# Topology Control in Wireless Ad Hoc and Sensor Networks

PAOLO SANTI
Istituto di Informatica e Telematica

Topology Control (TC) is one of the most important techniques used in wireless ad hoc and sensor networks to reduce energy consumption (which is essential to extend the network operational time) and radio interference (with a positive effect on the network traffic carrying capacity). The goal of this technique is to control the topology of the graph representing the communication links between network nodes, with the purpose of maintaining some global graph property (e.g., connectivity) while reducing energy consumption and/or interference, which are strictly related to the nodes' transmitting range. In this paper, we state several problems related to topology control in wireless ad hoc and sensor networks, and we survey state-of-the-art solutions which have been proposed to tackle them. We also outline several directions for further research, which we hope will motivate researchers to undertake additional studies on this field.

Categories and Subject Descriptors: C.2.1 [**Computer Systems Organization**]: Computer-Communication Networks—*Network Architecture and Design: Wireless Communication*

General Terms: Algorithms, Design

Additional Key Words and Phrases: Connectivity, Energy consumption, Topology control, Sensor networks, Wireless ad hoc networks

## 1. INTRODUCTION

Recent emergence of affordable, portable, wireless communication and computation devices, and concomitant advances in the communication infrastructure, have resulted in the rapid growth of mobile wireless networks. On one hand, this has led to the exponential growth of cellular networks, which are based on the combination of wired and wireless technologies. On the other hand, this has renewed the interest of the scientific and industrial community in the more challenging scenario in which a group of mobile units equipped with radio transceivers communicate without the assistance of any fixed infrastructure.

Networks composed of mobile, untethered units communicating with each other via radio transceivers, typically along multi-hop paths, have been called *ad hoc networks* in the literature[1]. Ad hoc networks can be used wherever a wired backbone

[1] Sometimes, ad hoc networks are also called *packet radio networks*, which is the name used in the early papers in the field.

Author's address: P. Santi, Istituto di Informatica e Telematica del CNR, Via G.Moruzzi 1, 56124 Pisa – Italy. e-mail: paolo.santi@iit.cnr.it.

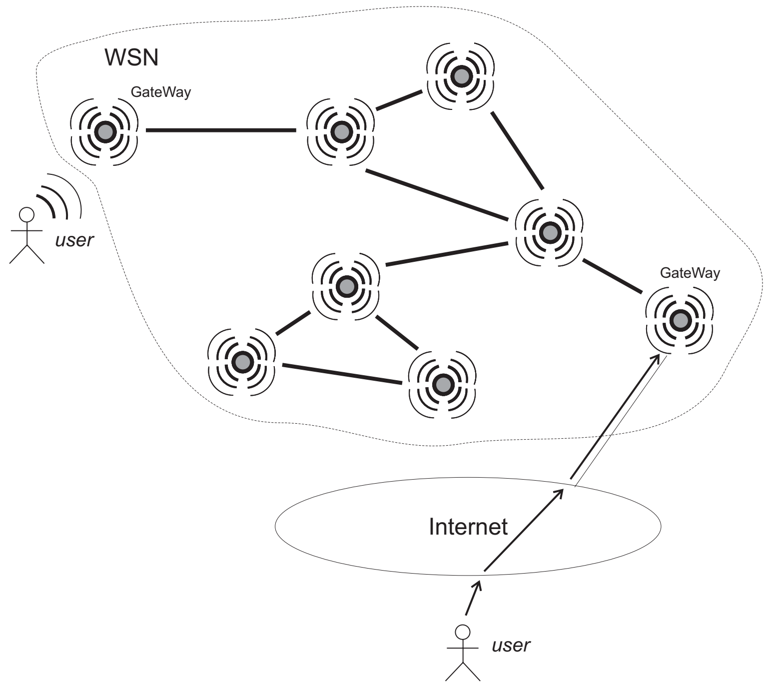


Fig. 1. Example of Wireless Sensor Network.

is infeasible and/or economically not convenient, e.g., to provide communications during emergencies, special events (expos, concerts, and so on) or in hostile environments.

Wireless sensor networks (WSNs for short) are a special class of ad hoc networks. In a WSN, the interconnected units are battery-operated microsensors, each of which is integrated in a single package with low-power signal processing, computation, and a wireless transceiver. Sensor nodes collect the data of interest (e.g., temperature, pressure, soil makeup, and so on), and transmit them, possibly compressed and/or aggregated with those of neighboring nodes, to the other nodes. This way, every node in the network acquires a global view of the monitored area, which can be accessed by the external user connected to the WSN through one or more gateway nodes (see Figure 1). Potential applications of sensor networks abound: they can be used to monitor remote and/or hostile geographical regions, to trace animals movement, to improve weather forecast, and so on. Examples of scenarios where WSN can be used are described in [Estrin et al. 1999; Heinzelman et al. 1999; Khan et al. 2000; Mainwaring et al. 2002; Pottie and Kaiser 2000; Sadler et al. 2004; Schwiebert et al. 2001; Srivastava et al. 2001; Steere et al. 2000; Szewczyk et al. 2004].

The following aspects, which have to be carefully taken into account in the design stage, are peculiar of wireless ad hoc networks:

– *energy conservation*: contrary to the case of wired networks, units in ad hoc networks are typically equipped with limited energy supplies. Hence, one of the primary goals of the design is to use this limited energy as efficiently as possible. Energy efficiency is especially important in WSNs, where replacing/refilling sensor batteries is in general infeasible. If energy conservation techniques are used at different levels of the wireless architecture, the functional lifetime of both individual units and the network can be extended considerably.

– *limited bandwidth*: typically wireless multi-hop networks are characterized by a limited bandwidth available to the nodes: although the theoretical bandwidth in industrial standards such as IEEE 802.11 can be as high as 54 Mb/sec [IEEE 1999], the situation is far worse in practical situations, mainly because of the radio interference caused by simultaneous communications. Thus, a major problem in the design of ad hoc networks is to keep the network traffic carrying capacity at a reasonable level, even in presence of dense node deployments.

– *unstructured and time-varying network topology*: nodes in the network may in principle be arbitrarily placed in the deployment region; hence, the graph representing the communication links between the nodes is usually unstructured. Furthermore, due to node mobility and/or failure, the network topology may vary with time. As a consequence, determining the appropriate value of fundamental network parameters (e.g., the critical transmitting range for connectivity – see Section 5.1) is a difficult task.

– *low-quality communications*: communication on wireless channels is in general much less reliable than in wired channels. Furthermore, the quality of communication is strongly influenced by environmental factors, which can be time-varying. Considering that ad hoc networks, and especially WSNs, are likely to be deployed in hostile environments, low communication quality is to be expected in general, with non-negligible off-service time intervals.

In case of WSNs, the following aspects must also be considered:

– *operation in hostile environments*: as in many scenarios WSNs are expected to operate in hostile environments, sensors must be explicitly designed to work under extreme conditions, which may make individual unit failure a likely event. Hence, resilience to sensor faults must be explicitly addressed at different network layers.

– *data processing*: given the energy constraints and the expected poor communication quality, sensed data must be compressed and/or aggregated with data of neighboring sensors before sending them to the gateway node(s).

– *scalability*: depending on the scenario considered, WSNs might be composed of several thousands of sensors. Thus, the scalability of the proposed protocols is an important issue.

Several solutions have been proposed in the literature that address at least some of the issues raised above. In particular, great efforts have been devoted to the design of energy-efficient and mobility resilient routing, broadcast and multicast protocols [Basagni et al. 1999; Gerla and Tsai 1995; Ko and Vaidya 1998; Michail and Ephremides 2003; Murthy and Garcia-Luna-Aceves 1996; Papadimitriou and Georgiadis 2004; Rajaraman 2002; Seada et al. 2004].

Routing and broadcast protocols are usually concerned with energy-efficient message delivery on a given communication graph, which is considered as an input to the protocol. However, contrary to the case of wired networks, the network topology in wireless networks is not fixed, and can be changed by varying the nodes' transmitting range. So, further energy can be saved if the network topology used to route/broadcast messages is energy-efficient itself. The goal of *topology control* is to dynamically change nodes' transmitting range in order to maintain some property of the communication graph (e.g., connectivity), while reducing the energy consumed by node transceivers (which is strictly related to the transmitting range). Since transceivers are one of the primary source of energy consumption in the wireless unit, especially in WSNs, topology control mechanisms are fundamental to achieve a good network energy efficiency.

Besides reducing energy consumption, topology control has the positive effect of reducing contention when accessing the wireless channel. In general, when the nodes' transmitting ranges are relatively short, many nodes can transmit simultaneously without interfering with each other, and the network capacity is increased. Ideally, the nodes' transmitting range should be set to the minimum value such that the graph that represents the communication links between units is connected. How to compute this value, under different hypotheses on the initial node distribution, presence and type of mobility, and so on, is the matter of this survey.

Before proceeding, some observations regarding terminology are in order. The term 'topology control' has been used with at least two different meanings in the ad hoc and sensor networks literature. For instance, several authors consider as topology control also techniques aimed at super-imposing a hierarchy on an otherwise flat network organization in order to reduce, typically, energy consumption. This is the case, for instance, or clustering algorithms, which select some of the nodes in the network as clusterheads, whose purpose is to optimize energy and communication efficiency in their cluster. Although, in a sense, clustering algorithms can be seen as a way of 'controlling' the network topology, they cannot be classified as topology control mechanisms according to the informal definition above, since typically the transmit power of the nodes is not modified by a clustering algorithm.

Also, the terms 'power control' and 'topology control' are often confused with each other in the current literature. In our view, we classify as 'power control' those techniques that, by acting on the transmit power level of the nodes, aim at optimizing a single wireless transmission. Although this transmission might in general be multi-hop, the focus of power control is on the efficiency of a single (possibly multi-hop) wireless channel. Again, this feature of power control does not fulfill our informal definition of topology control, in which nodes adjust their transmitting range in order to achieve a certain *network-wide* target goal (e.g., network connectivity).

The rest of this paper is organized as follows. In Section 2 we introduce a simplified but widely accepted model of wireless ad hoc network, which will be used in the rest of the paper. In Section 3 we propose a taxonomy to classify the many approaches to the topology control problem appeared in the literature. In Section 4, we review the probabilistic theories that have been used in the derivation of theoretical results concerning topology control. In Section 5, we introduce several problems related to topology control in stationary networks, and we survey state-of-the-art solutions which have been proposed to tackle them. In Section 6, we will discuss how does node mobility affect the picture drawn in Section 5. Finally, in Section 7 we outline several directions for further research.

