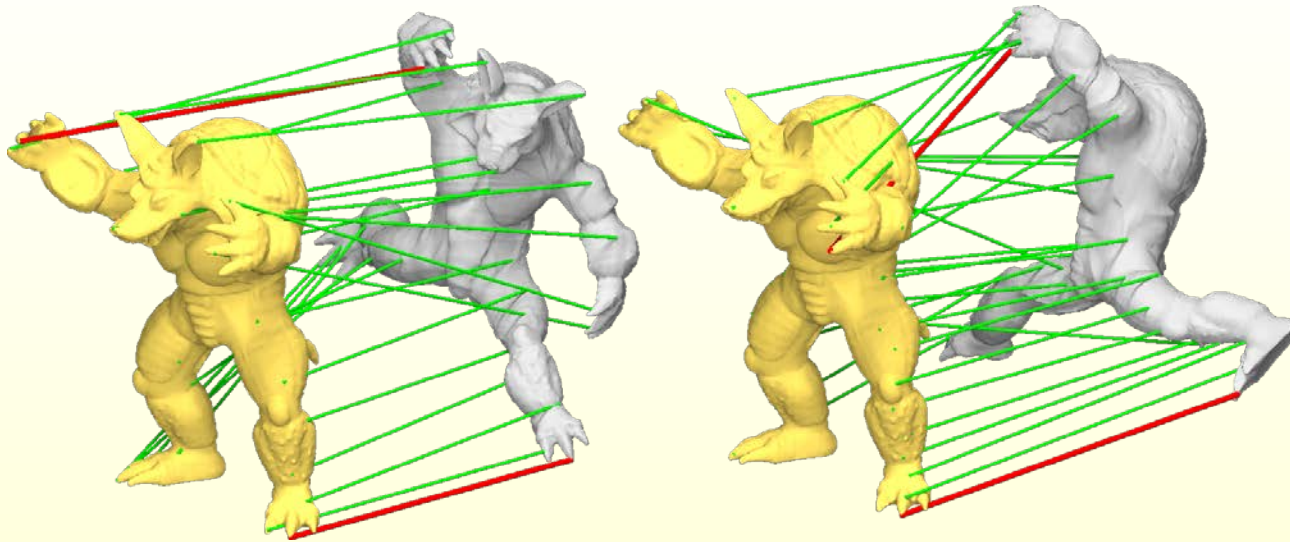


# Results – Isometric Matching



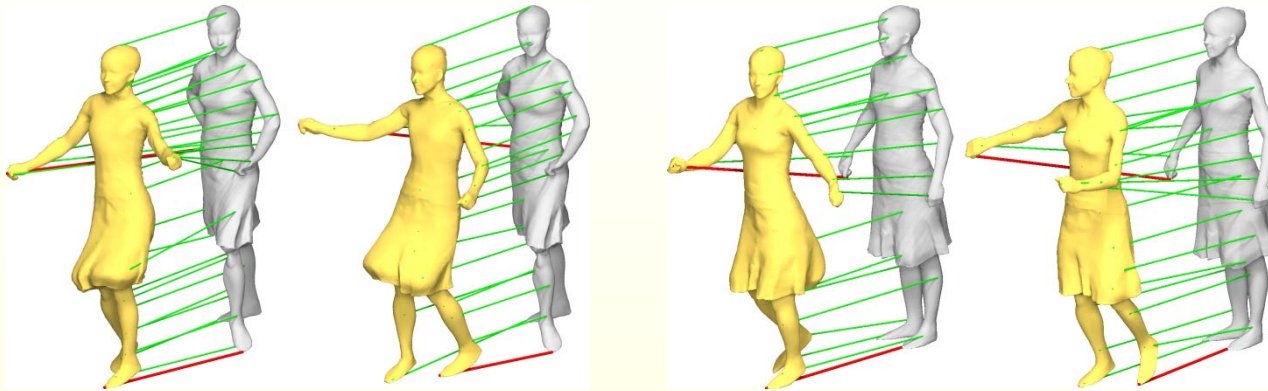
Geodesic distortion: 5.3% of the shape diameter

# Results – Isometric Matching

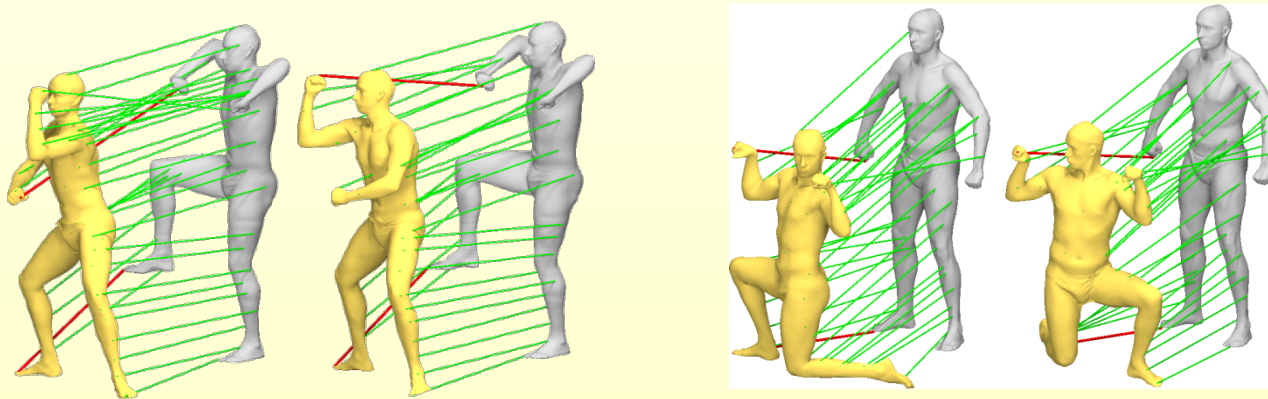


Geodesic distortion: 13.2% of the shape diameter

# Results – Isometric Matching

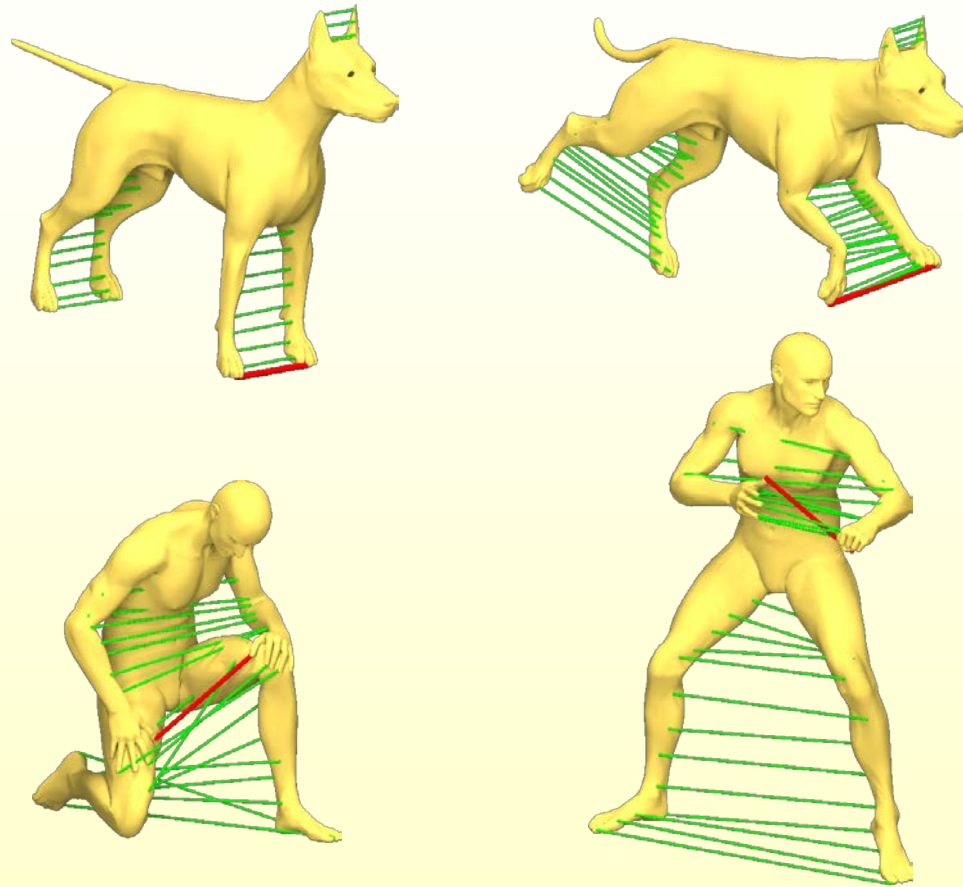


data: Vlastic et al.



data: Anguelov et al. (SCAPE)

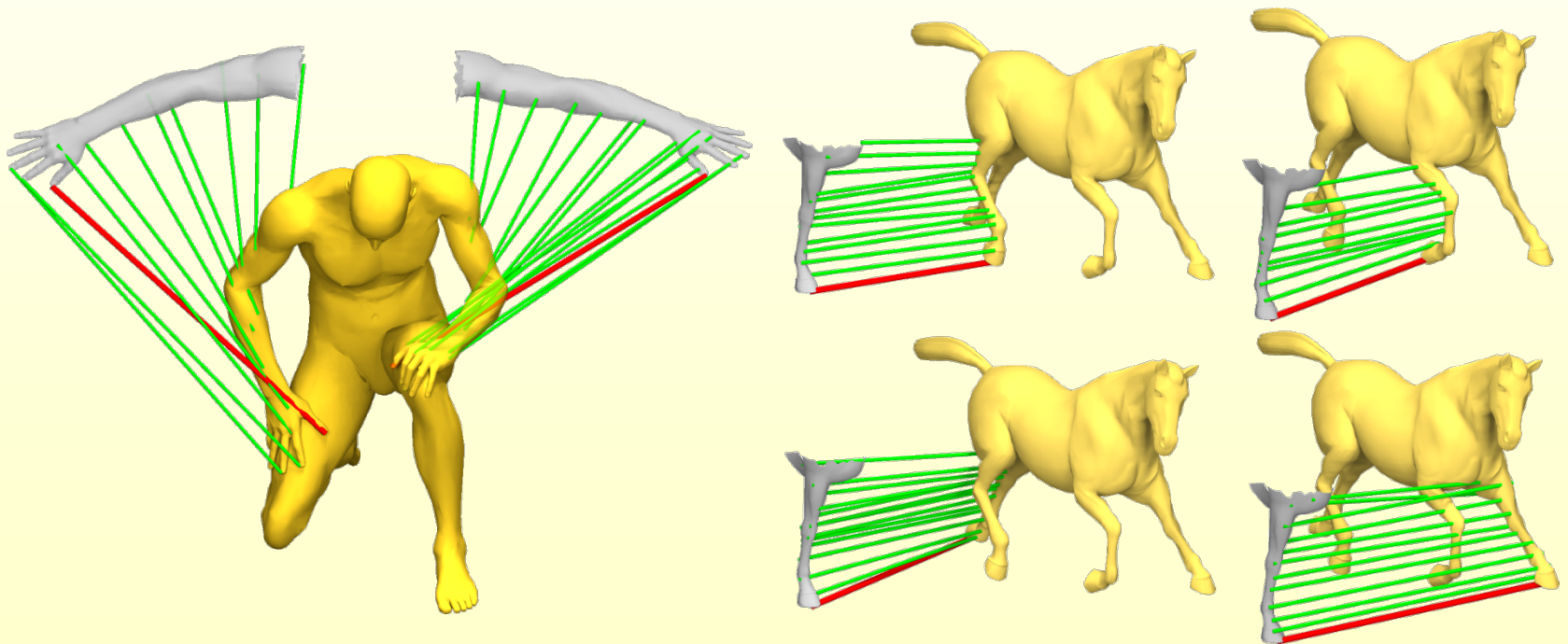
# Results – Symmetry Detection



data: Bronstein et al. (Nonrigid World)

# Results – Partial Isometric Matching

For small values of  $t$ , heat kernel  $k_t^M(p, \cdot)$  is well approximated by the heat kernel of a neighborhood of  $p$ .  
By limiting the HKM to small  $t$ , we can do partial matching.

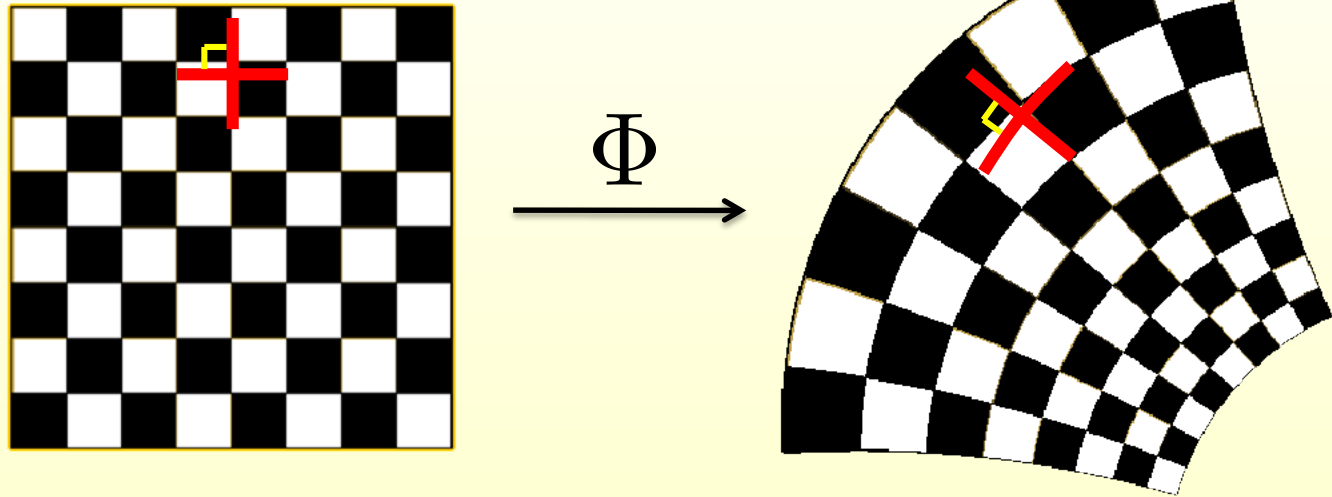


# Conformal Geometry: Angle-Preserving Maps

# Conformal Maps

Conformal maps are analytic maps that preserve **angles**.

$$(x, y) \longrightarrow (u(x, y), v(x, y))$$



$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

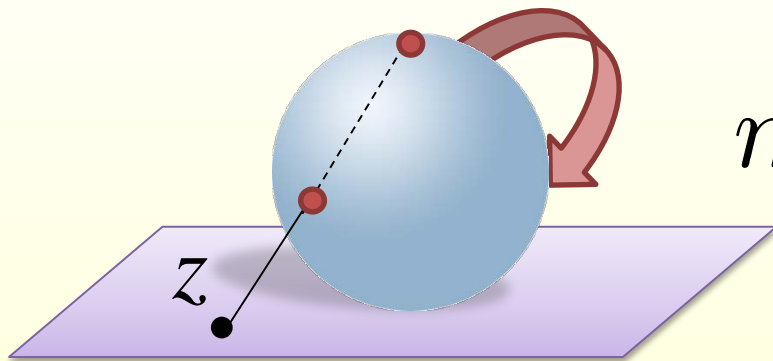
$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

$$\begin{aligned} u_x &= v_y \\ u_y &= -v_x \end{aligned}$$

Satisfy  
Cauchy-Riemann  
equations

# Bijjective Conformal Maps of Simple Domains: Möbius Transforms

- **Unit sphere**



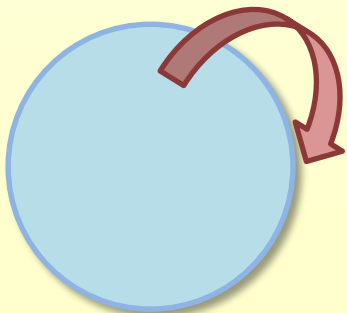
**6 real DOF**

$$m(z) = \frac{az+b}{cz+d} \quad ad - bc \neq 0$$

Best understood via complex numbers

Low dimensional

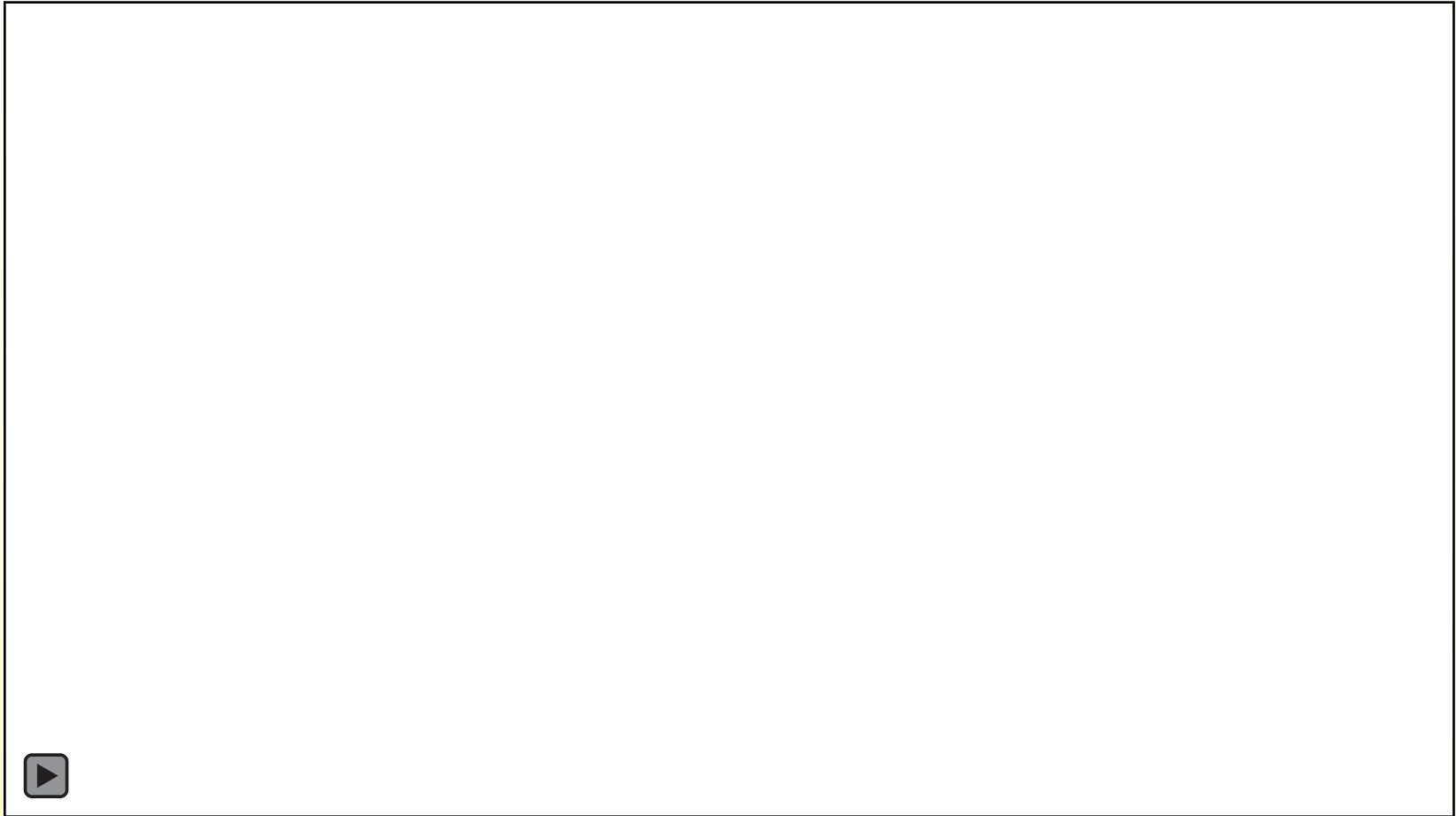
- **Unit disk**



**3 real DOF**

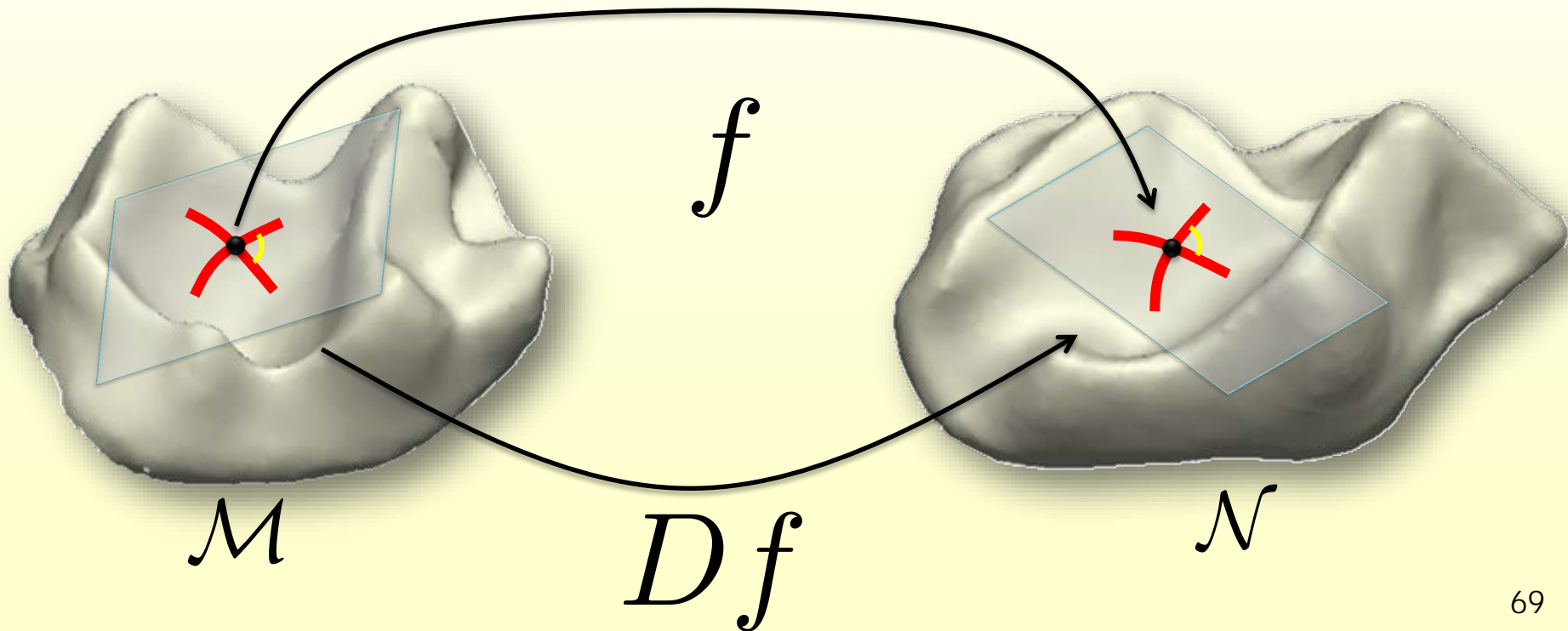
$$m(z) = e^{i\theta} \frac{z-a}{1-\bar{a}z} \quad a \in \mathcal{D}, \theta \in [0, 2\pi)$$

# Möbius Transforms

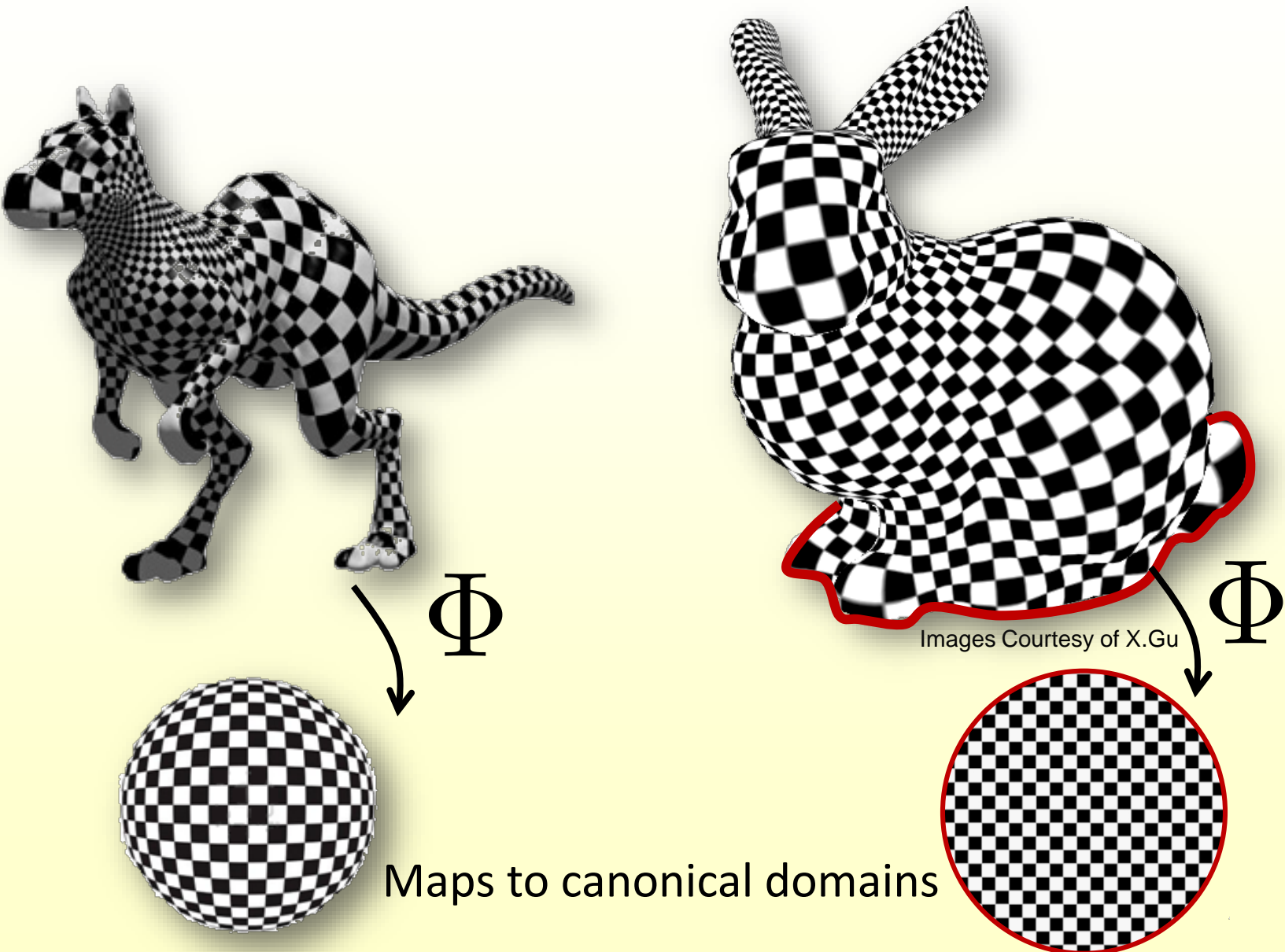


# Conformal Maps Between General Surfaces

More generally,  
conformal maps can be defined between **surfaces**.

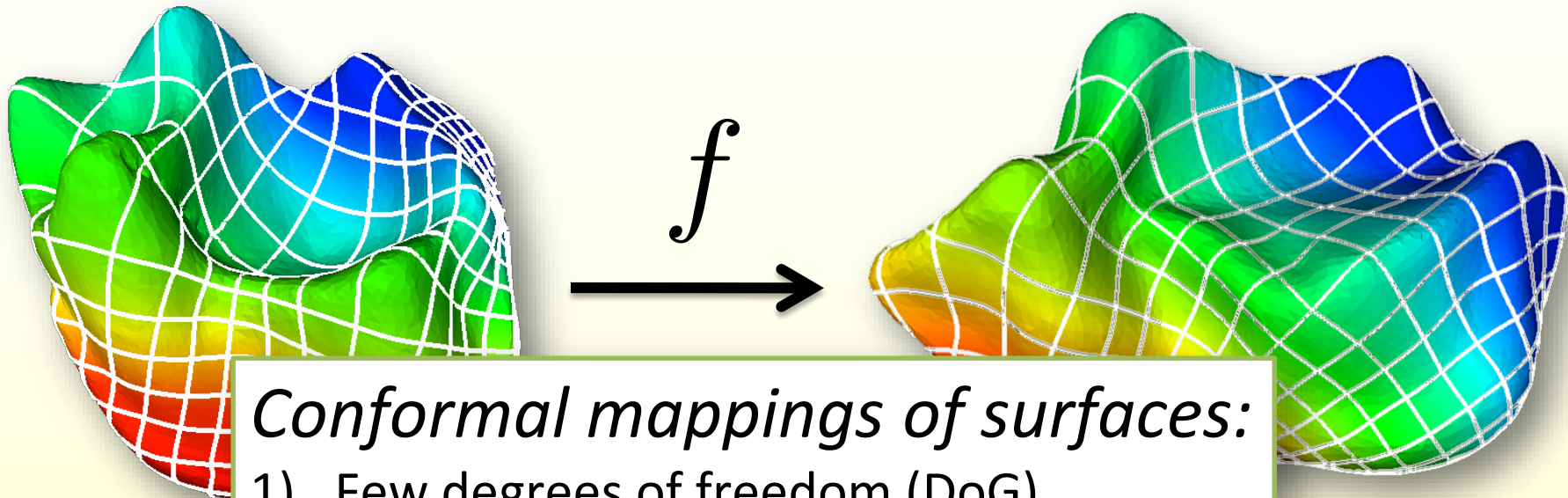


# Uniformization



Maps to canonical domains

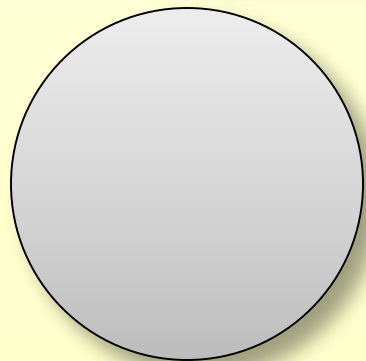
# The Power of Map Composition



*Conformal mappings of surfaces:*

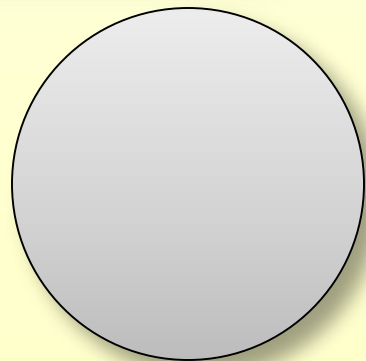
- 1) Few degrees of freedom (DoF)
- 2) easy to compute on canonical domains