## 2. A WIRELESS AD HOC NETWORK MODEL

In this section, we introduce a simplified but widely accepted model of wireless ad hoc network, which will be used in the definition of the various problems related to topology control considered in the literature.

The node configuration of a $d$-dimensional mobile wireless ad hoc network, with $d=1,2,3$, is represented by a pair $M_d=(N,P)$, where $N$ is the set of nodes, with $|N| = n$, and $P: N \times T \rightarrow [0,l]^d$, for some $l > 0$, is the placement function. The placement function assigns to every element of $N$ and to any time $t \in T$ a set of coordinates in the $d$-dimensional cube of side $l$, representing the node's physical position at time $t$. The choice of limiting the admissible physical placement of nodes to a bounded region of $\mathbb{R}^d$ of the form $[0,l]^d$, for some $l > 0$, is realistic and eases the treatment of some of the problems considered in the following.

Node $i \in N$ is said to be *stationary* if its physical placement does not vary with time. If all the nodes are stationary, the network is said to be stationary, and function $P$ can be represented simply as $P: N \rightarrow [0,l]^d$.

A *range assignment* for a $d$-dimensional node configuration $M_d = (N,P)$ is a function $RA: N \rightarrow (0, r_{max}]$ that assigns to every element of $N$ a value in $(0, r_{max}]$, representing its transmitting range. Parameter $r_{max}$ is called the *maximum transmitting range* of the nodes in the network and depends on the features of the radio transceivers equipping the mobile nodes. A common assumption is that all the nodes are equipped with transceivers having the same features; hence, we have a single value of $r_{max}$ for all the nodes in the network.

It is known [Rappaport 2002] that the power $p_i$ required by node $i$ to correctly transmit data to node $j$ must satisfy inequality

$$\frac{p_i}{\delta_{i,j}^{\alpha}} \geq \beta \ , \qquad (1)$$

where $\alpha \geq 2$ is the *distance-power gradient*, $\beta \geq 1$ is the *transmission quality* parameter, and $\delta_{i,j}$ is the Euclidean distance between the nodes. While the value of $\beta$ is usually set to 1, the value of $\alpha$ depends on environmental conditions. In the ideal case, we have $\alpha = 2$; however, $\alpha$ is typically 4 in realistic situations. A value of $\alpha$ in the interval $[2,6]$ is commonly accepted. Given the formula above, we can define the *energy cost* of a range assignment $RA$ as $c(RA) = \sum_{i \in N} (RA(i))^{\alpha}$.

Formula (1) holds for free-space environments with non-obstructed line of sight, and it does not consider the possible occurrence of reflections, scattering and diffraction caused by buildings, terrain, and so on. Although more complicated formulae of the radio signal attenuation with distance are known, such as that recently derived in [Bruck et al. 2002], inequality (1) is widely accepted in the ad hoc network community.

Note that inequality (1) accounts only for the power consumed by the sender node (transmit power). In practice, in a radio communication a non-negligible amount of energy is consumed also at the receiver node to receive and decode the transmitted signal. Most of current literature does not account for the receiver energy, and the design of topology control protocols based on more realistic energy models is one of the main open issues in the field (see Section 7).

Given a node configuration $M_d = (N, P)$ and a range assignment $RA$, the *communication graph* induced by $RA$ on $M_d$ at time $t$ is defined as the directed graph $G_t = (N, E(t))$, where the directed edge $[i, j]$ exists if and only if $RA(i) \geq \delta_{P(i,t),P(j,t)}$. In words, the directed edge $[i, j]$ exists if and only if nodes $i$ and $j$ are at distance at most $RA(i)$ at time $t$. In this case node $j$ is said to be a *neighbor* of $i$. A range assignment $RA$ is said to be *connecting at time t* if the resulting communication graph at time $t$ is strongly connected[2]. If the network is stationary, we simply say that the range assignment $RA$ is connecting. A range assignment in which all the nodes have the same transmitting range $r$, for some $0 < r \leq r_{max}$, is called *r-homogeneous* range assignment[3]. Observe that the communication graph generated by a homogeneous range assignment can be considered as undirected, since $[i, j] \in E(t) \Leftrightarrow [j, i] \in E(t)$.

In general, the range assignment may vary with time in order to ensure target properties (e.g., strong connectivity, a given network diameter $h < n$) of the communication graph. Hence, a sequence of range assignments $RA_{t_1}, RA_{t_2}, \ldots$ can be defined, where $RA_{t_i}$ is the range assignment at time $t_i$, and the transition between range assignments is determined by the topology control mechanism.

The communication graph as defined here is essentially the *point graph* model introduced in [Sen and Huson 1996], but it is more often called the *unit disk graph* model in the TC literature. If node positions are chosen according to some probability distribution, the point graph model coincides with the concept of Random Geometric Graph (RGG), which is a generalization of the notion of Random Graph introduced in the applied probability community (see Section 4 for details).

The main weakness of the point graph model is the assumption that the radio coverage area is a perfect circle. This assumption is quite realistic in open air flat environments, but it is critical in indoor or urban scenarios, where the presence of objects, walls, buildings, and so on, renders the radio coverage area extremely irregular. Further, the area and shape of the radio coverage is influenced by weather conditions and by the interference with pre-existing infrastructure (e.g., power lines, base stations, and so on). Including all these details in the network model would make it extremely complicated and scenario-dependent, hampering the derivation of meaningful and sufficiently general analytical results. For this reason, the point graph model described above, although quite simplistic, is widely used in the analysis of ad hoc networks.

## 3. A TAXONOMY OF TOPOLOGY CONTROL

In this section we try to organize the various approaches to the topology control problem appeared in the literature into a coherent taxonomy.

Our taxonomy is depicted in Figure 2. The first distinction is between *homogeneous* and *non-homogeneous* approaches. In the former case, which is the simpler (and easier to analyze) type of TC, nodes are assumed to use the same transmit-

[2]A directed graph $G = (N, E)$ is strongly connected if and only if, for any two nodes $u, v \in N$, there exists a directed path from $u$ to $v$ in $G$.

[3]When the value of $r$ is not relevant, the $r$-homogeneous range assignment is simply called homogeneous range assignment.

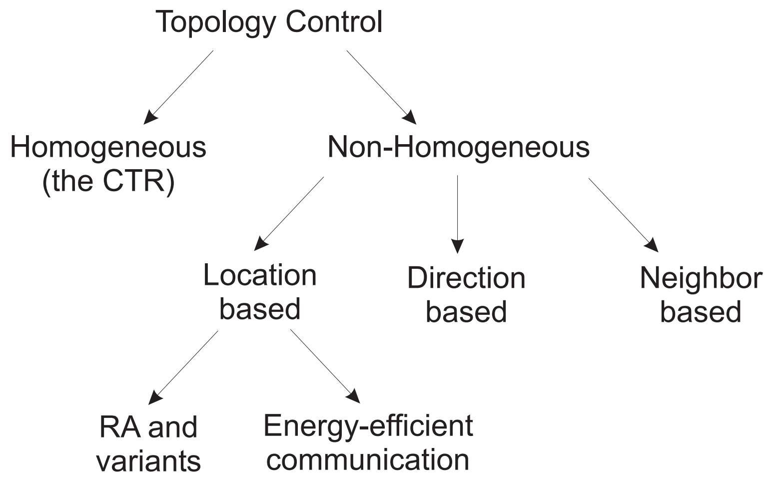


Fig. 2. A taxonomy of topology control techniques.

ting range, and the topology control problem reduces to the one of determining the minimum value of $r$ such that a certain network-wide property is satisfied (the Critical Transmitting Range). In the latter case, nodes are allowed to choose different transmitting ranges (provided they do not exceed the maximum range).

Non-homogeneous topology control is classified into three categories, depending on the type of information which is used to compute the topology. In location-based approaches, exact node positions are known. This information is either used by a centralized authority to compute a set of transmitting range assignments which optimizes a certain measure (this is the case of the Range Assignment problem and its variants) , or it is exchanged between nodes and used to compute an "almost optimal" topology in a fully distributed manner (this is the case of protocols for building energy-efficient topologies for unicast or broadcast communication). In direction-based approaches, it is assumed that nodes do not know their position, but they can estimate the relative direction of each of their neighbors. Finally, in neighbor-based techniques, nodes are assumed to know only the ID of the neighbors, and to be able to order them according to some criterion (e.g., distance, or link-quality).

Besides classifying topology control approaches based on the constraints we put on the range assignment (homogeneous or non-homogeneous), and on the type of information which is available to the network nodes, we can also distinguish the various approaches proposed in the literature based on the properties of the network topology resulting from the application of topology control techniques.

Most of the approaches presented in the literature are concerned with building and maintaining a *connected* network topology, as network partitioning is highly undesirable. More recently, some authors have considered the problem of building a $k$-connected network topology (with $k > 1$), i.e. a topology in which there exist at least $k$ distinct paths between any two network nodes. Guaranteeing $k$-connectivity of the communication graph is fundamental in all those applications in which a certain degree of *fault-tolerance* is needed: since there exist at least $k$ paths between any two network nodes, network connectivity is guaranteed in presence of up to $k-1$ node failures. Other authors have recently considered also the topology control problem in a context (typical of wireless sensor networks) in which nodes alternate between active and sleeping times, and the goal is to build a network topology such that the subnetwork composed of the active nodes is connected at any time (see Section 5.1.4).

## 4. PROBABILISTIC TOOLS

Some of the analytical results presented in this paper are based on a probabilistic approach. In this Section, we survey the probabilistic theories that have been used to derive them.

The main difficulty that arises in the probabilistic analysis of wireless ad hoc networks is that the well-established theory of random graphs [Bollobás 1985; Palmer 1985] cannot be used. In fact, a fundamental assumption in this model is that the probabilities of edge occurrence in the graph are independent, which is not the case in wireless ad hoc networks. As an example, consider three nodes $i, j, k$ such that $\delta_{i,j} < \delta_{i,k}$. With common wireless technologies that use omni-directional antennas, and disregarding the effect of shadowing and fading on radio signal propagation, if $i$ has a link to $k$, then it has also a link to $j$. Hence, the occurrences of edges $(i, j)$ and $(i, k)$ are correlated.