$\Psi$



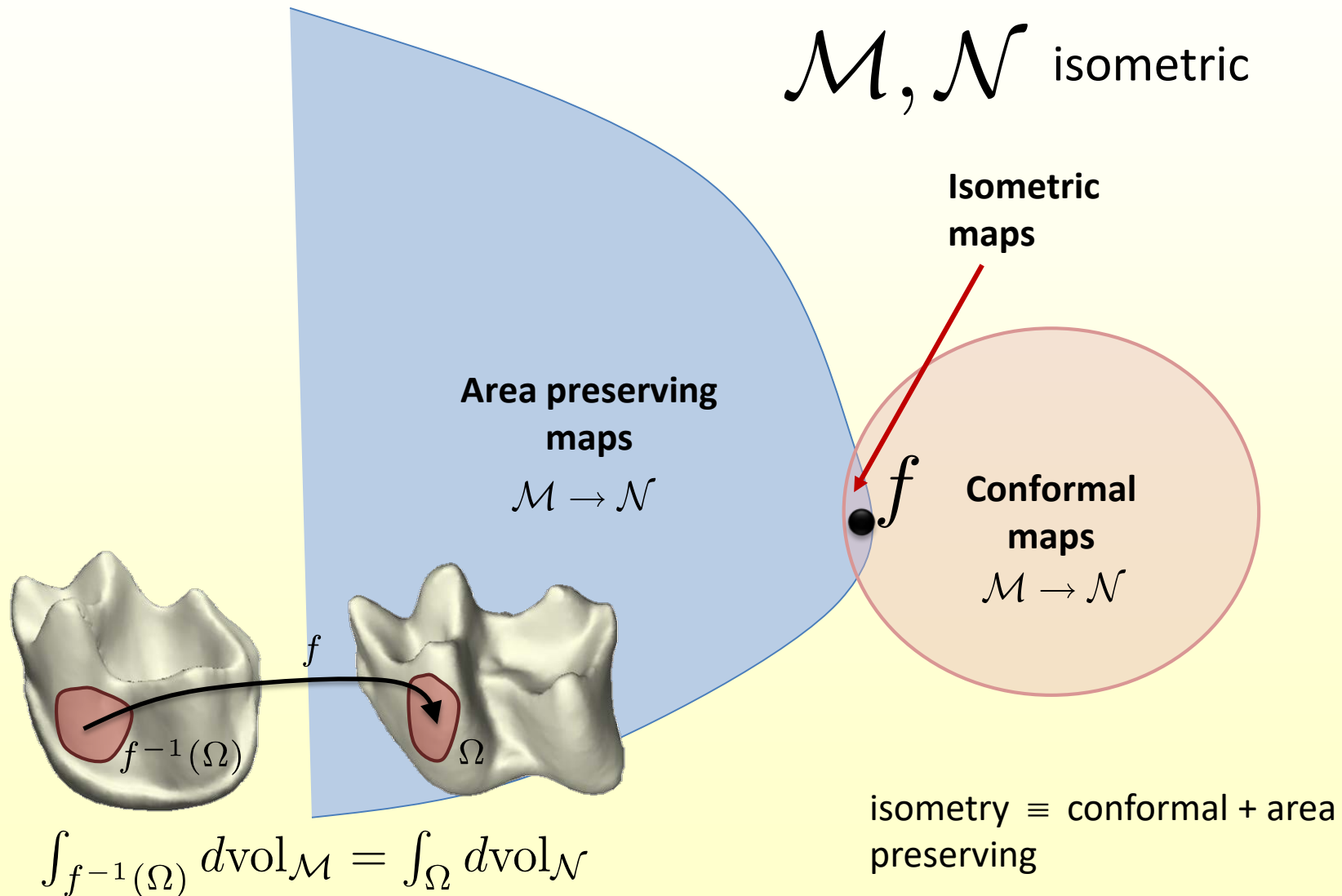
$$\Phi^{-1} \circ m \circ \Psi$$

$$m_{\text{Möbius}}$$

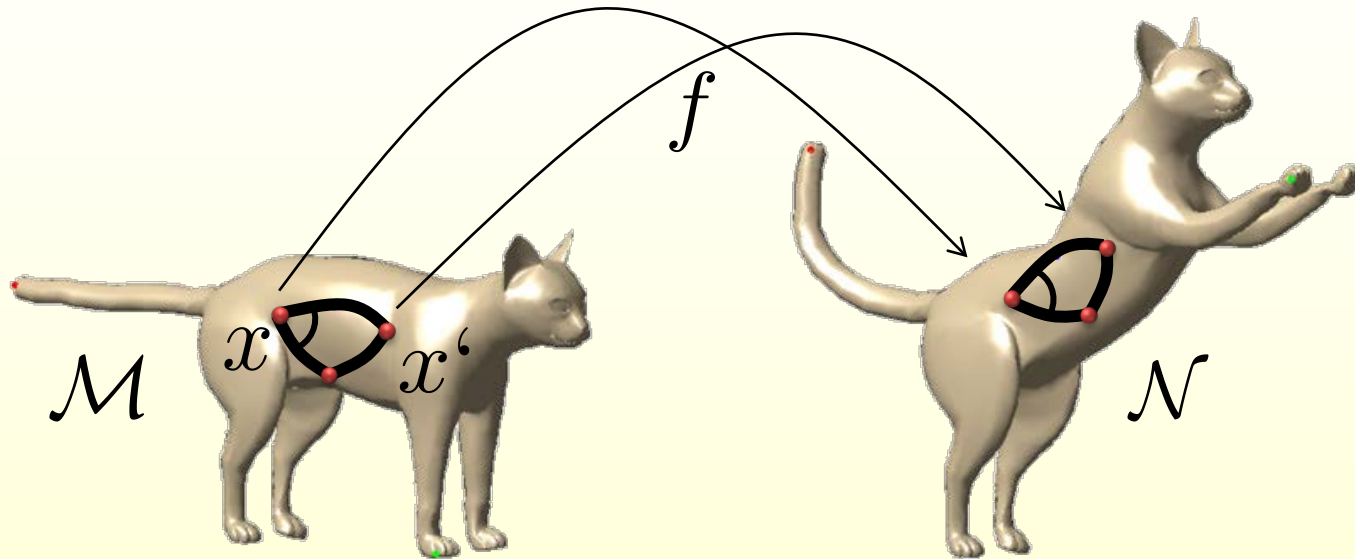


$\Phi$

# Isometries are Conformal Maps



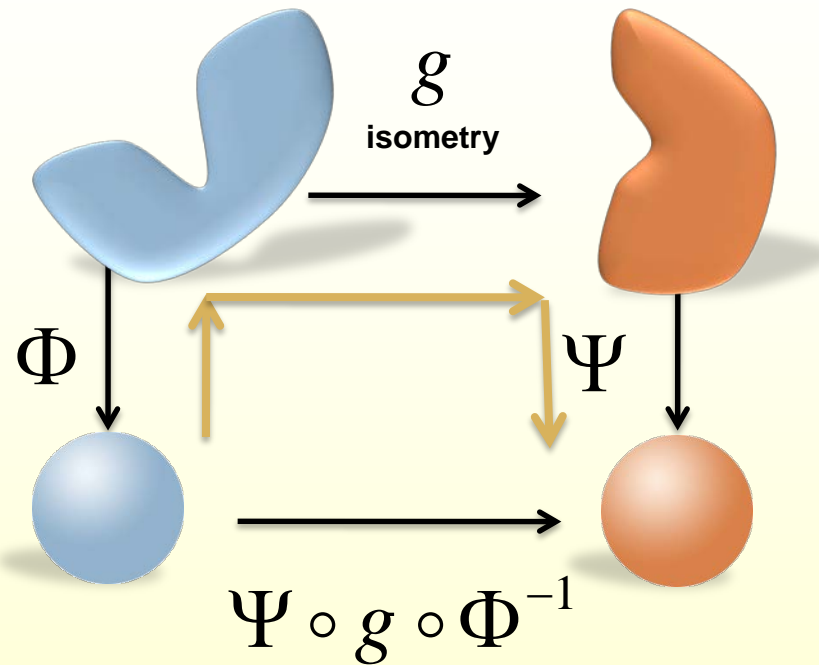
# Search for Conformal Maps

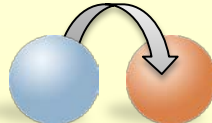


$$d_{\mathcal{M}}(x, x') = d_{\mathcal{N}}(f(x), f(x'))$$

and try to minimize are distortion

# Algorithm

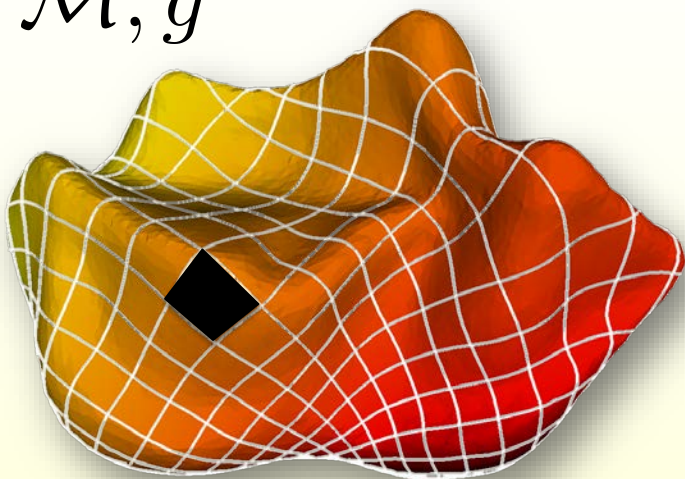


$$\Psi \circ g \circ \Phi^{-1} \in \text{Conformal} \Rightarrow \Psi \circ g \circ \Phi^{-1}(z) = \frac{az + b}{cz + d}$$


search the **Möbius** group (6 DOF) for a good correspondence

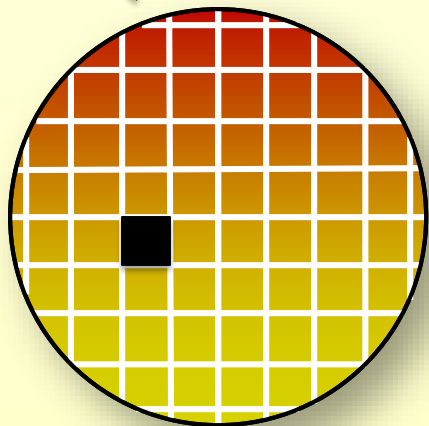
# Area Info: Conformal Factors

$\mathcal{M}, g$

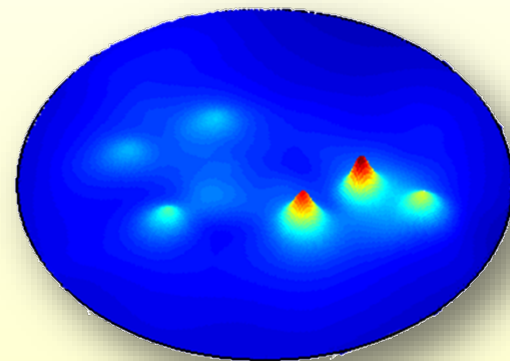


Conformal Factor:

$\Phi$

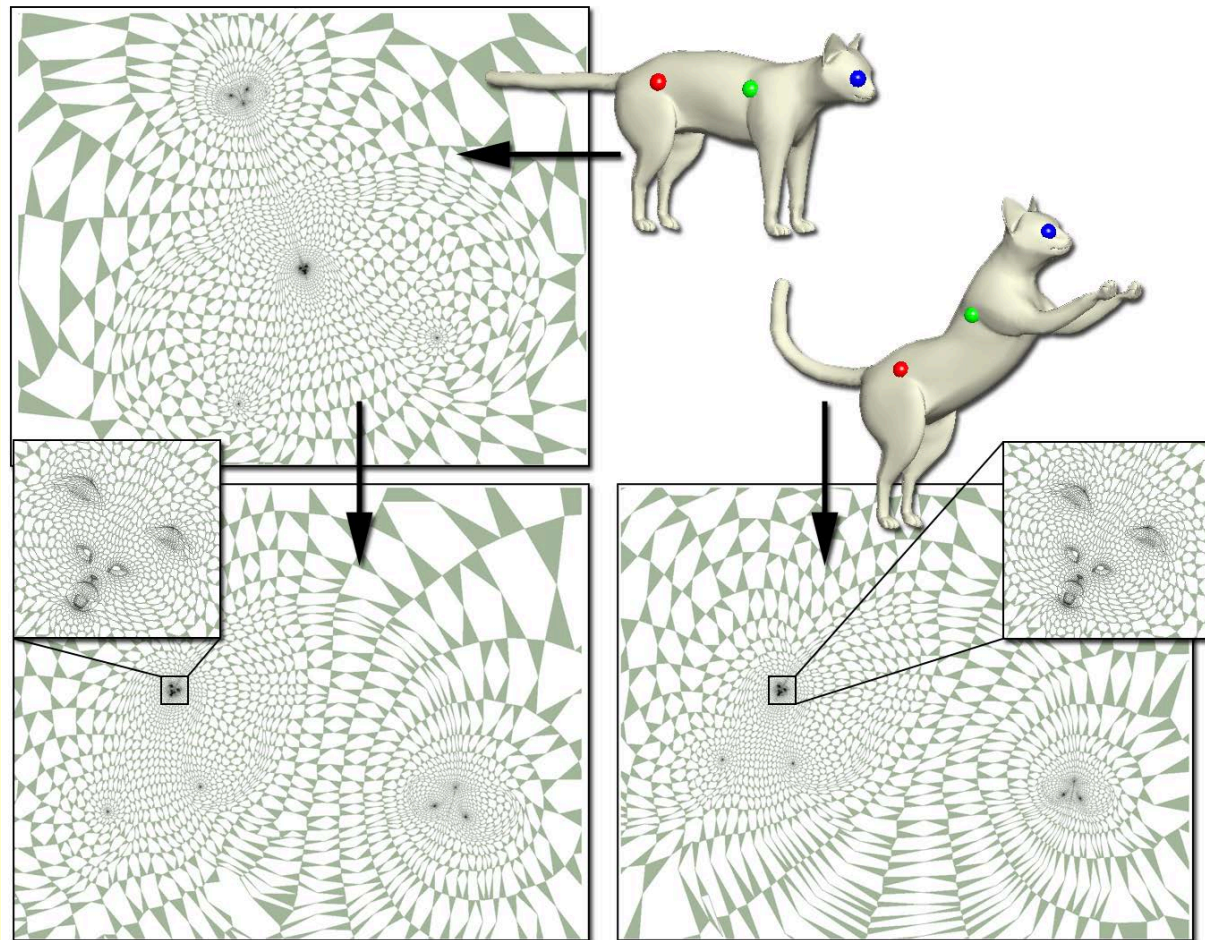
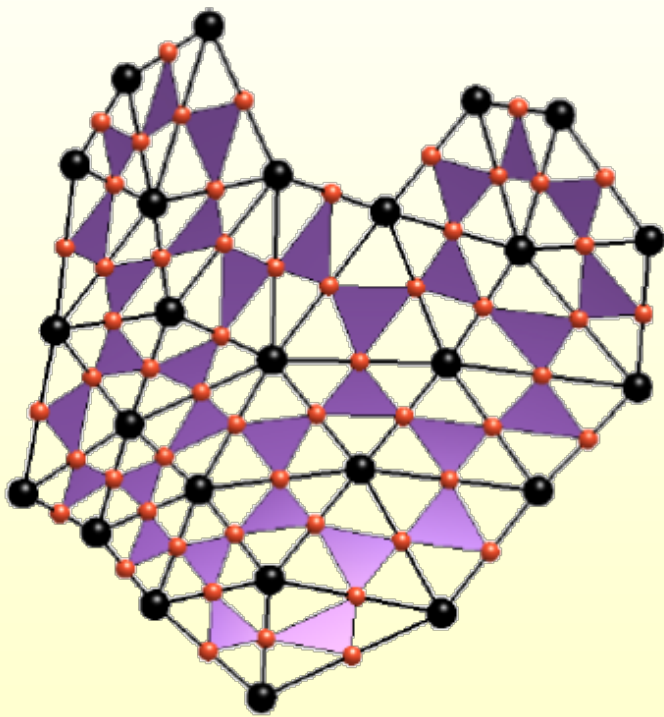


$$\mu(z) = \frac{\text{Area}(\blacklozenge)}{\text{Area}(\blacksquare)}$$



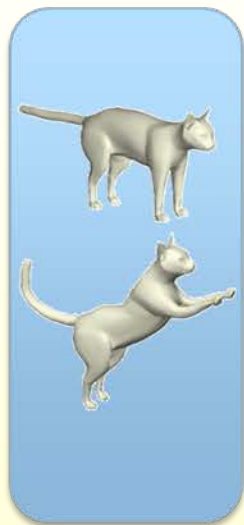
$$\Phi_*g = \mu(z) \left( dx^2 + dy^2 \right)$$

# Mid-Edge Uniformization / Flattening

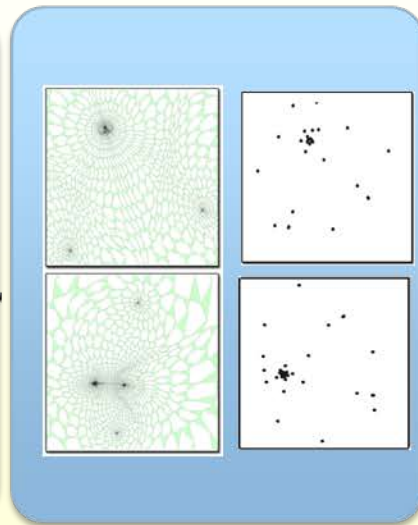


# Möbius Voting

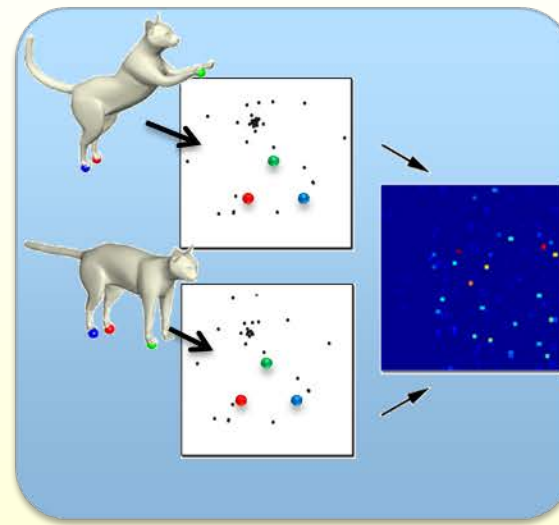
# Möbius Voting Algorithm



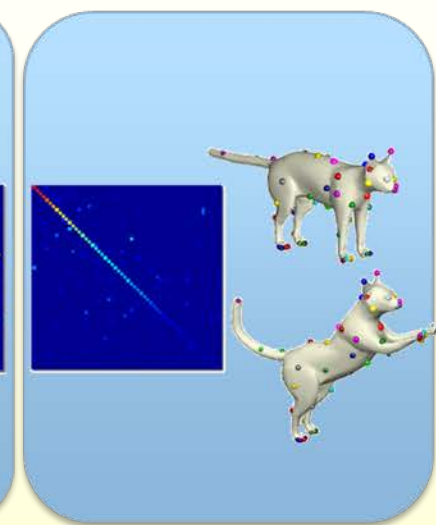
sample points



Uniformization



Voting



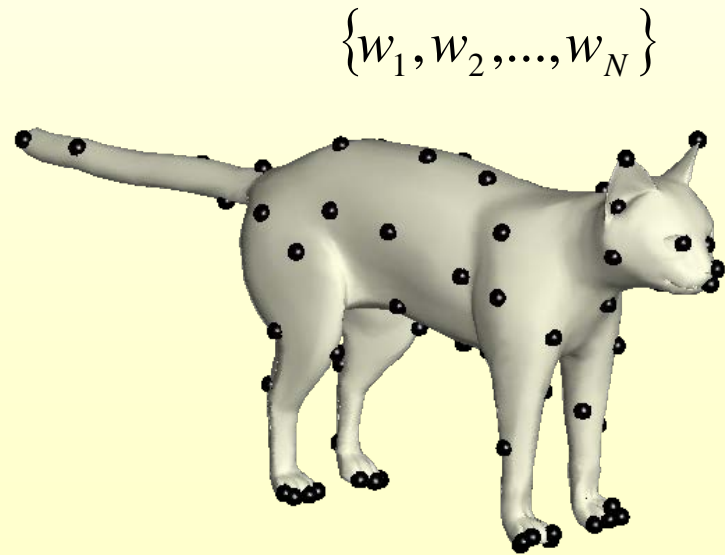
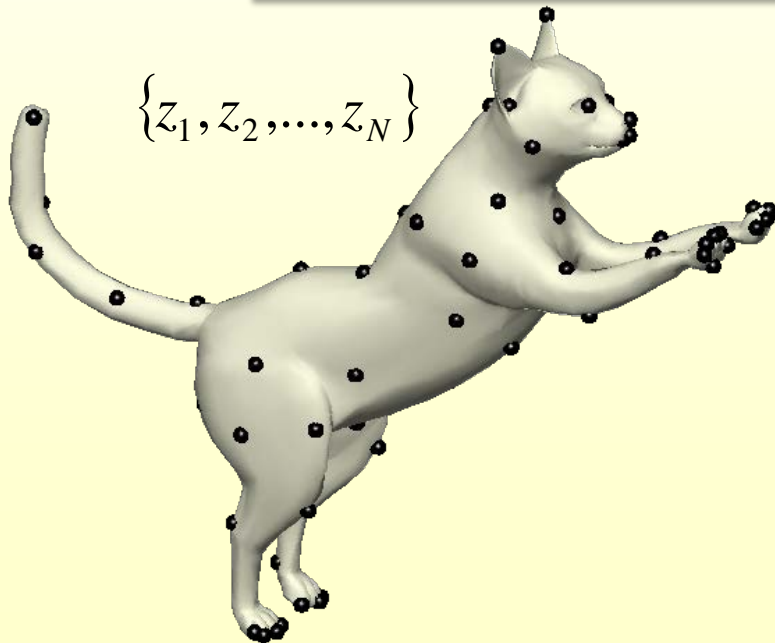
Extracting Correspondences

# Sampling Points

Sample by:

- 1) Extrema of Gauss curvature or HKS / WKS (isometry invariant)
- 2) Complete by furthest point sampling

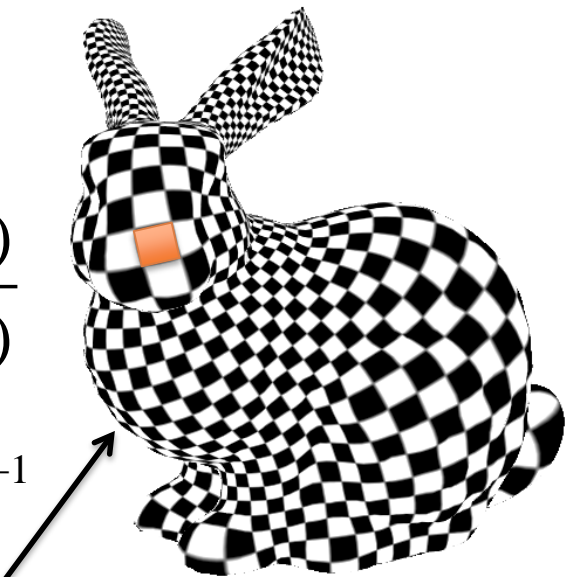
Each point represents a surface patch of “equal importance”.



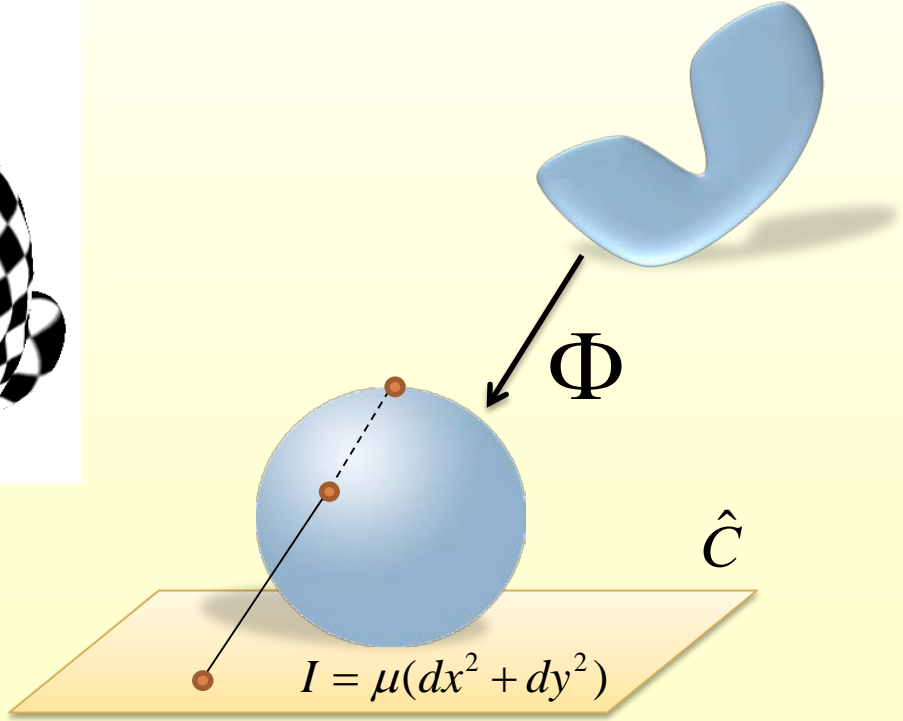
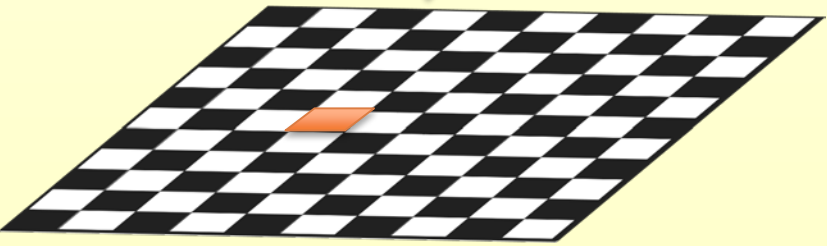
# Uniformization

Conformal factor:

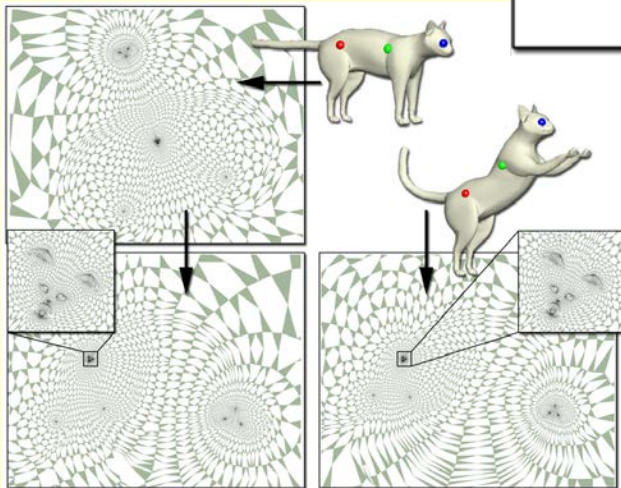
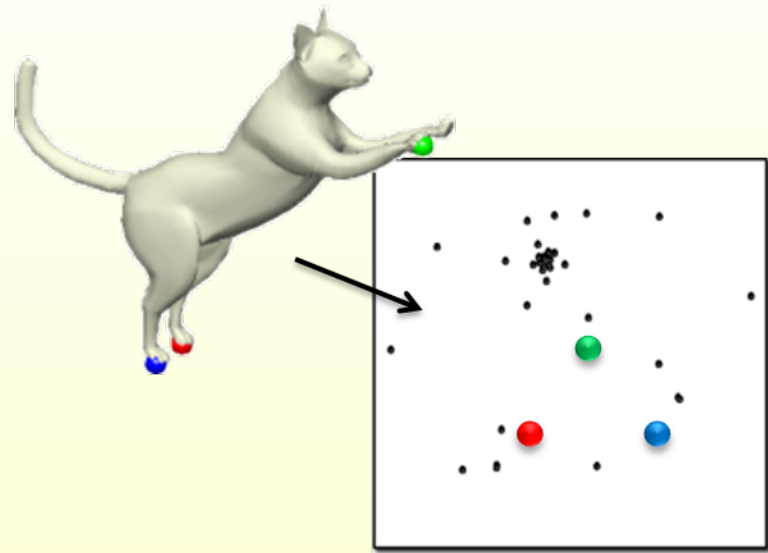
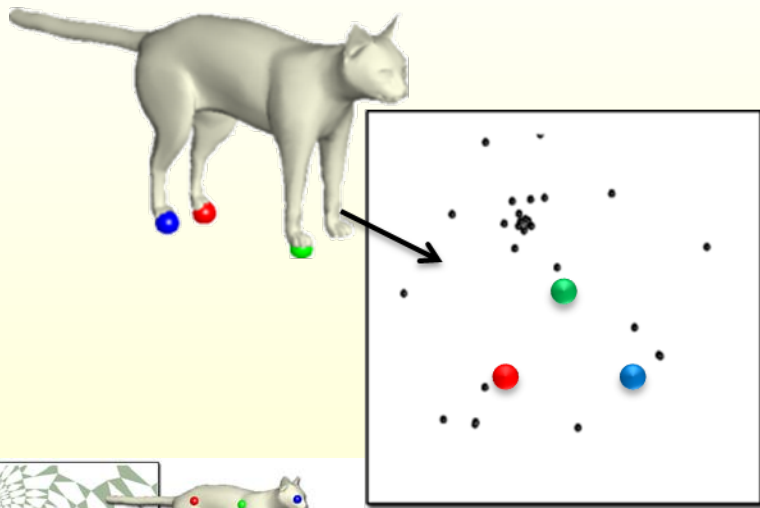
$$\mu(z) \approx \frac{\text{area}(\blacksquare)}{\text{area}(\blacktriangleright)}$$



Gu and S-T Yau 2003

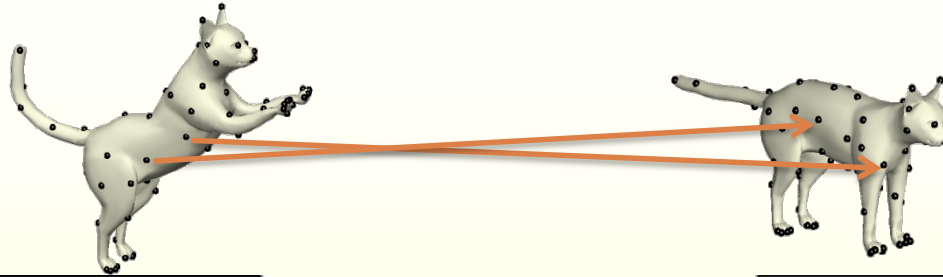


# Canonicalization of Triplet Pairing

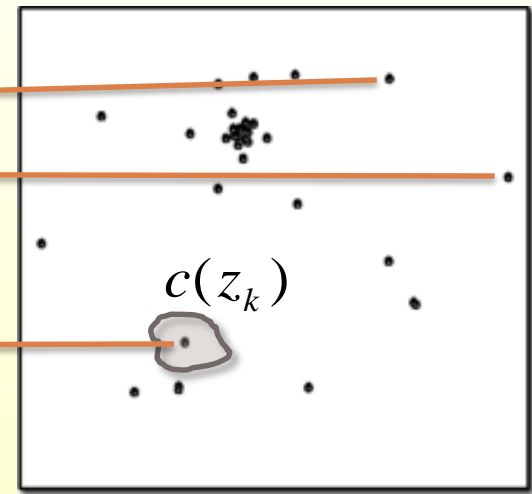
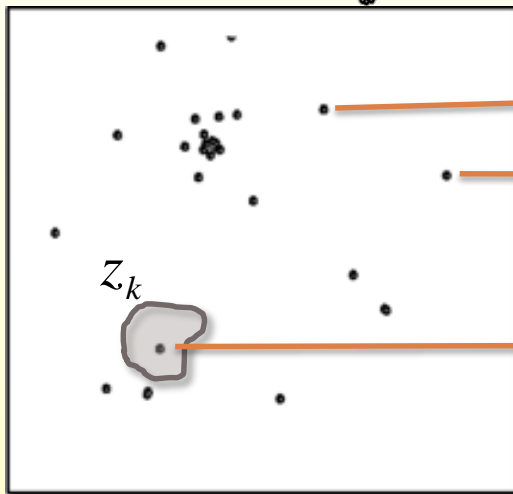