In order to circumvent this problem, Chlamtac and Faragó [Chlamtac and Faragó 1999] introduced the *Random Network* (RN) model as a generalization of the uniform random graph model, in which graphs are selected according to a more general probability distribution. We recall that in the uniform random graph model, each element of a given class of graphs with $n$ vertices is assigned an equal probability of being chosen. Examples of uniform models are random graphs with a given number $m$, with $0 \leq m \leq \binom{n}{2}$, of edges, random trees, random $k$-regular graphs, and so on. In the RN model, graphs can be chosen according to an arbitrary non-degenerate distribution, where a non-degenerate distribution is a distribution that does not concentrate (in a probabilistic sense) on a class of graphs of relatively small size. Based on the RN model and using the theory of Kolmogorov complexity, Chlamtac and Faragó analyzed the performance of a randomized distributed algorithm aimed at connecting clusterheads in a Virtual Cellular Architecture. The authors claim that the RN model, relying on an arbitrary non-degenerate probability distribution, can account for correlations between edges, which were not allowed in the uniform model. Unfortunately, the actual probability distribution of the graphs describing ad hoc networks might be degenerate. In fact, the actual distribution is the uniform distribution over the class of point graphs, which is degenerate if the size of the class of point graphs is relatively small as compared to the class of all possible graphs. Since the class of point graphs has not yet been characterized, its size is unknown, and determining whether this distribution is degenerate or not is still an open problem.

A more recent theory, which is still in development, is the theory of *geometric random graphs* (GRG). In the theory of GRG, a set of $n$ points is distributed according to some density in a $d$-dimensional region $R$, and some property of the resulting node placement is investigated. For example, the longest nearest neighbor link [Penrose 1999a], the longest edge of the Euclidean Minimum Spanning Tree (MST) [Penrose 1999c; 1997], and the total cost of the MST have been investigated [Aldous and Steele 1992; Steele 1988; Yukich 2000]. For a survey of GRG, the reader is referred to [Diaz et al. 2000]. Recently, several papers [Bettstetter 2002b; Blough et al. 2002; Panchapakesan and Manjunath 2001; Santi 2005; Wan and Yi 2004; Yi et al. 2003; Yi and Wan 2005] have used the theory of GRG to analyze fundamental properties (typically, connectivity) of wireless ad hoc networks.

Two others theories have been used in the probabilistic analysis of ad hoc networks: the theory of *continuum percolation* and the *occupancy theory*.

In the theory of continuum percolation [Meester and Roy 1996], nodes are assumed to be distributed with Poisson density $\lambda$ in $\mathbb{R}^2$, and two nodes are connected to each other if the distance between them is at most $r$. It has been proven that for each $\lambda > 0$, there exists at most one infinite-order component with high probability. However, the existence of an infinite-order component is not sufficient to ensure the connectivity of the network. In fact, there could exist (infinitely many) nodes which do not belong to the giant component, thus leading to a disconnected communication graph. Hence, the quality of connectivity is related to the fraction $\theta$ of nodes belonging to the giant component [Janson et al. 1993], which in turn depend on the *percolation probability*. The percolation probability is the probability that an arbitrary node belongs to a connected component of infinite order. The main result of the theory of continuum percolation is that there exists a finite, positive value $\lambda_c$ of $\lambda$, called *critical density*, under which the percolation probability is zero, and above which it is non zero. However, no explicit expression of the percolation probability is known to date. The theory of continuum percolation have been used in [Dousse et al. 2002; Gupta and Kumar 1998] to analyze the connectivity of ad hoc networks.

In the occupancy theory [Kolchin et al. 1978], it is assumed that $n$ balls are thrown independently at random into $C$ cells. The allocation of balls into cells can be characterized by means of random variables describing some property of the cells. The occupancy theory is aimed at determining the probability distribution of such variables as $n$ and $C$ grow to infinity (i.e., the *limit distribution*). The most studied random variable is the number of empty cells after all the balls have been thrown, which we denote $\mu(n, C)$. Of course, the limit distribution of $\mu(n, C)$ depends on the relative magnitude of $n$ and $C$, i.e., on the asymptotic behavior of $\rho = n/C$. Depending on the asymptotic behavior of $\rho$, five domains such that $n, C \to \infty$ for which the limit distribution of $\rho(n, C)$ is different have been determined. Depending on the domain, the limit distribution can be either Poisson or Normal with different parameters. The occupancy theory can be used to analyze connectivity in ad hoc networks by subdividing the deployment region $R$ into equal subregions (cells) of size $\approx r^d$, and by determining under which conditions all the cells are filled with at least one node (ball). This technique has been used in [Santi and Blough 2003; 2002].

## 5. STATIONARY NETWORKS

In this Section, we will consider several problems related to topology control in stationary ad hoc networks. The generalization of some of these problems to the

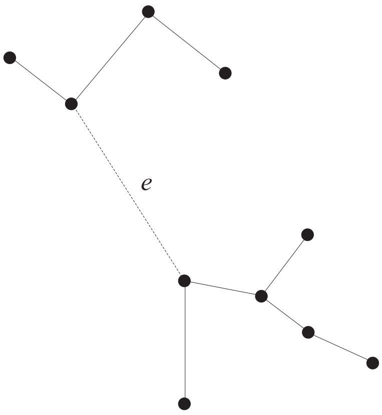


Fig. 3. The CTR for connectivity is the length of the longest edge of the Euclidean MST (edge $e$).

more complicated scenario of mobile networks is presented in Section 6.

### 5.1 Homogeneous topology control

First, we consider the following problem concerning homogeneous range assignments:

*Definition* 5.1 CTR. Suppose $n$ nodes are placed in $R = [0, l]^d$, with $d = 1, 2, 3$; what is the minimum value of $r$ such that the $r$-homogeneous range assignment for this placement is connecting?

The minimum value of $r$ such that the $r$-homogeneous range assignment is connecting is known as the *critical transmitting range for connectivity* in the literature.

The motivation for studying CTR (Critical Transmitting Range) stems from the fact that in many situations dynamically adjusting node transmitting range is not feasible. In fact, inexpensive radio transceivers might not allow the transmission range to adjusted [Ramanathan and Rosales-Hain 2000]. In this scenario, setting the same transmitting range $r$ for all the units is a reasonable choice, and the only option to reduce power consumption and increase network capacity is to set $r$ to the minimum possible value that ensures connectivity.

Characterizing the critical transmitting range helps the system designer to answer fundamental questions, such as: given a number of nodes $n$ to be deployed in a region $R$, which is the minimum value of the transmitting range that ensures network connectivity? Conversely, for a given transmitter technology, how many nodes must be distributed over a given region to ensure network connectivity?

The solution to CTR depends on the information we have about the physical placement of nodes. If the node placement is known in advance, the critical transmitting range is the length of longest edge of the Euclidean MST [Penrose 1997; Sanchez et al. 1999] built on the nodes (see Figure 3). Unfortunately, in many realistic scenarios of ad hoc or sensor networks the node placement is not known in advance. For example, in WSNs sensors could be spread from a moving vehicle (airplane, ship, or spacecraft). If node positions are not known, the minimum value of $r$ ensuring connectivity in all possible cases is $r \approx l\sqrt{d}$, which accounts for the fact that nodes could be concentrated at opposite corners of the deployment region. However, this scenario is unlikely in most realistic situations. For this reason, CTR has been studied under the assumption that nodes are distributed in $R$ according to some probability distribution. In this case, the goal is to characterize the minimum value of $r$ which provides connectivity with high probability (w.h.p.), i.e., with a probability that converges to 1 as the number of nodes (or the side $l$ of the deployment region) increases.

5.1.1 *Dense networks.* The probabilistic theory that is most suited to the analysis of CTR is the theory of geometric random graphs (see Section 4). Since the critical transmitting range coincides with the length of the longest edge in the Euclidean MST, probabilistic solutions to CTR can be derived by using results concerning the asymptotic distribution of the longest MST edge [Penrose 1999c; 1997]. This approach has been used in [Panchapakesan and Manjunath 2001] to prove that, under the hypothesis that nodes are uniformly distributed in $[0,1]^2$, the critical transmitting range for connectivity w.h.p. is $r = c\sqrt{\frac{\log n}{n}}$, for some constant $c > 0$. The characterizations of the critical range for connectivity in one- and three-dimensional networks can be obtained by combining some results derived in [Dette and Henze 1989; Holst 1980; Penrose 1999c; 1997; 1999a], and are as follows. In one-dimensional networks, it is shown that if $n$ nodes are distributed uniformly at random in $[0,1]$, then the critical range for connectivity w.h.p. is $r = \frac{\log n}{n}$. In three-dimensional networks, it is shown that if $n$ nodes are distributed uniformly at random in $[0,1]^3$, then the critical range for connectivity w.h.p. is

$$r = \sqrt[3]{\frac{\log n - \log\log n}{n\pi} + \frac{3}{2} \cdot \frac{1.41 + g(n)}{\pi n}} \,,$$

where $g(n)$ is an arbitrary function such that $\lim_{n\to\infty} g(n) = +\infty$.

A notable result of the theory of GRG is that, under the assumption of uniformly distributed points and $d \geq 2$, the longest nearest neighbor link and the longest MST edge have the same value (asymptotically, as $n \to \infty$) [Penrose 1999c]. In terms of the resulting communication graph, this means that connectivity occurs (asymptotically) when the last isolated node disappears from the graph. This result reveals an interesting analogy with non-geometric random graphs, which display the same behavior (known as the *giant component* phenomenon).

Although interesting, the theory of GRG can be used only to derive results concerning *dense* ad hoc networks. In fact, a standard assumption in this theory is that the deployment region $R$ is fixed, and the asymptotic behavior of $r$ as $n$ grows to infinity is investigated, i.e., the node density is assumed to grow to infinity. A similar limitation applies to the model of Gupta and Kumar [Gupta and Kumar 1998]. In their case, $R$ is the disk of unit area, and the authors show that if the units' transmitting range is set to $r = \sqrt{\frac{\log n + c(n)}{\pi n}}$, then the resulting network is connected w.h.p. if and only if $c(n) \to \infty$. This result is obtained making use of the theory of continuum percolation [Meester and Roy 1996], which is also used in [Dousse et al. 2002] to investigate the connectivity of hybrid ad hoc networks, in which base stations can be used to improve connectivity.

5.1.2 *Sparse networks.* Given the discussion above, the applicability of theoretical results concerning connectivity in ad hoc networks to realistic scenarios could be impaired. In fact, it is known that real wireless networks cannot be too dense, due to the problem of spatial reuse: when a node is transmitting, it interferes with all the nodes within its interference range, which is typically larger than the transmitting range. If the node density is very high, the level of interference is very high as well, and the overall network capacity is compromised [Gupta and Kumar 2000].