Any three points can be taken to any three points

# Scoring



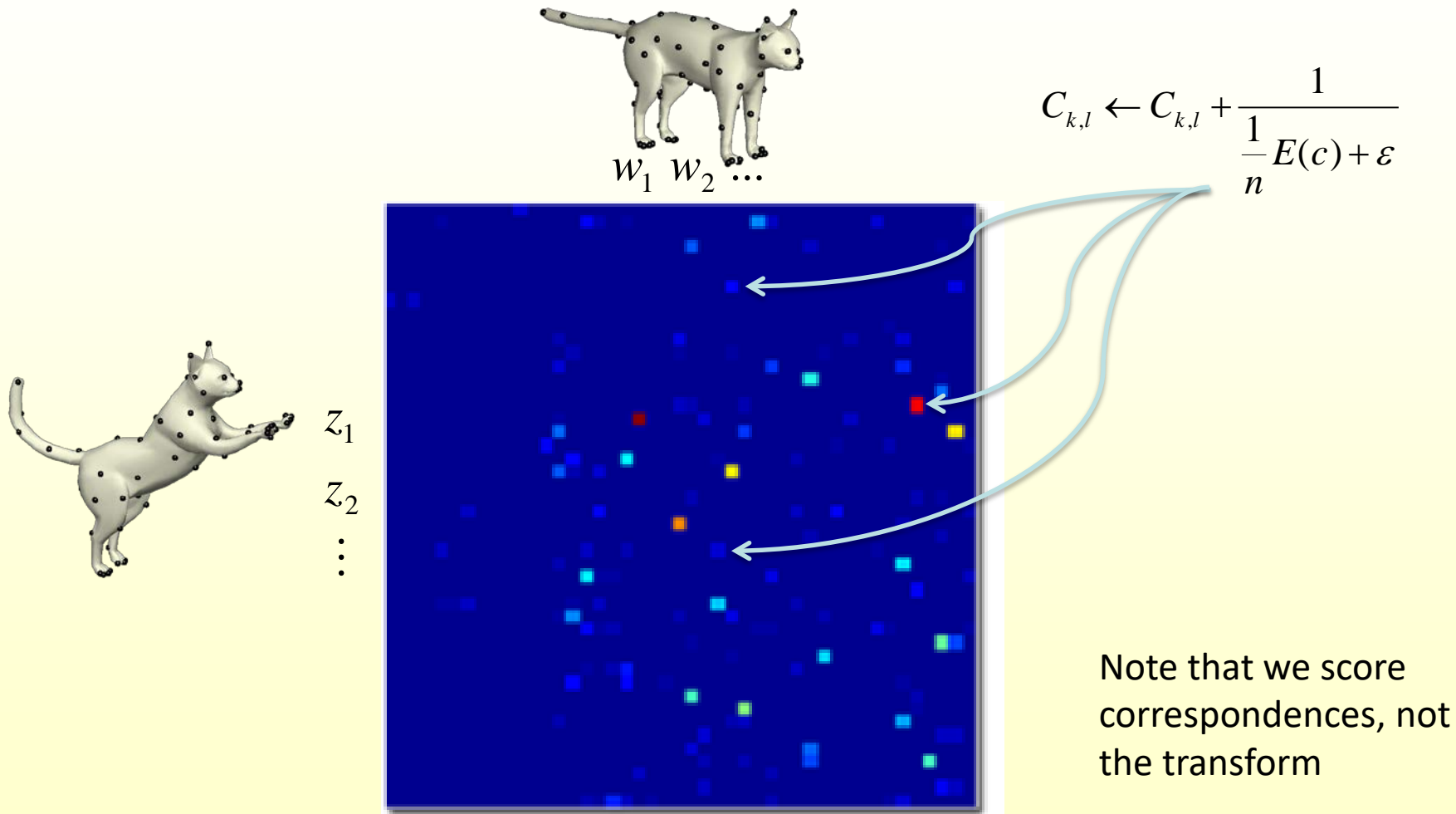
Closest points:  
at least  $K$



$$d(z, w) = \begin{cases} |z - w| \\ |z - w| / |1 + \bar{z}w| \end{cases}$$

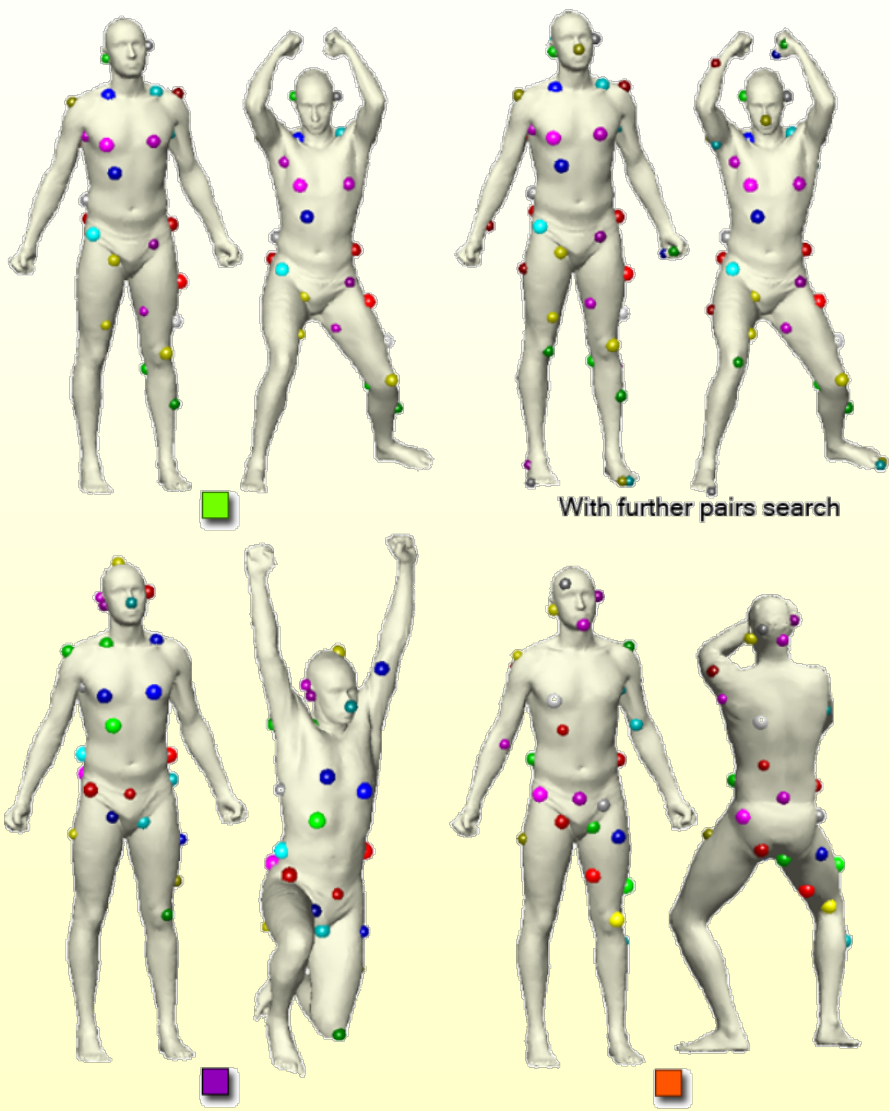
$$E(c) = \int_{\hat{c}} d(z, c(z)) d\mu$$

# Updating a Correspondence Matrix

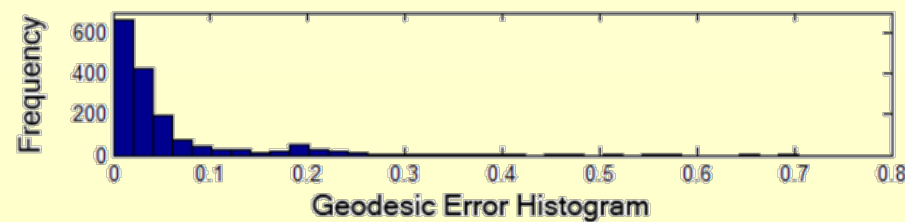
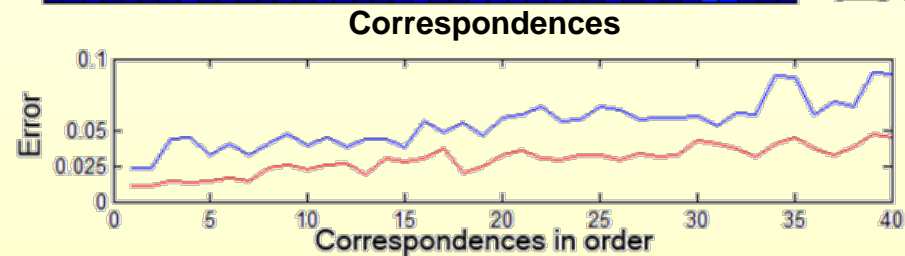
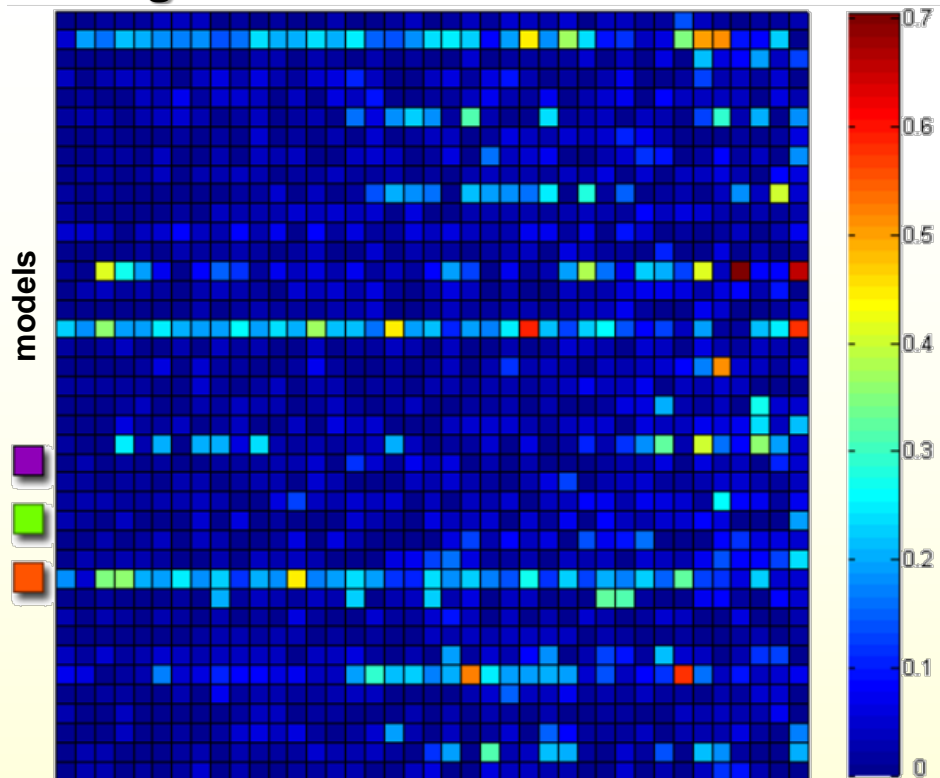


Use this  $C$  matrix to greedily extract correspondences and score them

# Results (SCAPE)



## Measuring deviation of correspondences from ground truth

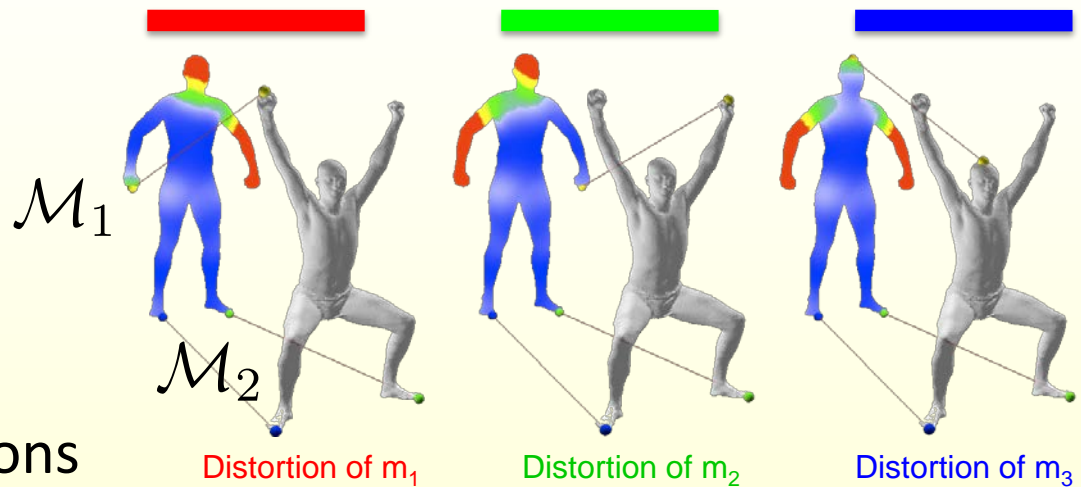


# Möbius Voting Summary

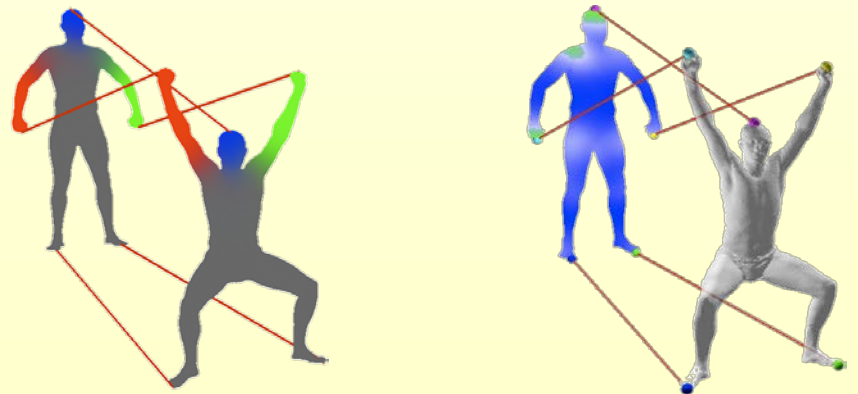
- ◆ RANSAC in the space is conformal maps -- works well for near isometric shapes
- ◆ Running time is  $\Theta(n^4 \log n)$  -- not great for more than >100 pts
- ◆ Limited to genus 0 surfaces

# Blended Intrinsic Maps

# Blended Intrinsic Maps



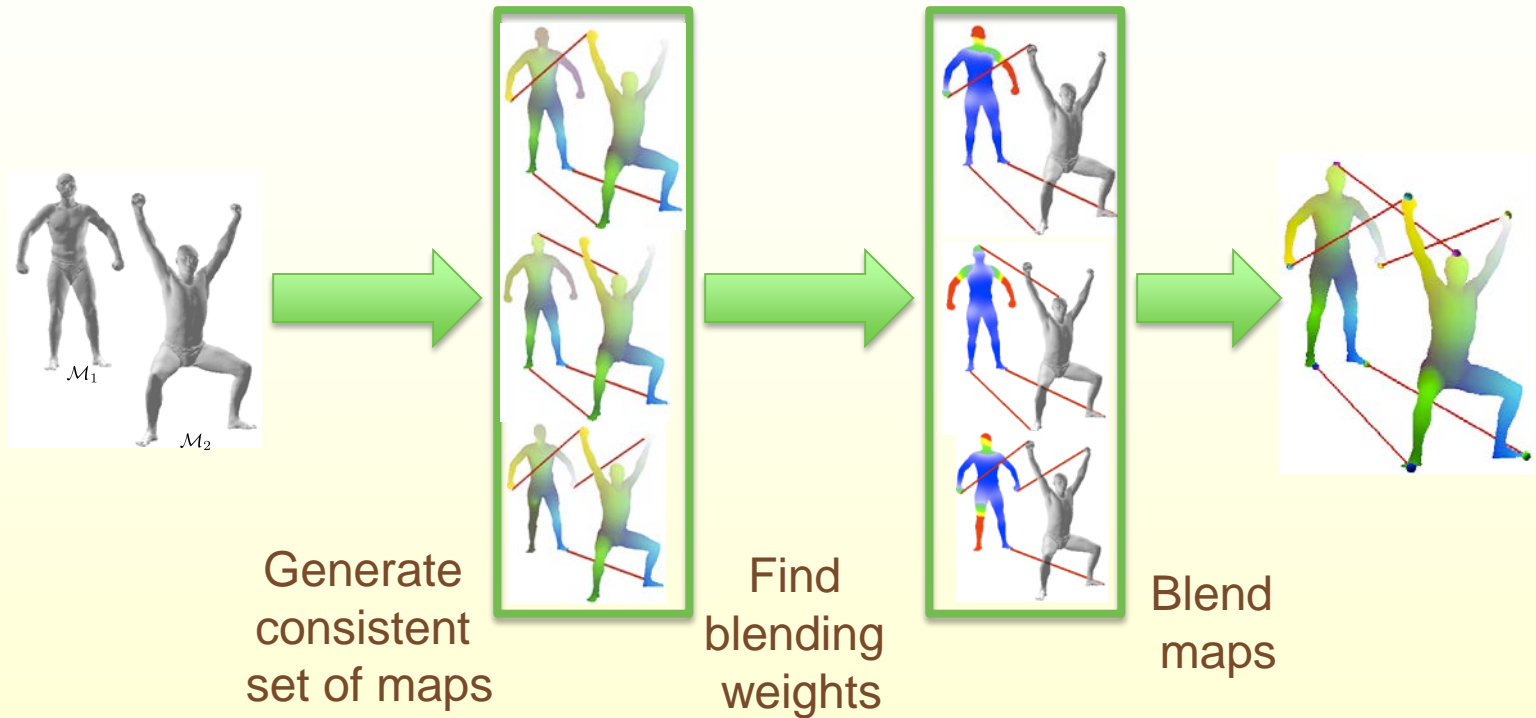
Take weighted combinations  
of Möbius maps



Blending weights for  $m_1, m_2$ , and  $m_3$

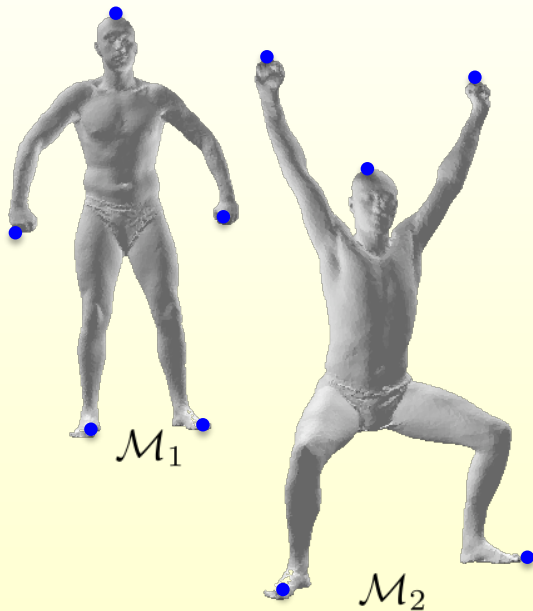
Distortion of the blended map <sup>87</sup>

# Pipeline



# Generating Consistent Maps

Generate a set of candidate conformal maps by enumerating triplets of feature points

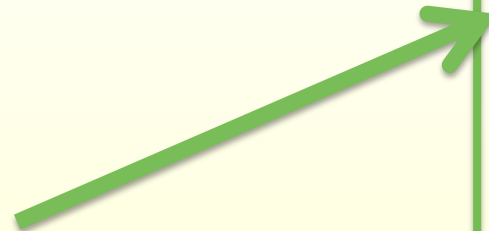
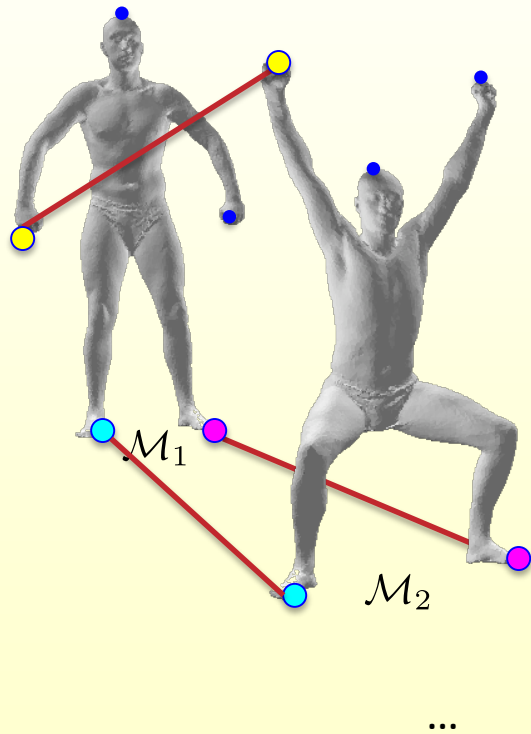


Set of candidate maps



# Generating Consistent Maps

Generate a set of candidate conformal maps by enumerating triplets of feature points



Set of candidate maps

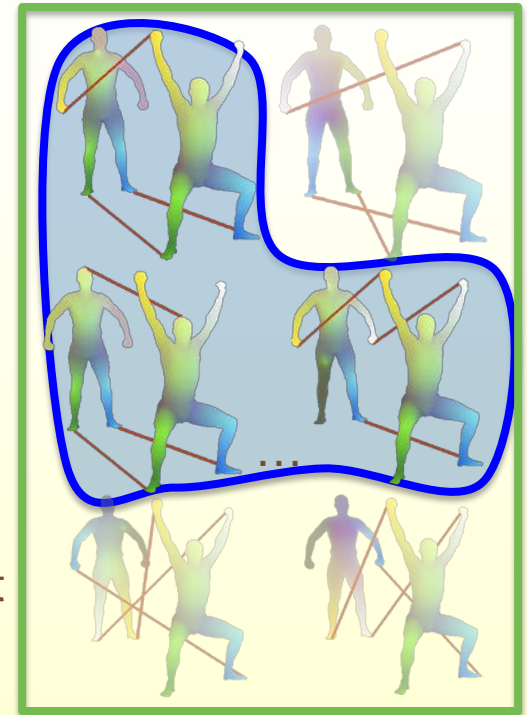


# Generating Consistent Maps

Find consistent set(s) of candidate maps



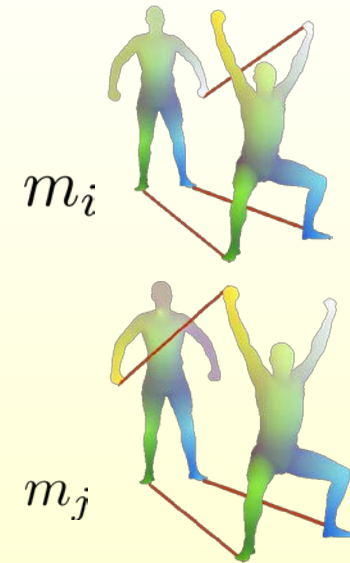
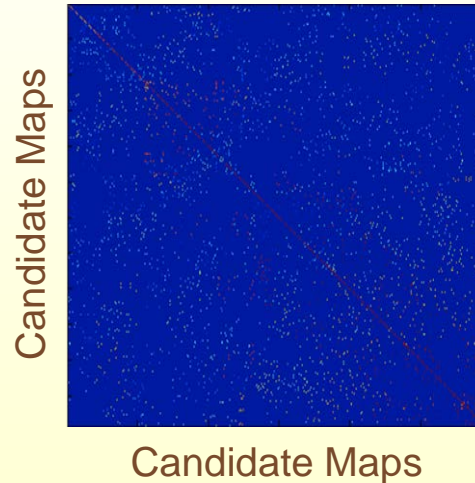
Set of consistent candidate maps



# Generating Consistent Maps

Define a matrix  $B$  where every entry  $(i,j)$  indicates the distortion of  $m_i$  and  $m_j$  as well as their pairwise similarity  $S_{i,j}$

$$B_{i,j} = \int_{M_1} c_i(p)c_j(p)S_{i,j}(p)dA(p)$$

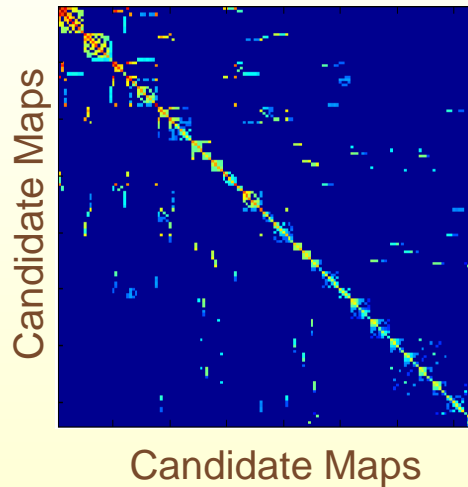


$c_i(p)$  “confidence” of map  $i$  at  $p$  – use conformal factor

$S_{i,j}(p)$  Agreement of maps  $i$  and  $j$  at  $p$  – use geodesic distance  $d(m_i(p), m_j(p))$

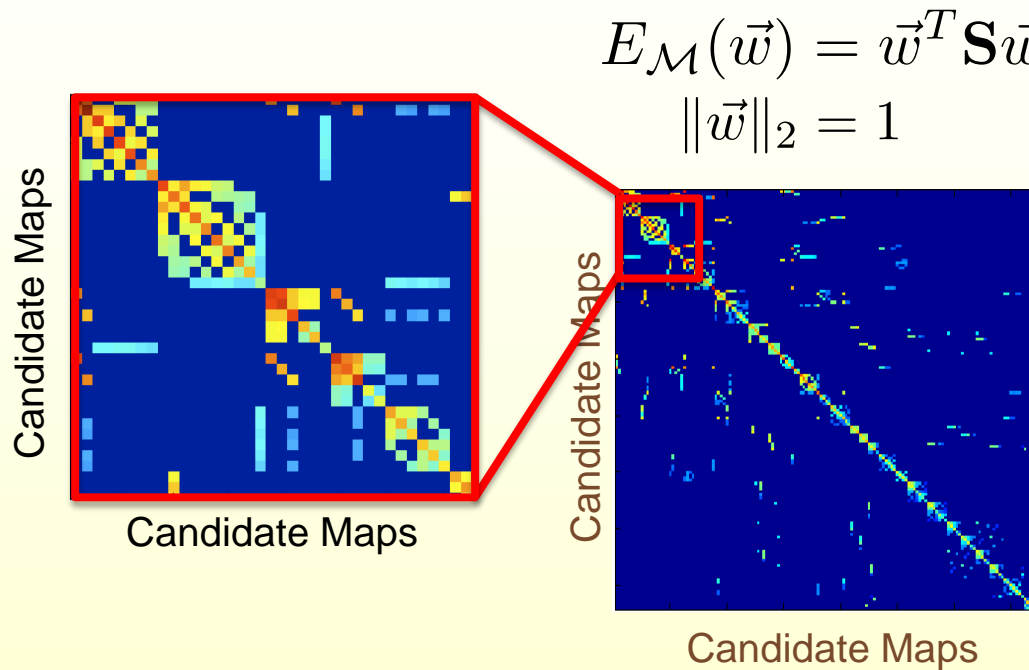
# Generating Consistent Maps

$$E_{\mathcal{M}}(\vec{w}) = \vec{w}^T \mathbf{S} \vec{w}$$
$$\|\vec{w}\|_2 = 1$$



Find blocks of low-distortion and mutually similar maps

# Generating Consistent Maps

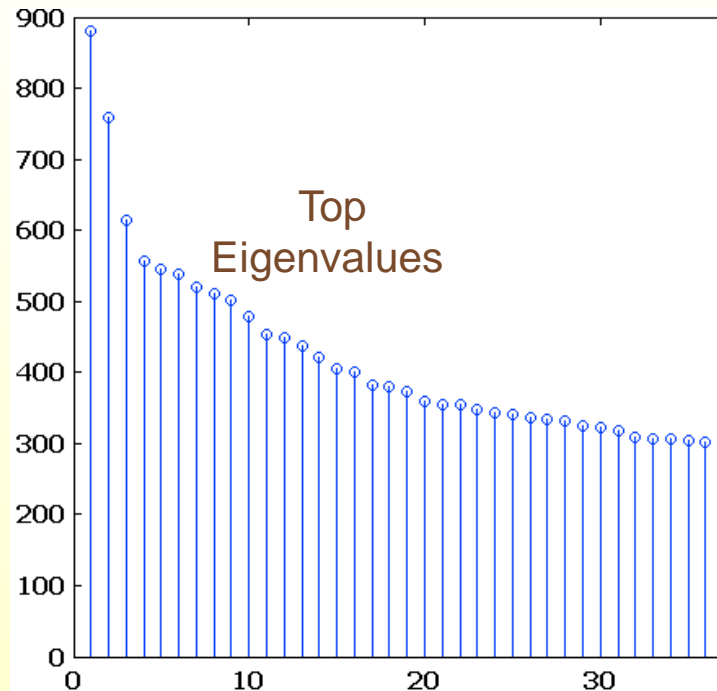
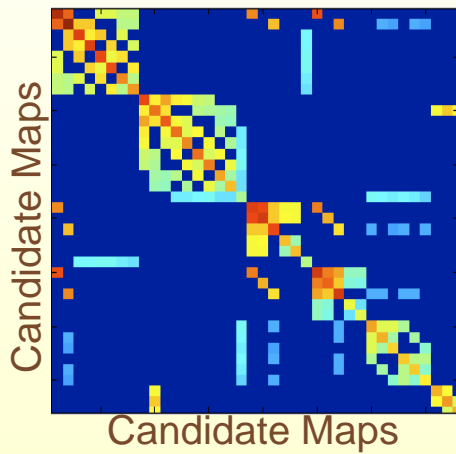


We use eigenanalysis,  
as in spectral clustering

Find blocks of low-distortion and mutually similar maps

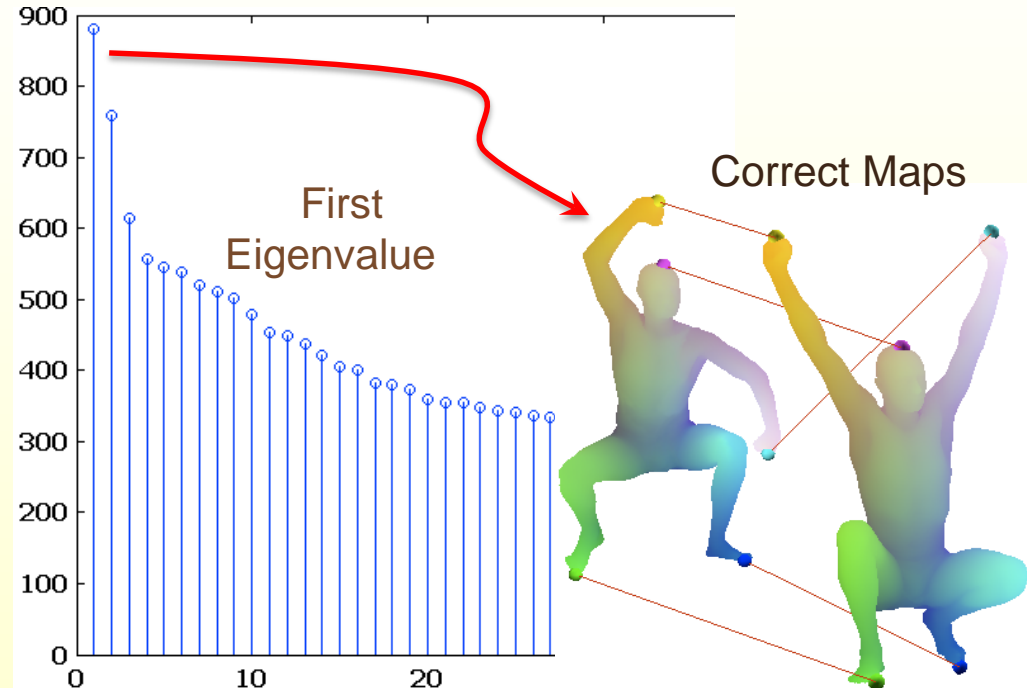
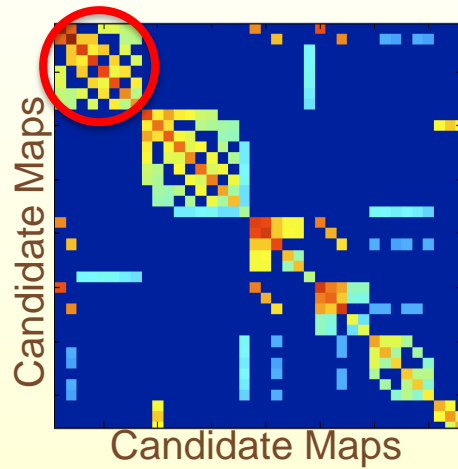
# Generating Consistent Maps