In order to circumvent this problem, other authors have characterized the critical transmitting range in the more general model in which the side $l$ of the deployment region is a further parameter, and $n$ and $r$ can be arbitrary functions of $l$. In this case, the critical transmitting range is analyzed asymptotically, as $l \to \infty$. Note that using this model, the node density might either converge to 0, or to a constant $c > 0$, or diverge as the size of the deployment region grows to infinity. Thus, results based on this framework can be applied to dense, as well as *sparse*, ad hoc networks.

The critical transmitting range for connectivity in sparse ad hoc networks have been analyzed in [Santi et al. 2001; Santi and Blough 2003; 2002] using the occupancy theory. It has been proven that, under the assumption that $n$ nodes are distributed uniformly at random in $R = [0, l]^d$, the $r$-homogeneous range assignment is connecting w.h.p. if $r = l \sqrt[d]{c \frac{\log l}{n}}$, for some constant $c > 0$. The authors also prove that, if $r \in O\left(l \sqrt[d]{\frac{1}{n}}\right)$, then the $r$-homogeneous range assignment is not connected w.h.p.

5.1.3 *More practical characterizations of the CTR.* Besides analytical characterization, the critical transmitting range has been investigated from a more practical viewpoint. In [Narayanaswamy et al. 2002], Narayanaswamy et al. present a distributed protocol, called COMPOW, that attempts to determine the minimum common transmitting range needed to ensure network connectivity. The authors show that setting the transmitting range to this value has the beneficial effects of maximizing network capacity, reducing the contention to access the wireless channel, and minimizing energy consumption. In [Bettstetter 2002a], Bettstetter analyzes network connectivity under the assumption that some of the nodes have transmitting range $r_1$, and the remaining ones have transmitting range $r_2 \neq r_1$. In [Santi and Blough 2003], Santi and Blough investigate through simulation the trade off between the transmitting range and the size of the largest connected component in the communication graph. The experimental results presented in [Santi and Blough 2003] show that, in sparse two and three-dimensional networks, the transmitting range can be reduced significantly if weaker requirements on connectivity are acceptable: halving the critical transmitting range, the largest connected component has average size of $0.9n$, approximately. This means that a considerable amount of energy is spent to connect relatively few nodes. This behavior is not displayed in case of one-dimensional networks, in which a small decrease of the transmitting range with respect to the critical value split the network into at least two connected components of moderate size. Quite interestingly, the experimental analysis of [Santi and Blough 2003] is coherent with the theoretical result of the theory of GRG (which, we recall, can be applied only to dense ad hoc networks) concerning the giant component phenomenon occurring in two and three-dimensional networks. This seems to indicate that, also in the case of sparse ad hoc networks, connectivity occurs (asymptotically) when the last isolated node disappears from the communication graph.

5.1.4 *Other characterizations of the critical range.* The critical transmitting range for connectivity has been studied also under the assumption of non-uniform node distribution. In particular, Penrose has characterized the critical range when nodes are distributed according to the two-dimensional Normal distribution [Penrose 1998], and to arbitrary probability density functions [Penrose 1999b] (provided certain technical conditions are satisfied).

Other authors have considered the critical transmitting range for $k$-connectivity of the communication graph[4], i.e. the critical range for ensuring a certain degree of fault-tolerance in the network. By exploiting a result due to Penrose [Penrose 1999a] showing that when the minimum node degree in a GRG becomes $k$ the graph becomes $k$-connected w.h.p. (this result holds only for two- and three-dimensional networks), Wan and Yi [Wan and Yi 2004] have derived the following characterization of the critical range for $k$-connectivity in two-dimensional networks with uniformly distributed points:

$$r = \sqrt{\frac{\log n + (2k-3)\log\log n + f(n)}{\pi n}} ,$$

where $k > 1$ is an arbitrary *constant*, and $f(n)$ is a function such that $\lim_{n\to\infty} f(n) = +\infty$. The problem of ensuring $k$-connectivity in ad hoc networks has been studied also in [Bettstetter 2002b].

Another model considers the problem of ensuring connectivity in networks with *Bernoulli nodes*. In this model, it is assumed that at any instant of time any node in the network is active with a certain, constant probability $p > 0$. Since node activation periods are independent events, the node active/inactive status can be modeled by a Bernoulli random variable of parameter $p$. The study of ad hoc networks with Bernoulli nodes finds its motivation in the fact that, in many application scenarios (especially for WSNs), nodes alternately shut down their radio to save energy. In this context, it is important that the subnetwork composed of active nodes is connected (*active connectivity*). Furthermore, it is desirable that any inactive node is adjacent to at least one active node (*active domination*), so that it can quickly propagate alarm messages in case an anomalous condition is detected (we recall that inactive nodes still sense the environment – it is only the radio that is turned off). Denoting with $G(n,r)$ the GRG graph with $n$ nodes and transmitting range $r$, with $A(n,r,p)$ the subgraph of $G(n,r)$ induced by the set of active nodes, and with $I(n,r,p)$ the subgraph of $G(n,r)$ obtained by removing all edges whose endpoints are both inactive nodes, active connectivity is obtained when $A(n,r,p)$ is connected, and active domination when $I(n,r,p)$ is connected. By combining the results presented in [Yi et al. 2003] and in [Yi and Wan 2005], it can be shown that w.h.p. the critical range for connectivity in $A(n,r,p)$ and in $I(n,r,p)$ under the assumption of uniformly distributed nodes is the same, and it

[4]We recall that a graph is $k$-connected if removing any $k-1$ nodes does not disconnect the graph.

equals

$$r = \sqrt{\frac{\log n + f(n)}{\pi p n}} \ ,$$

where $f(n)$ is a function such that $\lim_{n\to\infty} f(n) = +\infty$.

Another problem considered is the characterization of the critical coverage range. Network coverage is defined as follows: every node covers a circular area of radius $r_c$, and the monitored area $R$ is *covered* if every point of $R$ is at distance at most $r_c$ from at least one node. The goal is to find the critical value of $r_c$ that ensures coverage w.h.p. This problem has been investigated in [Philips et al. 1989] for the case of nodes distributed in a square with side of length $l$ according to a Poisson process of fixed density. The critical transmitting and coverage range for Poisson distributed points on a line of length $l$ is derived in [Piret 1991].

### 5.2 Non-homogeneous topology control

In the previous Section, we have analyzed the problem of determining a minimum common value of the transmitting range that generates a connected communication graph, under the hypothesis that only probabilistic information about node positions is available. In this Section, we survey the considerable body of results obtained for the more general problem in which nodes are allowed to have different transmitting ranges. As in the case of homogeneous topology control, in this Section we only report results concerning the stationary case. Non-homogeneous topology control techniques for mobile networks will be discussed in Section 6.

5.2.1 *The range assignment problem.* The problem of assigning transmitting range to nodes in such a way that the resulting communication graph is strongly connected and the energy cost is minimum is called the *range assignment problem* (RA), and was first studied in [Kirousis et al. 2000]. More formally, the problem is defined as follows.

*Definition* 5.2 RA. Let $N = \{u_1, \ldots, u_n\}$ be a set of points in the $d$-dimensional space ($d = 1, 2, 3$), denoting the positions of the network nodes. Determine a connecting range assignment RA such that $c(RA) = \sum_{u_i \in N} (RA(u_i))^\alpha$ is minimum.

The computational complexity of RA has been analyzed in [Kirousis et al. 2000]. The problem is solvable in polynomial time (more specifically, in time $O(n^4)$) in the one-dimensional case (i.e., nodes in a line), while it is shown to be NP-hard in the case of three-dimensional networks. In a later paper, Clementi et al. [Clementi et al. 1999] have shown that RA is NP-hard also in the two-dimensional case. Thus, computing the optimal range assignment in two and three-dimensional networks is a virtually impossible task. However, the optimal solution can be approximated within a factor of 2 using the range assignment generated as follows [Kirousis et al. 2000]: let $T$ be the MST built on $N$, where the weight of edge $(u_i, u_j)$ is the power $\delta^\alpha_{u_i,u_j}$ needed to transmit a message between $u_i$ and $u_j$; for every node $u_i \in N$, define $RA(u_i)$ as the maximum of distances $\delta_{u_i,u_j}$, for all nodes $u_j$ which are neighbors of $u_i$ in $T$. In the following, we will denote this range assignment with $RA_{MST}$.

Several variants of RA have been considered in the literature. In [Clementi et al. 1999; 2000; Clementi et al. 2000; Kirousis et al. 2000], the focus is on a constrained version of RA, in which the additional requirement of having a communication graph with diameter at most $h$, for some constant $h<n$, is imposed on the communication graph. However, we believe this version of the problem is less interesting from a practical point of view. In fact, imposing a topology which is "too connected" would often cause communication interference to occur even between nodes that are far apart, thus decreasing the network capacity. This phenomenon is confirmed by theoretical as well as experimental results [Grossglauser and Tse 2001; Gupta and Kumar 2000; Li et al. 2001], which show that the communication graph in wireless ad hoc networks should be as sparse as possible, while preserving connectivity.

Two important variants of RA which have been recently studied are based on the concept of *symmetry* of the communication graph. In general, the communication graph generated by a range assignment is not symmetric, i.e., it might contain unidirectional links. Although implementing wireless unidirectional links is technically feasible (see [Bao and Garcia-Luna-Aceves 2001; Kim et al. 2001; Pearlman et al. 2000; Prakash 2001; Ramasubramanian et al. 2002] for unidirectional link support at different layers), the actual advantage of using unidirectional links is questionable. For example, in [Marina and Das 2002] Marina and Das have shown that the high overhead needed to handle unidirectional links in routing protocols outweighs the benefits that they can provide, and better performance can be achieved by simply avoiding them. The high overhead is due to the fact that low level protocols, such as the MAC (Medium Access Control) protocol, are naturally designed to work under the symmetric assumption. For instance, the MAC protocol defined in the IEEE 802.11 standard [IEEE 1999] is based on RTS - CTS message exchange: when node $u_i$ wishes to send a message to one of its neighbors $u_j$ (at this level, communication is only between immediate neighbors), it sends a RTS (Request To Send) to $u_j$, and waits for a CTS (Clear To Send) message from $u_j$. If the CTS message is not received within a certain time, then message transmission is aborted and it is tried again after a backoff interval. Hence, for the protocol to work $u_i$ must be within the transmitting range of $u_j$, i.e., the range assignment must be symmetric.