Eigenanalysis



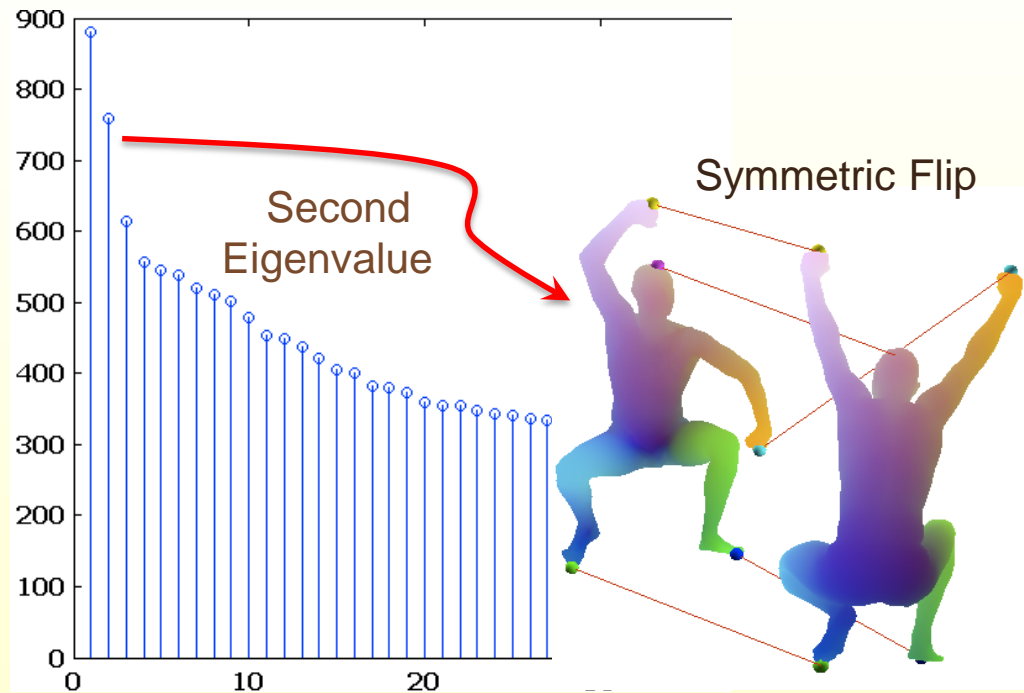
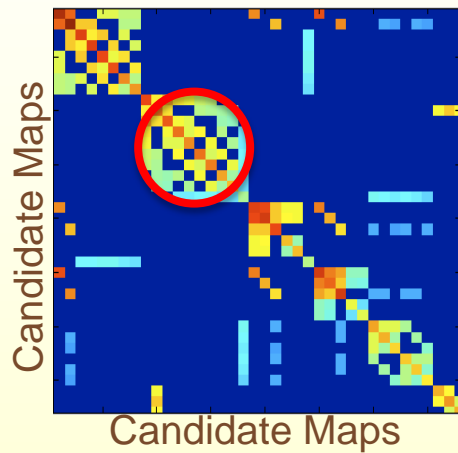
# Generating Consistent Maps

## Eigenanalysis

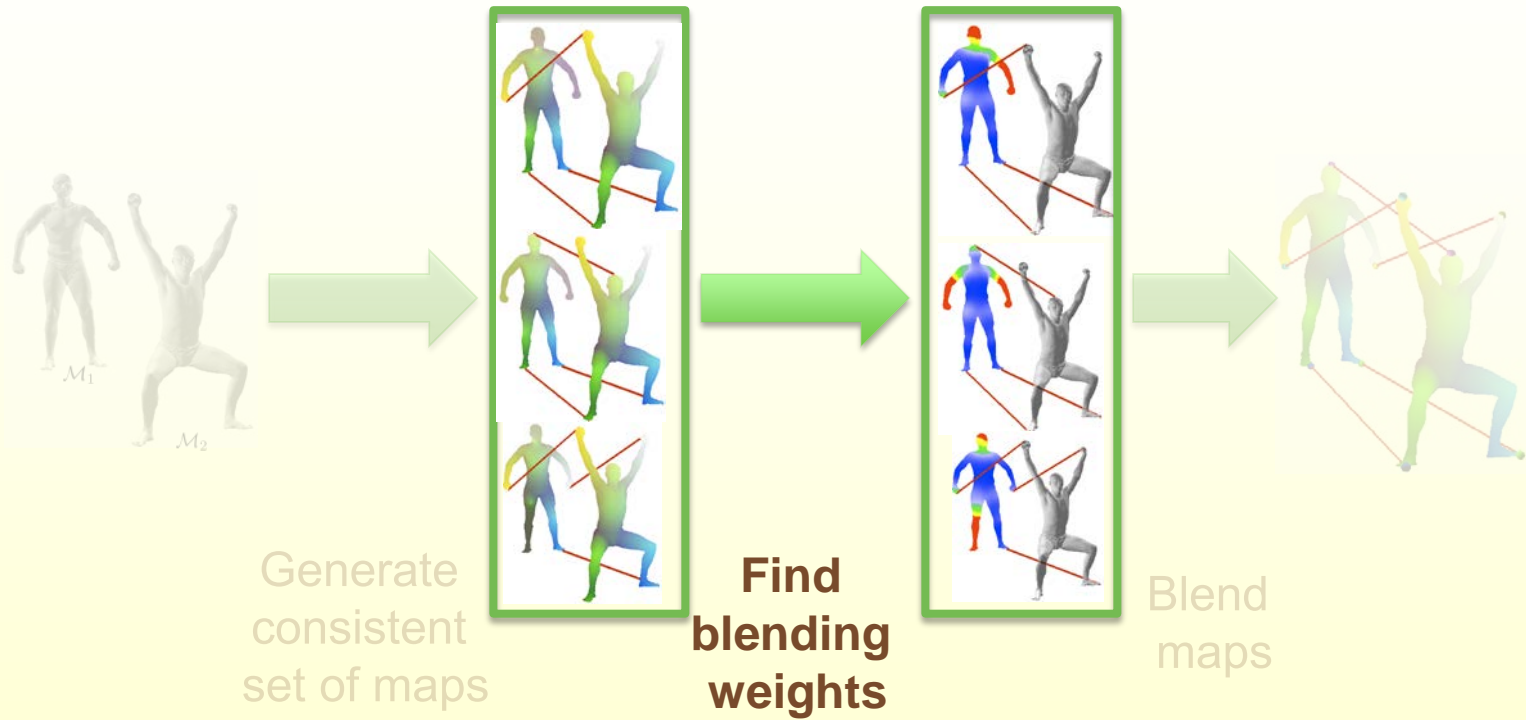


# Generating Consistent Maps

## Eigenanalysis

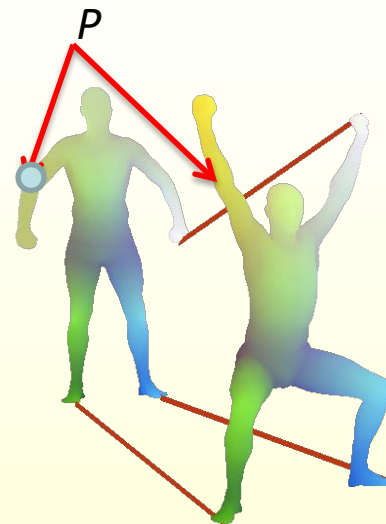


# Pipeline

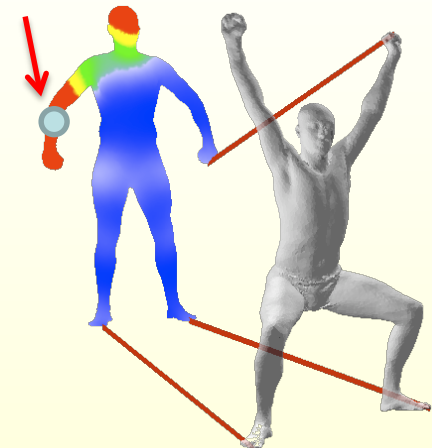


# Finding the Blending Weights

- For every **point**  $p$ 
  - Compute a weight of each map  $m_i$  at  $p$
- We model the weight with deviation from isometry
  - Area distortion for conformal maps

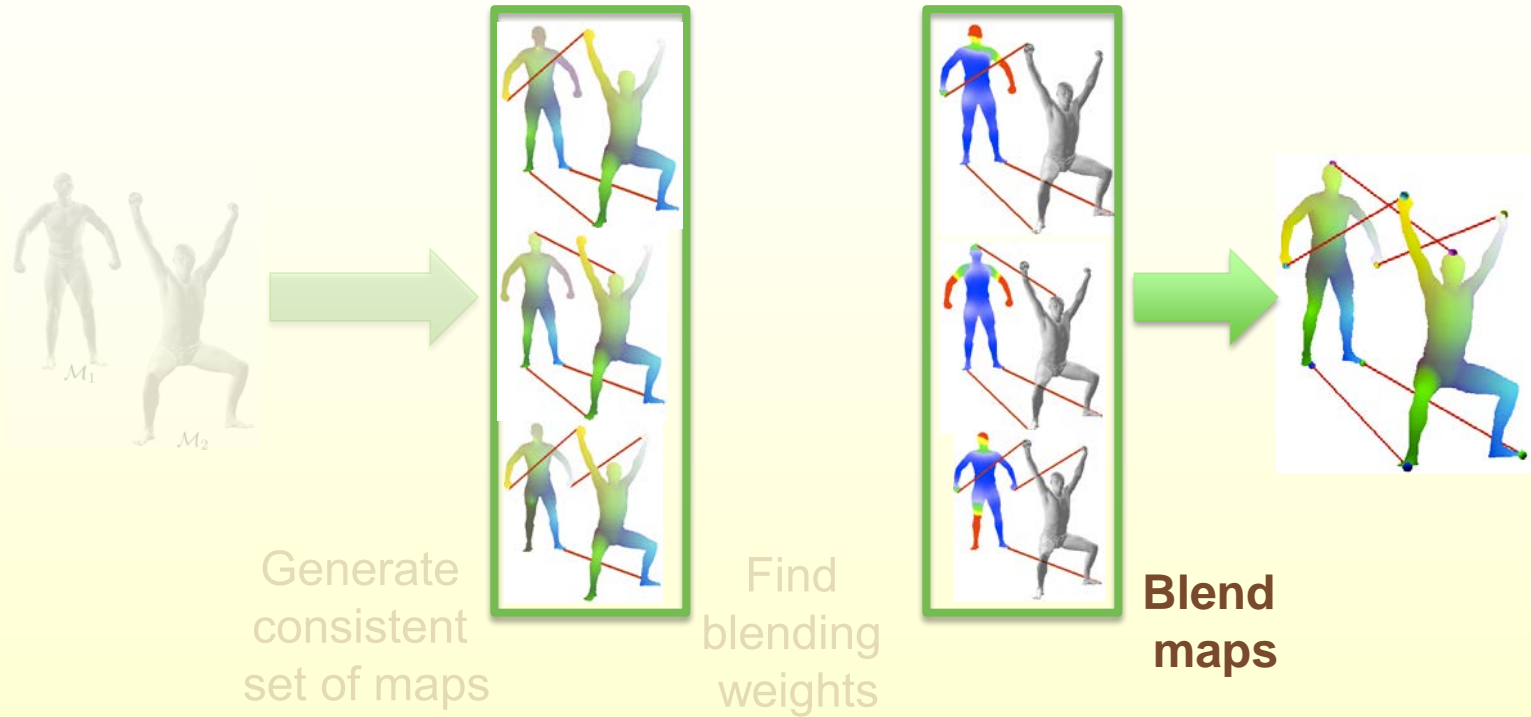


Candidate Map



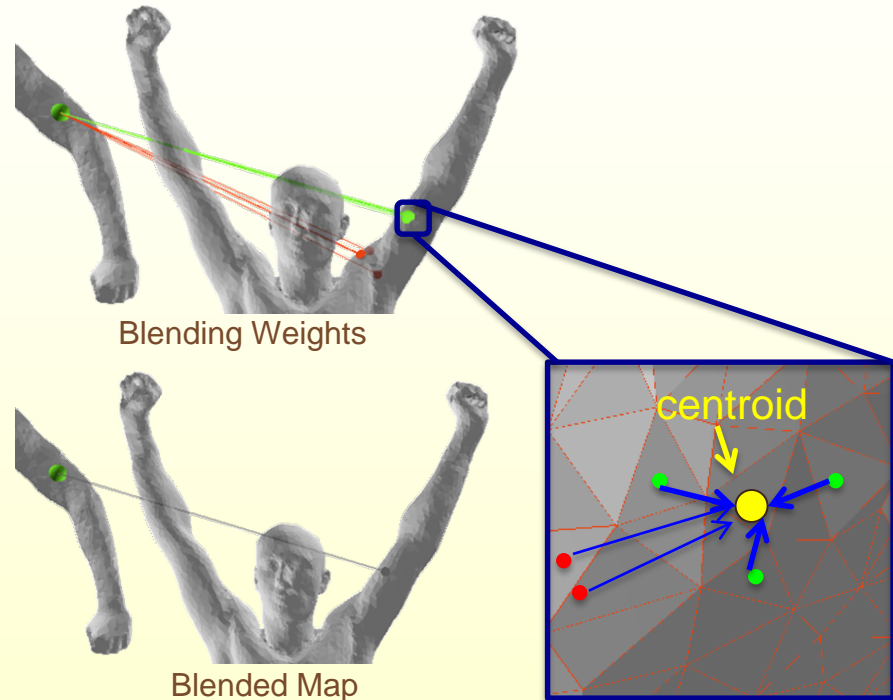
Blending Weight  
 $c_i(p)$

# Pipeline

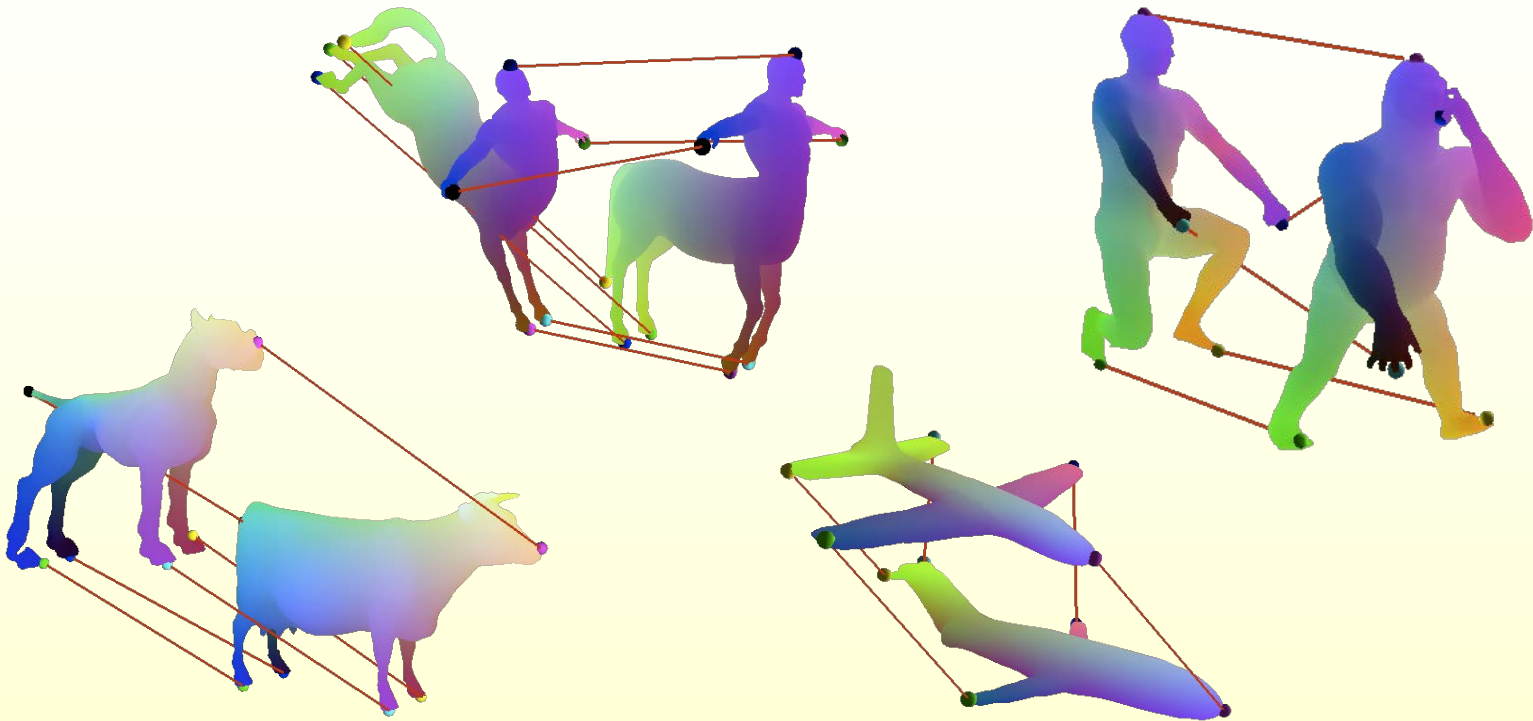


# Blending the Maps

- Input for each point  $p$ :
  - An image  $m_i(p)$  after applying each map  $m_i$
  - A blending weight for each map
- Output for each point:
  - Weighted **geodesic centroid** of  $\{ m_i(p) \}$