The symmetric range assignment problem have been independently defined and studied in [Blough et al. 2002; Calinescu et al. 2002]. In [Blough et al. 2002], two different symmetric restrictions of RA are considered:

*Definition* 5.3 *Symmetric subgraph.* Let $G=(N,E)$ be an arbitrary communication graph. The *symmetric subgraph* of $G$, denoted $G_S$, is obtained from $G$ by deleting all the unidirectional links, i.e. all the edges such that $(u,v) \in E$, but $(v,u) \notin E$.

*Definition* 5.4 WSRA. Let $N=\{u_1,\ldots,u_n\}$ be a set of points in the $d$-dimensional space ($d=1,2,3$), denoting the positions of the network nodes. Determine a connecting range assignment $RA$ such that the symmetric subgraph $G_S$ of the communication graph resulting from $RA$ is connected and $c(RA)=\sum_{u_i\in N}(RA(u_i))^\alpha$ is minimum.

*Definition* 5.5 SRA. Let $N=\{u_1,\ldots,u_n\}$ be a set of points in the $d$-dimensional space ($d=1,2,3$), denoting the positions of the network nodes. A range assignment $RA$ is said to be *symmetric* if it generates a communication graph which contains only bidirectional links, i.e., $RA(u_i) \geq \delta_{u_i,u_j} \Leftrightarrow RA(u_j) \geq \delta_{u_i,u_j}$. Determine a connecting symmetric range assignment $RA$ such that $c(RA) = \sum_{u_i \in N} (RA(u_i))^\alpha$ is minimum.

In Sra (Symmetric Range Assignment), it is required that the communication graph contains only bidirectional links. This requirement is weakened in Wsra (Weakly Symmetric Range Assignment), in which unidirectional links may exist, but they are not essential for connectivity. The motivation for studying Wsra stems from the observation that what is really important in the design of ad hoc networks is the existence of a connected backbone of symmetric edges. In other words, there could exist further edges for which symmetry is not guaranteed, but these links can be ignored without compromising network connectivity.

In [Blough et al. 2002], it is shown that Sra remains NP-hard in two and three-dimensional networks. Hence, imposing (weak) symmetry does not reduce the computational complexity of the problem. The authors of [Blough et al. 2002] have also investigated the relations between the energy cost of the optimal solutions of Ra, Wsra and Sra. Denoting with $c_{RA}$, $c_{WS}$ and $c_S$ these costs, respectively, we have $c_{WS} - c_{RA} \in O(1)$, and $c_S - c_{RA} \in \Omega(n)$. In words, this means that the requirement for weak symmetry has only a marginal effect on the energy cost of the range assignment, while it eases significantly the integration of topology control mechanisms with existing higher level protocols (e.g., routing). On the other hand, imposing the stronger requirement of symmetry incurs a considerable additional energy cost. Overall, we can conclude that weak symmetry is a desirable property of the range assignment.

In [Calinescu et al. 2002], Calinescu et al. introduce two polynomial approximation algorithms for Wsra, which improve on the approximation ratio of 2 previously known[5]. The first algorithm has an approximation ratio of $1 + \ln 2 \approx 1.69$, while the second, which is more computationally efficient, has an approximation ratio of $\frac{15}{8}$. These ratios have been recently improved to $\frac{5}{3} + \epsilon$, for any positive constant $\epsilon$, and to $\frac{11}{6}$, respectively [Althaus et al. 2003]. Further, the authors of [Althaus et al. 2003] present an exact branch and cut algorithm for solving Wsra based on a new integer linear program formulation of the problem. Experimental results show that the branch and cut algorithm solves instances with up to 35-40 nodes (with randomly generated positions) in 1 hour. Most importantly, the experimental results show that the average improvement of the exact solution over $RA_{MST}$, which can be easily calculated, is in the range 4–6%. This means that the average case approximation ratio of $RA_{MST}$ is much smaller than the worst case ratio of 2.

The problem of ensuring $k$-connectivity (i.e., fault-tolerance) of the communication graph has also been considered in the literature. It has first been studied in [Loyd et al. 2002] in the weakly symmetric version when $k = 2$, and further analyzed in [Calinescu and Wan 2003]. In particular, Calinescu and Wan prove that the weakly symmetric version of the problem with $k = 2$ is NP-hard, and they provide approximation algorithms for both the weakly symmetric and asymmetric version of the problem.

[5]It can be easily observed that the $RA_{MST}$ range assignment used in [Kirousis et al. 2000] to approximate Ra within a factor of 2 is weakly symmetric. This observation has been used in [Blough et al. 2002] to prove that $c_{WS} - c_{RA} \in O(1)$.

5.2.2 *Minimum energy unicast and broadcast.*

5.2.2.1 **Unicast**. In the previous Section, the emphasis was on finding a range assignment that generates a connected topology of minimum energy cost. Another branch of research focused on computing topologies which have energy-efficient paths between potential source-destination pairs. More specifically, the following problem has been considered (see [Li et al. 2002; Rajaraman 2002]).

Let $G$ be the communication graph obtained when all the nodes transmit at maximum power (the *maxpower graph*), and assume $G$ is connected. Every edge $(u_i, u_j)$ in $G$ is weighted with the power $\delta^{\alpha}_{u_i,u_j}$ needed to transmit a message between $u_i$ and $u_j$. Given any path $P = u_1, u_2, \ldots, u_k$ in $G$, the *power cost* of $P$ is defined as the sum of the power costs of the single edges, i.e., $pc(P) = \sum_{i=1}^{k-1} \delta^{\alpha}_{u_i,u_{i+1}}$. Let $pc_G(u,v)$ denote the minimum of $pc(P)$ over all paths $P$ that connect nodes $u$ and $v$ in $G$. A path in $G$ connecting $u$ and $v$ and consuming the minimum power $pc_G(u,v)$ is called a *minimum-power path* between $u$ and $v$. Let $G'$ be an arbitrary subgraph of $G$. The *power stretch factor* of $G'$ with respect to $G$ is the maximum over all possible node pairs of the ratio between the cost of the minimum-power path in $G'$ and in $G$. Formally, $\rho_{G'} = \max_{u,v \in N} \frac{pc_{G'}(u,v)}{pc_G(u,v)}$.

The power stretch factor is a generalization of the concept of *distance stretch factor*, which is well known in computational geometry. Another similar concept is the *hop stretch factor*, which measures the ratio of the hop-counts rather than that of power or distance.

In general, we would like to identify a subgraph $G'$ (also called *routing graph* in the following) of the maxpower graph $G$ which has a low power stretch factor, and which is sparser than the original graph. The routing graph can be used to compute routes between nodes, with the guarantee that the power needed to communicate along these routes is "almost minimal". The advantage of using $G'$ instead of $G$ is that computing the optimal routes in $G'$ is easier than in $G$ and generates little message overhead, and that a sparse communication graph requires little maintenance in presence of node mobility.

Given the maxpower graph $G$, the problem of computing a subgraph $G'$ with low power stretch factor has been widely studied in the literature. Ideally, the routing graph should have the following features:

*a.* constant power stretch factor, i.e., $\rho_{G'} \in O(1)$. Using the terminology of geometric graphs, $G'$ should be a *power spanner* of $G$.

*b.* linear number of edges; in other words, $G'$ should be *sparse*.

*c.* bounded node degree.

*d.* easily computable in a distributed and localized fashion. By localized, we mean that every node should be able to compute the set of its neighbors in $G'$ using only information provided by its neighbor nodes in $G$.

Property *a.* ensures that the routes calculated on $G'$ are at most a constant factor away from the energy-optimal routes. Property *b.* eases the task of finding routes in $G'$ and of maintaining the routing graph in presence of node mobility, and it reduces the routing overhead. The requirement of bounded node degree is motivated by the fact that nodes with high degree are likely to be bottlenecks in the communication

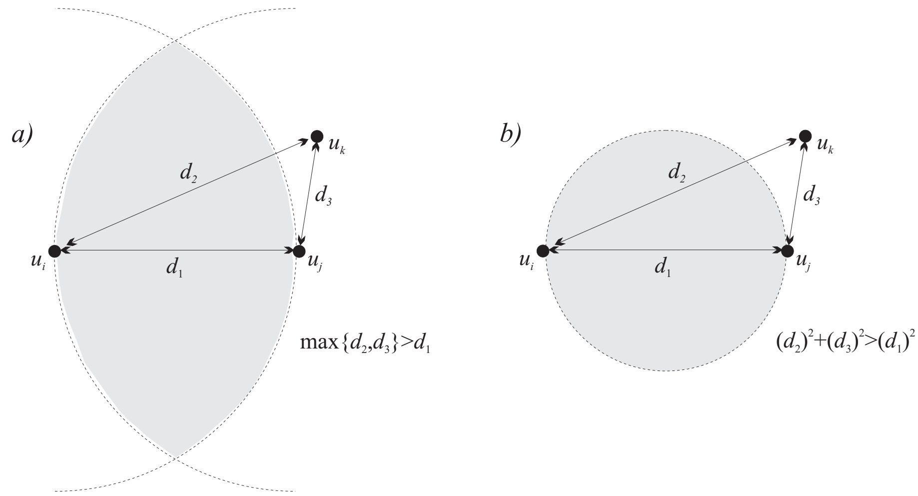


Fig. 4. Edges in the Relative Neighborhood Graph (left) and in the Gabriel Graph (right).

graph. Finally, property *d.* is fundamental for a fast and effective computation of the routing graph in a real wireless ad hoc network.

Several routing graphs that satisfy some of the requirements above have been proposed in the literature. Most of them are based on subgraphs of $G$ which have been shown to be good distance spanners. In fact, it can be easily seen that if a subgraph $G'$ is a distance spanner of graph $G$, then it is also a power spanner of $G$ (note that the reverse implication in general is not true). Thus, the considerable body of research devoted to distance spanners in computational geometry can be used to design good routing graphs.

The following geometric graphs have been considered in the literature:

*Definition* 5.6. Let $N$ be a set of points in the Euclidean two-dimensional space;

RNG. the Relative Neighborhood Graph of $N$ has an edge between two nodes $u_i$ and $u_j$ if there is no node $u_k$ such that $\max\{\delta_{u_i,u_k}, \delta_{u_j,u_k}\} \leq \delta_{u_i,u_j}$ (see Figure 4–a);

GG. the Gabriel Graph of $N$ has an edge between two nodes $u_i$ and $u_j$ if there is no node $u_k$ such that $\delta^2_{u_i,u_k} + \delta^2_{u_j,u_k} \leq \delta^2_{u_i,u_j}$; in words, $(u_i, u_j) \in GG(N)$ if and only if the disk obtained using $\overline{u_i u_j}$ as diameter does not contain any node from $N$ (see Figure 4–b);

DG. the Delaunay Graph of $N$ is the unique triangulation such that the circumcircle of every triangle contains no points of $N$ in its interior;

YG. the Yao Graph of $N$ of parameter $c$, for any integer $c \geq 6$, is denoted $YG_c$, and is defined as follows. At each node $u_i \in N$, any $c$ equally separated rays originated at $u_i$ define $c$ equal cones. In each cone, choose the shortest directed edge $(u_i, u_j) \in G$, if any, and add the correspondent directed edge in $YG_c$. If we add the reverse directed link from $u_j$ to $u_i$, we obtain the Reverse Yao Graph. If we ignore the direction of edges, we have the Undirected Yao Graph.

Note that in general the $DG$ of a set of points may include edges much longer than the maximum node transmitting range. For this reason, a restricted version of $DG$ has been introduced in [Gao et al. 2001], in which a limit on the maximum edge length is imposed. We denote the restricted $DG$ graph of a set of points $N$ with $RDG(N)$.

The graphs defined above are called *proximity graphs*, since the set of neighbors of any node $u$ in the computed graph can be calculated based on the position of the neighbor nodes in the original graph. Thus, proximity graphs satisfy requirement *d.* above.

The following relationships between proximity graphs have been proven [Goodman and O'Rourke 1997; Li et al. 2002]: for any set of points $N$, $RNG(N) \subseteq GG(N)$, and $RNG(N) \subseteq YG_c(N)$, for any $c \geq 6$. Furthermore, $MST(N)$ is contained in $RNG(N)$, $GG(N)$, $DG(N)$ and $YG_c(N)$, for any $c \geq 6$.

The distance stretch factor, the power stretch factor, and the maximum node degree of the proximity graphs defined above have been analyzed in [Gao et al. 2001; Li et al. 2002; Wang et al. 2003], and are reported in Table I. As it is seen, the Gabriel Graph is energy-optimal, since it has a power stretch factor of 1.

| | Distance | Power | Degree |
|---|---|---|---|
| RNG | $n-1$ | $n-1$ | $n-1$ |
| GG | $\sqrt{n-1}$ | 1 | $n-1$ |
| RDG | $\frac{1+\sqrt{5}}{2}\pi$ | $\left(\frac{1+\sqrt{5}}{2}\pi\right)^\alpha$ | $\Theta(n)$ |
| $YG_c$ | $\frac{1}{1-2\sin\frac{\pi}{c}}$ | $\frac{1}{1-\left(2\sin\frac{\pi}{c}\right)^\alpha}$ | $n-1$ |

Table I. Distance stretch factor, power stretch factor, and maximum node degree of different proximity graphs.

All the graphs defined above have been shown to be sparse, which implies that they have a constant *average* node degree. However, the *maximum* node degree is not constant in any of the considered graphs. For this reason, several variants of these proximity graphs have been proposed, with the purpose of bounding the maximum node degree. Unfortunately, it has been shown that no geometric graph with constant node degree contains the minimum-power path for any pair of nodes [Wang et al. 2002]. Thus, no energy-optimal spanner with a constant bounded maximum node degree exists. To date, the routing graph with constant maximum node degree which has the best power stretch factor is the $OrdYaoGG$ graph of [Song et al. 2004], which is obtained by building the $YG_c$ graph, with $c > 6$, on top of the $GG$. The $OrdYaoGG$ graph has power stretch factor of $\rho = \frac{1}{1-(2\sin\frac{\pi}{c})^\alpha}$, and maximum node degree of $c+5$, where $c > 6$ is the parameter of the Yao graph. For example, setting $c = 9$ and $\alpha = 2$ we have a power stretch factor of 1.88 with a bound on the maximum node degree of 14.

5.2.2.2 **Broadcast**. Another relevant problem that has been considered in the literature is the determination of energy-efficient *broadcast graphs*. Here, the emphasis is on the one-to-all communication scheme typical of broadcast, rather than on point–to–point communications.

Similarly to the case of unicast, the concept of *broadcast stretch factor* can be defined. More precisely, let us consider a connected maxpower graph $G$. Any broadcast generated by node $u$ can be seen as a directed spanning tree $T$ rooted at $u$, which we call a *broadcast tree.* The power cost of the broadcast tree $T$ is defined as follows. Denoting with $pc_T(v)$ the power consumed by node $v$ to broadcast the message along $T$, we have that $pc_T(v) = 0$ for any leaf node of $T$, and $pc_T(v) = \max_{(v,w)\in T} \delta^{\alpha}_{v,w}$ otherwise. Thus, the total power needed to broadcast the message along the broadcast tree $T$ is $pc(T) = \sum_{v\in N} pc_T(v)$. A tree in $G$ rooted at $u$ and consuming the minimum power is called a *minimum-power broadcast tree* of $u$. Let $G'$ be an arbitrary subgraph of $G$. The *broadcast stretch factor* of $G'$ with respect to $G$ is the maximum over all possible nodes of the ratio between the cost of the minimum-power broadcast tree in $G'$ and in $G$. Formally, $\beta_{G'} = \max_{u\in N} \frac{pc_{G'}(u)}{pc_G(u)}$, where $pc_{G'}(u)$ and $pc_G(u)$ denote the cost of the minimum-power broadcast tree of $u$ in $G'$ and in $G$, respectively.

As in the case of unicast, the goal is to find sparse broadcast spanners[6] that can be computed in a distributed and localized fashion. Unfortunately, this task is more difficult than in the case of unicast.

The problem of computing a minimum-power broadcast tree rooted at a node $u$ has been proven to be NP-hard independently in [Cagali et al. 2002] and [Liang 2002], under the hypothesis that nodes can transmit at different power levels $P = \{p_1, \ldots, p_k\}$, where the $p_i$ are arbitrary power levels and $k$ is an arbitrary positive constant. Thus, the task of finding the energy-optimal broadcast tree of a given communication graph $G$ is virtually impossible in any realistic scenario.

In [Wieselthier et al. 2000], Wieselthier et al. introduce three greedy heuristics for the minimum-power broadcast problem based on the construction of the MST, and evaluate them by means of simulation. The broadcast stretch factor of the graphs generated by these heuristics are formally derived in [Wan et al. 2002], in which it is shown that the MST has constant broadcast stretch factor $c$, for some $6 \leq c \leq 12$. Thus, the MST is a broadcast spanner of the original graph. Unfortunately, the construction of the MST, as well as of the other graphs proposed in [Wieselthier et al. 2000], requires global information, which can be a major difficulty in implementing it in a real ad hoc network. In order to circumvent this problem, Li et al. [Li et al. 2004] have recently proposed a localized, fully distributed algorithm called $\text{LMST}_k$ that builds a local approximation of the MST. $\text{LMST}_k$ requires exchanging $O(n)$ messages (although the hidden constant is larger than 225), and builds a $O(n^{\alpha-1})$ approximation of the energy-optimal broadcast tree. Thus, $\text{LMST}_k$ cannot be used to compute a broadcast spanner of $G$. To date, no distributed and localized algorithm that constructs a broadcast spanner is known.

Before ending this Section, we want to outline the similarities between the range assignment problem discussed in Section 5.2.1 and the problem of energy-efficient broadcast. Suppose $G$ is the maxpower graph on the set of points $N$. In the RA problem, the goal is to find the energy-optimal range assignment that generates a connected communication graph. Suppose an arbitrary node $u \in N$ wants to

[6] A subgraph $G'$ of graph $G$ is a *broadcast spanner* of $G$ if it has $O(1)$ broadcast stretch factor.

broadcast a message $m$, and let $RA$ be the optimal range assignment. A very simple broadcast scheme is the following. Node $u$ transmits $m$ at distance $RA(u)$, and every other node $v$, upon receiving $m$ for the first time, re-transmits it at distance $RA(v)$. It is immediate that, after all nodes in $N$ have transmitted the message once, $m$ has been broadcast in all the network. Thus, the energy cost of $RA$ is an upper bound to the power cost of any broadcast tree in $G$. We recall that the energy cost of the optimal range assignment (and of the optimal weakly symmetric range assignment) differs from the cost of the MST at most for a factor 2. Since the MST is a broadcast spanner of $G$, this implies that the communication graph generated by the optimal (weakly symmetric) range assignment is a broadcast spanner of $G$. Unfortunately this does not help very much, since computing this graph in two and three-dimensional networks is NP-hard.

5.2.3 *Distributed topology control protocols.* In Sections 5.2.1 and 5.2.2, we have reviewed several problems related to energy-efficient communication in wireless ad hoc networks. In these approaches to TC, it was assumed that exact node positions are known (location-based topology control), and the problem is one of finding a range assignment (and, thus, a network topology) which is optimal with respect to a certain measure. Hence, in these approaches the emphasis is on the quality of the topology produced, rather than on the process of building the topology itself. Another branch of research focused on more practical approaches to the TC problem, trying to design simple, fully distributed protocols that build and maintain a "reasonably good" topology. We call these protocols *topology control protocols.*

Ideally, a topology control protocol should be fully distributed, asynchronous and localized. As discussed above, these requirements are vital for an effective implementation of the protocol, especially in presence of node mobility. Another aspect to be considered is the "quality" of the information needed by the topology control protocol. In general, there is a trade off between information quality and energy consumption and/or interference reduction: the more accurate is the information required (e.g., exact node positions), the more energy savings/interference reductions can be achieved. However, the price to be paid (in terms of additional hardware on the nodes, or of additional messages to be exchanged) to obtain high quality information must be carefully considered. For example, suppose protocol $P_1$ is based on location information, and protocol $P_2$ is based on distance estimation. Clearly, the cost of implementing $P_2$ in a real network is lower than that required by $P_1$, since the hardware needed to estimate distance between nodes is cheaper than that required to estimate node positions. So, if the energy savings provided by protocol $P_1$ are not considerably higher than those achieved by $P_2$, a solution based on protocol $P_2$ may be preferable in practice.

Summarizing, a topology control protocol should:

– be fully distributed and asynchronous;
– rely on local information only;
– generate a connected topology (at least with high probability) composed of bi-directional links[7];

– rely on 'low-quality' information.