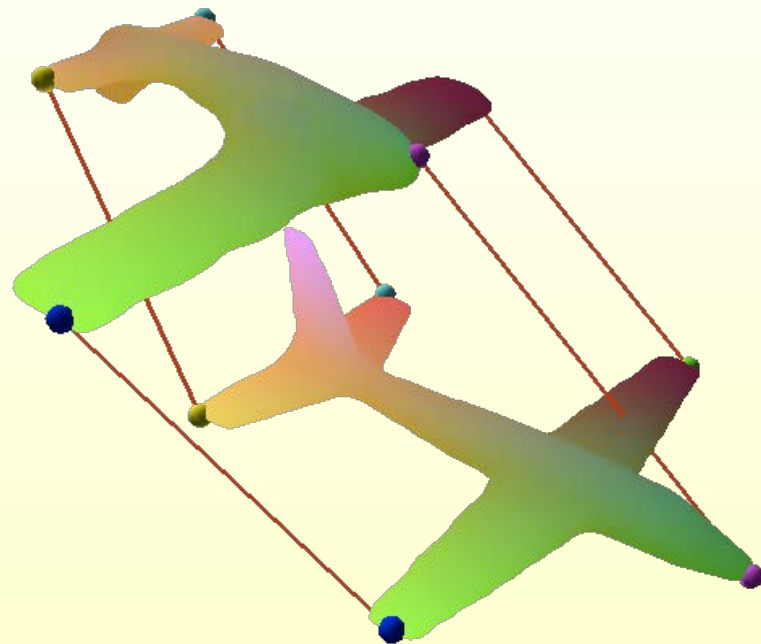


# Examples



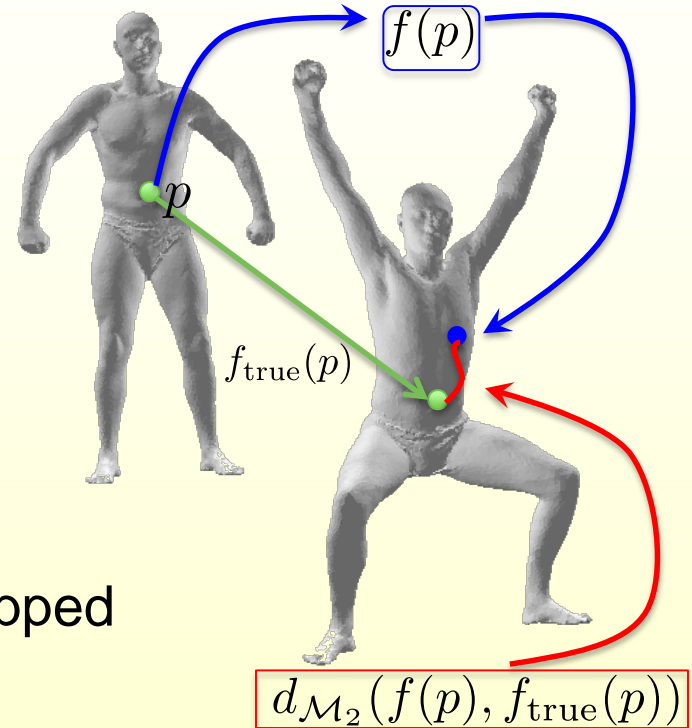
# Failures

Symmetric flip

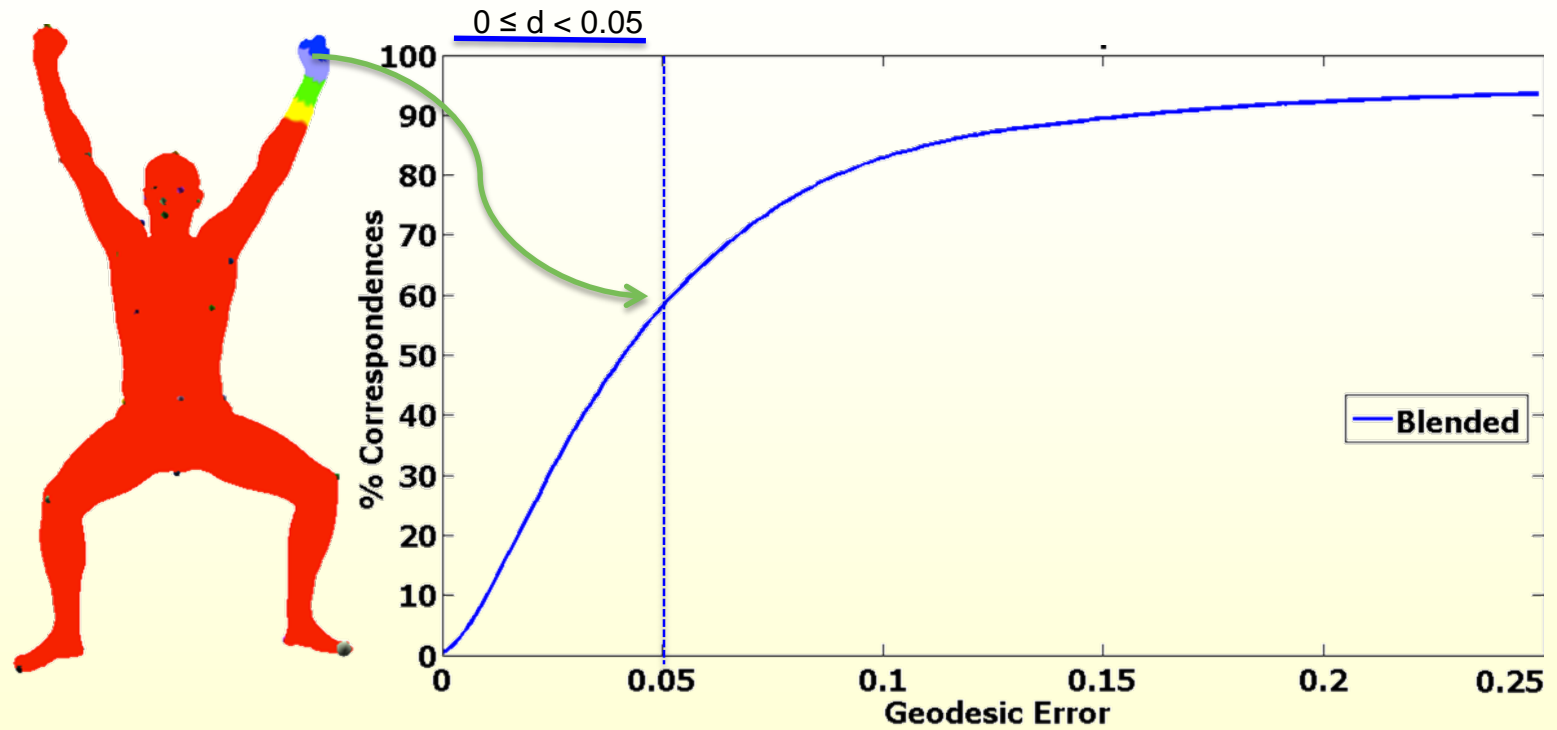


# Evaluation

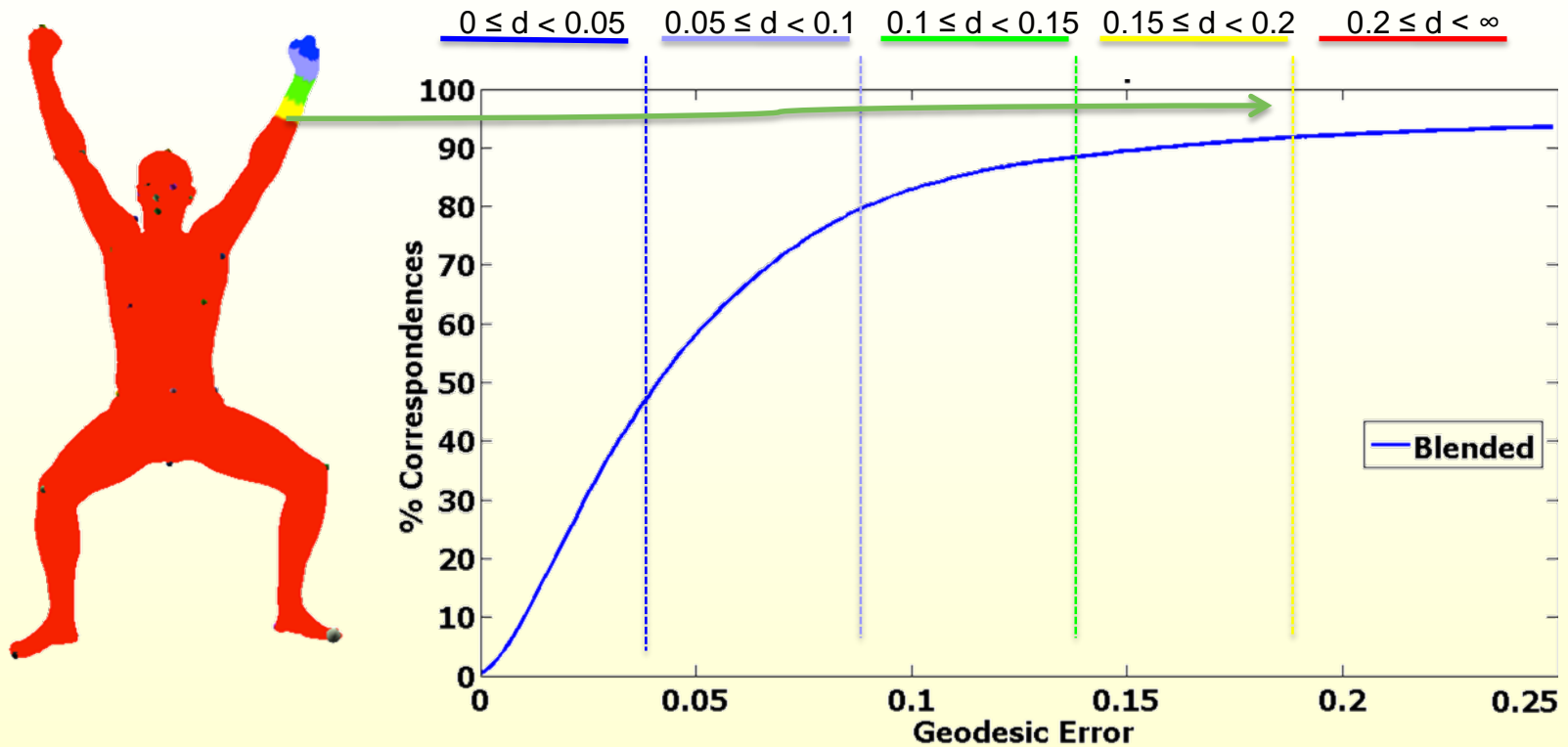
- Predict the map for every point with a ground truth correspondence
- Measure geodesic distance between prediction and the ground truth
- Record fraction of points mapped within geodesic error



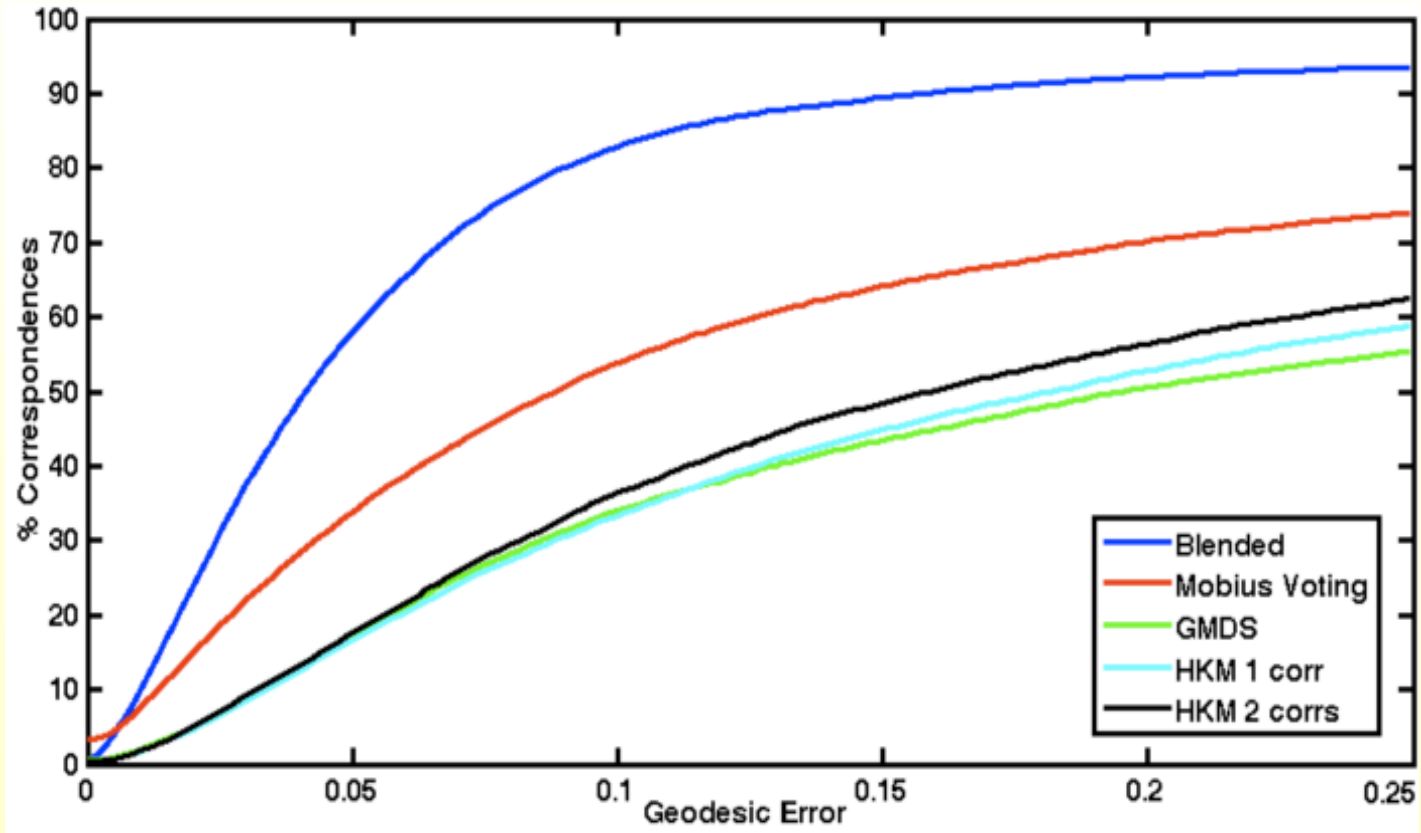
# Evaluation

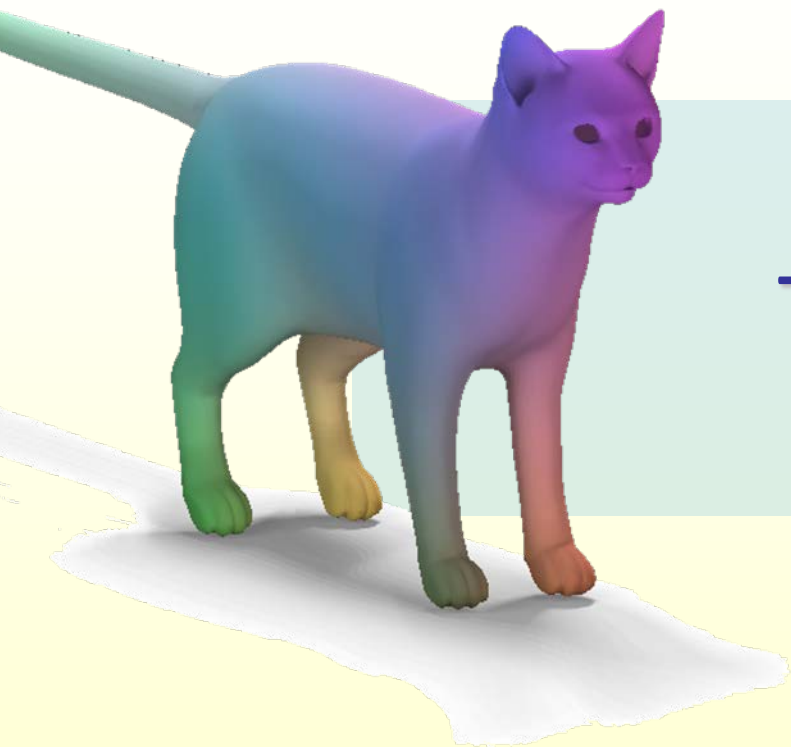


# Evaluation



# Performance





The End