5.2.3.1 *Location-based TC protocols.* In [Rodoplu and Meng 1999], the authors presented a distributed topology control algorithm that leverages on location information (provided by low-power GPS receivers) to build a topology that is proven to minimize the energy required to communicate with a given master node. In [Li and Wan 2001], the authors described a more efficient implementation of the protocol which, however, computes only an approximation of the minimum energy topology.

In [Ramanathan and Rosales-Hain 2000], the authors considered the problem of minimizing the maximum of node transmitting ranges while achieving connectedness. They also considered the stronger requirement of 2-connectivity of the communication graph. They present centralized topology control algorithms that provide the optimal solution for both versions of the problem. The range assignment returned by the algorithm has the additional property of being per-node minimal, i.e., no transmitting range can be reduced further without impairing connectivity (or 2-connectivity).

In [Li et al. 2003], the authors introduced LMST, a fully distributed and localized protocol aimed at building an MST-like topology. The authors show that: (1) the protocol generates a strongly connected communication graph; (2) the node degree of any node in the generated topology is at most 6; and (3) the topology can be made symmetric by removing asymmetric links without impairing connectivity. Furthermore, the authors show through simulation that LMST outperforms CBTC (see below) and the protocol of Rodoplu and Meng in terms of both average node degree and average node transmitting range. A drawback of LMST is that it requires location information, which can be provided only with a considerable hardware and/or message cost. Recently, some of the authors of LMST introduced a fault-tolerant version of this algorithm which generates a $k$-connected topology [Li and Hou 2004].

5.2.3.2 *Direction-based TC protocols.* In [Wattenhofer et al. 2001], the authors introduced a distributed topology control protocol based on directional information, called CBTC (Cone Based Topology Control). The basic idea is similar to the one inspiring the Yao graph $YG$: a node $u$ transmits with the minimum power $p_{u,\rho}$ such that there is at least one neighbor in every cone of angle $\rho$ centered at $u$. The obtained communication graph is made symmetric by adding the reverse edge to every asymmetric link. The authors show that setting $\rho \leq 2\pi/3$ is a sufficient condition to ensure connectivity. A set of optimizations aimed at pruning energy-inefficient edges without impairing connectivity (and symmetry) is also presented. Further, the authors prove that if $\rho \leq \pi/2$, every node in the final communication graph has degree at most 6. A more detailed analysis of CBTC, along with an improved set of optimizations (which, however, rely on distance estimation), can be found in [Li et al. 2001]. The CBTC protocol has been extended to the case of nodes in the three-dimensional space in [Bahramgiri et al. 2002]. The authors of [Bahramgiri et al. 2002] also presented a fault-tolerant version of the protocol that guarantees $k$-connectivity. In [Huang et al. 2002], the CBTC protocol is implemented using directional antennas[8].

[7]The motivation for using bi-directional links is given in Section 5.2.1.

In [Borbash and Jennings 2002], the authors introduced a distributed protocol which is also based on directional information. The goal of the protocol is to build the Relative Neighbor Graph of the network in a distributed fashion. The choice of the RNG as the target graph of the protocol is due to the fact that it guarantees connectivity and it shows good performance in terms of average transmitting range, node degree and hop diameter.

5.2.3.3 *Neighbor-based TC protocols.* Another class of topology control protocols is based on the simple idea of connecting each node to its $k$ closest neighbors.

The MobileGrid protocol of [Liu and Li 2002] and the LINT protocol of [Ramanathan and Rosales-Hain 2000] try to keep the number of neighbors of a node within a low and high threshold centered around an optimal value. When the actual number of neighbors is below (above) the threshold, the transmitting range is increased (decreased), until the number of neighbors is in the proper range. However, for both protocols no characterization of the optimal value of the number of neighbors is given, and, consequently, no guarantee on the connectivity of the resulting communication graph is provided. Another problem of the MobileGrid and LINT protocols is that they estimate the number of neighbors by simply overhearing control and data messages at different layers. This approach has the advantage of generating no control message overhead, but the accuracy of the resulting neighbor number estimate heavily depends on the traffic present in the network. In the extreme case, a node which remains silent is not detected by any of its actual neighbors.

The problem of characterizing the minimum value of $k$ such that the resulting communication graph is connected (the *Critical Neighbor Number*) has been investigated in [Xue and Kumar 2004], where it is shown that $k \in \Theta(\log n)$ is a necessary and sufficient condition for connectivity w.h.p. Recently, Wan and Yi [Wan and Yi 2004] have improved the upper bound on the CNN for connectivity derived in [Xue and Kumar 2004].

Based on Xue and Kumar's theoretical result, Blough et al. propose the $k$-NEIGH protocol [Blough et al. 2003]. The goal of $k$-NEIGH is to keep the number of neighbors of a node equal to, or slightly below, a given value $k$. The communication graph that results is made symmetric by removing asymmetric edges. Given the characterization of the critical neighbor number presented in [Xue and Kumar 2004], Blough et al. prove that the communication graph generated by $k$-NEIGH when $k \in \Theta(\log n)$ is connected with high probability. From a practical viewpoint, Blough et al. show through simulation that setting $k = 9$ is sufficient to obtain connected networks with high probability for networks with $n$ ranging from 50 to 500. Furthermore, the authors analyze the time and message complexity of the protocol, and present simulation results that show that the topology generated by $k$-NEIGH is, on the average, 20% more energy efficient than that generated by CBTC.

A protocol that shares many similarities with $k$-NEIGH is the XTC protocol presented in [Wattenhofer and Zollinger 2004]: the neighbors of a node $u$ are ordered according to some metric (e.g. distance, or link quality), and $u$ decides which nodes are kept as immediate neighbors in the final network topology based on a

[8] Directional antennas have the ability to propagate the radio signal only in specific directions.

simple rule. Contrary to $k$-NEIGH, which achieves connectivity w.h.p., XTC builds a topology which is connected whenever the maxpower communication graph is connected. To achieve this, the requirement of having an upper bound $k$ on the number of neighbors of a node is dropped: contrary to $k$-NEIGH, in XTC a node can have as much as $n-1$ neighbors in the final topology.

The main features of the distributed topology control protocols presented in this section are summarized in Table II.

| **Protocol** | **Approach** | **Connectivity** | **Fault-tolerance** |
|---|---|---|---|
| R&M | loc-based | yes | no |
| LMST | loc-based | yes | yes |
| CBTC | dir-based | yes | yes |
| RNG | dir-based | yes | no |
| LINT/LILT | neigh-based | unknown | no |
| MobileGrid | neigh-based | unknown | no |
| KNeigh | neigh-based | w.h.p. | no |
| XTC | neigh-based | yes | no |

Table II. Main features of the distributed topology control protocols presented in this paper.

### 5.3 Discussion of energy cost

The results of Sections 5.1 and 5.2 can be used to evaluate the potential benefit (in terms of energy cost) achieved by topology control protocols. In fact, the solution to the range assignment problem RA can be seen, at least to a certain extent, as the best possible result of the execution of a topology control protocol. On the other hand, the critical transmitting range for connectivity considered in Section 5.1 is representative of the scenario in which only a straightforward type of topology control is feasible.

The following theorem is a consequence of the results presented in [Santi and Blough 2003].

THEOREM 5.7. *Let $l$ be a positive real number sufficiently large, and let $N$ be a set of $n$ nodes positioned uniformly and independently at random in $R=[0,l]^d$, with $d=1,2,3$. Assume the distance-power gradient $\alpha$ is 2, and denote by $c_{min}(N)$ the cost of the $r$-homogeneous range assignment such that $r$ is minimum and the resulting communication graph is connected. Then, with high probability:*

$$c_{min}(N) = \begin{cases} O(\frac{l^2 \log^2 l}{n}) & \text{for } d=1 \\ O(l^2 \log l) & \text{for } d=2 \\ O(l^2 n^{1/3} \log^{2/3} l) & \text{for } d=3 \end{cases}$$

The bounds of Theorem 5.7 can be compared to similar bounds obtained in [Blough et al. 2002; Clementi et al. 1999; 2000; Clementi et al. 2000; Kirousis et al. 2000] for the range assignment problem. The following result for one-dimensional networks is an easy consequence of the results presented in [Clementi et al. 2000; Kirousis et al. 2000]:

PROPOSITION 5.8. *Let $N$ be a set of $n$ collinear points equally spaced at distance $\delta>0$. The energy cost of the solution of* RA *on input $N$ is $\Theta(\delta^2 n)$.*

Assuming that the $n$ nodes are placed along a line of length $l$, the bound of Theorem 5.8 can be restated as $\Theta\left(\frac{l^2}{n}\right)$. It is not difficult to show that equally spacing nodes is the most energy-efficient placement. It follows immediately that the energy cost of any instance (including a random one[9]) of RA is $\Omega\left(\frac{l^2}{n}\right)$. Comparing this bound with the upper bound reported in Theorem 5.7 for $d=1$, we have that the asymptotic gap between the energy cost of the optimal range assignment and that of the optimal homogeneous range assignment is at most $\log^2 l$. Hence, the asymptotic benefit of the adoption of a topology control mechanism in one-dimensional networks is at most a factor of $\log^2 l$.

Bounds on the energy cost of the solution of the random instance of RA in two and three dimensions have been obtained in [Blough et al. 2002], and are $\Theta(l^2)$ for $d=2$ and $\Theta(l^2 n^{1/3})$ for $d=3$. By Theorem 5.7, we can conclude that the asymptotic benefit of the adoption of a topology control mechanism is at most a factor of $\log l$ in two-dimensional networks, and at most a factor of $log^{2/3} l$ in three-dimensional networks.

The comparison of the bounds on the energy cost of the optimal solution of RA and CTR in one, two and three-dimensional networks indicates that the benefit, expressed in terms of energy cost, of the adoption of a topology control mechanism increases with the length $l$ of the side of the deployment region, but becomes less significant for networks of higher dimension.

## 6. MOBILE NETWORKS

In Section 5, we have analyzed several problems related to energy-efficient communication in stationary wireless ad hoc networks. In this Section, we will discuss how does node mobility affect topology control in general.

The impact of mobility on topology control is two-fold:

– *increased message overhead*: the implementation of any distributed topology control protocol causes a certain message overhead, which is due to the fact that nodes need to exchange messages in order to set the transmitting range to the appropriate value. In case of stationary networks, the topology control protocol is in general executed once at the beginning of the network operational time, and periodically to account for node join/leave. Thus, the "efficiency" of the protocol (expressed here in terms of message overhead) has relatively little importance, and the emphasis is more on the "quality" of the produced topology. In presence of mobility, the topology control protocol must be executed frequently, in order to account for the new positions of the nodes. Thus, reducing message overhead is fundamental when implementing topology control mechanisms in mobile networks (especially in the case of high mobility scenarios), even if reducing message overhead comes at the cost of a "lower-quality" of the constructed topology.

– *non-uniform node spatial distribution*: as it will be discussed in details later, some mobility patterns cause a non-uniform node spatial distribution. This fact should be carefully taken into account in setting important network parameters (e.g., the critical transmitting range) at the design stage.

[9]Here, with random instance we mean an instance of the problem in which node positions are chosen uniformly at random in the deployment region $R=[0,l]^d$.

From the discussion above, it is clear that the impact of mobility on the effectiveness of topology control techniques heavily depends on the mobility pattern. For this reason, we first present the mobility models which have been considered in the literature.

### 6.1 Mobility models

The most widely used mobility model in the ad hoc network community is the random waypoint model [Johnson and Maltz 1996]. In this model, every node chooses uniformly at random a destination in $[0,l]^d$ (the *waypoint*), and moves towards it along a straight line with a velocity chosen uniformly at random in the interval $[v_{min}, v_{max}]$. When it reaches the destination, it remains stationary for a predefined pause time $t_{pause}$, and then it starts moving again according to the same rule.

A similar model is the random direction model [Bettstetter 2001; Royer et al. 2001], in which nodes move with direction chosen uniformly in the interval $[0, 2\pi[$, and velocity chosen uniformly at random in the interval $[v_{min}, v_{max}]$. After a randomly chosen time, taken usually from an exponential distribution, the node chooses a new direction. A similar procedure is used to change velocity, using an independent stochastic process.

Contrary to the case of the random waypoint and the random direction model which resemble, at least to some extent, intentional motion, the class of Brownian-like mobility models resembles non-intentional movement. For example, in the model used in [Blough et al. 2002], mobility is modeled using parameters $p_{stat}$, $p_{move}$ and $m$. Parameter $p_{stat}$ represents the probability that a node remains stationary during the entire simulation time. Hence, only $(1 - p_{stat})n$ nodes (on the average) will move. Introducing $p_{stat}$ in the model accounts for those situations in which some nodes are not able to move. For example, this could be the case when sensors are spread from a moving vehicle, and some of them remain entangled, say, in a bush or tree. This can also model a situation where two types of nodes are used, one type that is stationary and another type that is mobile. Parameter $p_{move}$ is the probability that a node moves at a given step. This parameter accounts for heterogeneous mobility patterns, in which nodes may move at different times. Intuitively, the smaller is the value of $p_{move}$, the more heterogeneous is the mobility pattern. However, values of $p_{move}$ close to 0 result in an almost stationary network. If a node is moving at step $i$, its position in step $i+1$ is chosen uniformly at random in the square of side $2m$ centered at the current node location. Parameter $m$ models, to a certain extent, the velocity of the nodes: the larger $m$ is, the more likely it is that a node moves far away from its position in the previous step.

Observe that in case of random direction or Brownian-like motion, nodes may in principle move out of the deployment region. Since a standard approach in simulations is to keep the number of network nodes constant, we need a so called *border rule* [Bettstetter 2001], which defines what to do with nodes that are about to leave the deployment region. In this situation, a node can be:

– bounced back according to some rule;

– positioned at the point of intersection of the boundary with the line connecting the current and the desired next position;

– wrapped around to the other side of the region, which is considered as a torus;
– "deleted", and a new node is initialized according to the initial distribution;
– forced to choose another position, until the chosen position is inside the boundaries of the deployment region.

Depending on the choice of the border rule, non-uniformity in the node spatial distribution can be produced. For example, the second rule described above places nodes exactly on the boundary of the region with higher probability than at other points. In fact, the only two rules that do not appear to favor one part of the region over another are the torus rule and the one in which a node is eliminated when it would cross the boundary and a new node is created in its place. However, these rules appear quite unrealistic, and are used mainly to artificially generate a more uniform node spatial distribution.

For a more exhaustive survey of mobility models in wireless networks, the reader is referred to [Bettstetter 2001; Camp et al. 2002].

### 6.2 Homogeneous topology control

If deriving analytical results for stationary networks is difficult, even more challenging is deriving theoretical results regarding *mobile* ad hoc networks, even in the simpler case of topology control, i.e., in case of homogeneous range assignment.

When the range assignment is homogeneous, the message overhead is not an issue, since the nodes' transmitting range is set at the design stage, and it cannot be changed dynamically. However, the node spatial distribution generated by the mobility model could be an issue. For instance, it is known [Bettstetter 2001; Bettstetter and Krause 2001; Bettstetter et al. 2003; Blough et al. 2002] that the random waypoint model generates a node spatial distribution which is independent of the initial node positions, and in which nodes are concentrated in the center of the deployment region. This phenomenon, which is known as the *border effect*, is due to the fact that in the random waypoint model a node chooses a uniformly distributed destination point rather than a uniformly distributed angle. Therefore, nodes located at the border of the region are very likely to cross the center of the region on their way to the next waypoint. The intensity of the border effect mainly depends on the pause time $t_{pause}$. In fact, a longer pause time tends to increase the percentage of nodes that are "resting" at any given time. Since the starting and destination points of a movement are chosen uniformly in $[0, l]^d$, this implies that a relatively long pause time generates a "more uniform" node spatial distribution.

An immediate consequence of the fact that the node spatial distribution in presence of mobility is in general non-uniform is that results concerning the critical transmitting range in stationary networks (which are based on the uniformity assumption) cannot be directly used. For this reason, the relationship between the critical transmitting range without and with mobility must be carefully investigated.

In [Sanchez et al. 1999], Sanchez et al. analyze the probability distribution of the critical transmitting range in presence of different mobility patterns (random waypoint, random direction, and Brownian-like) through simulation. The simulation results seem to indicate that the mobility pattern has little influence on the distribution of the critical transmitting range. Unfortunately, the significance of the findings of [Sanchez et al. 1999] is partly impaired by the fact that the toroidal border rule is used in simulations, and that the values of the mobility parameters used in the experiments (such as $t_{pause}$ in the random waypoint model) are not reported.

In [Santi and Blough 2003; 2002], Santi and Blough investigate the relationship between the critical transmitting range in stationary and in mobile networks through extensive simulation. They consider random waypoint and Brownian-like motion, and analyze different "critical values" for the node transmitting range, which are representative of different requirements on network connectivity (for instance, connectivity during 100% and 90% of the simulation time). The simulation results show that a relatively modest increase of the transmitting range with respect to the critical value in the stationary case is sufficient to ensure network connectivity during 100% of the simulation time. The increase is about 21% in the random waypoint and about 25% in the Brownian-like model. Furthermore, the simulation results show that the transmitting range can be considerably reduced (in the order of 35–40%) if the requirement for connectivity is only on 90% of the simulation time.

Further insights on the relationship between the stationary and mobile critical transmitting range can be derived from the statistical analysis of the node spatial distribution of mobile networks reported in [Blough et al. 2002]. Again, the authors consider random waypoint and Brownian-like mobility, and perform several statistical tests on the node spatial distribution generated by these models. The results of these tests show that the distribution generated by Brownian-like motion is virtually indistinguishable from the uniform distribution, and confirm the occurrence of the border effect in random waypoint motion, whose intensity heavily depends on the value of $t_{pause}$. In the extreme case of $t_{pause} = 0$, the random waypoint model generates a node spatial distribution which is considerably different from uniform. Overall, the analysis of [Blough et al. 2002] indicate that Brownian-like mobility should have little influence on the value of the critical transmitting range, while the effect of random waypoint mobility on the critical transmitting range should heavily depend on the settings of the mobility parameters.

The quality of the observation above is confirmed by the probabilistic analysis reported in [Santi 2005], which is, to the best of our knowledge, the only theoretical result concerning the critical transmitting range in presence of mobility reported in the literature so far. Denoting with $r$ and $r_m^p$ the critical transmitting range in case of uniformly distributed nodes and of random waypoint mobile networks with $t_{pause} = p$, respectively, and with $v = v_{min} = v_{max}$ the node velocity, the author shows that

$$\frac{r_m^p}{r} = \frac{p + \frac{0.521405}{v}}{p} > 1$$

if $p > 0$, and that $\frac{r_m^p}{r} \to \infty$ otherwise (asymptotically, as $n \to \infty$). The author validates this result through simulations, whose results show an interesting "threshold phenomenon": for small values of $n$ ($n \leq 50$), $r_m^p$ is less than $r$, while for larger value of $n$ the situation is reversed. This phenomenon is caused by the border effect induced by random waypoint mobility, which tends to concentrate nodes in the center of the deployment region: when $n$ is small, the probability of finding at least one node close to the border is very low, and the critical transmitting range is smaller than in the stationary case. However, when $n$ is large enough, some of the nodes actually lie close to the border of the deployment region, forcing an higher value of $r_m^p$.

### 6.3 Non-homogeneous topology control

In case of non-homogeneous topology control, the more relevant effect of mobility is the message overhead generated to update nodes' transmitting range in response to node mobility. The amount of this overhead depends on the frequency with which the reconfiguration protocol used to restore the desired network topology is executed. In turn, this depends on several factors, such as the mobility pattern and the properties of the topology generated by the protocol. To clarify this point, let us consider two topology control protocols $P_1$ and $P_2$. Protocol $P_1$ builds the MST in a distributed fashion, and set the nodes' transmitting range accordingly, while protocol $P_2$ attempts to keep the number of neighbors of each node below a certain value $k$, as in the $k$-NEIGH protocol of [Blough et al. 2003]. Protocol $P_1$ is based on global and very precise information, since the MST can be built only if the exact position of every node in the network is known. In principle, $P_1$ should be reconfigured every time the relative position of any two nodes in the network changes, since this change could cause edge insertion/removal in the MST. On the other hand, $P_2$ can be easily computed in a localized fashion, and can be implemented using relatively inaccurate information such as distance estimation. In this case, the protocol should be re-executed only when the "relative neighborhood" relation of some node changes. It is quite intuitive that this occurs less frequently than edge insertion/removal in the MST. It should also be observed that having a non up-to-date topology is much more critical in case of the MST than in case of the $k$-neighbors graph: in fact, a single edge removal in the MST is sufficient to disconnect the network, while several edges can in general be removed from the $k$-neighbors graph without impairing connectivity. Overall, we can reasonably state that $P_1$ should be re-executed much more frequently than $P_2$. Further, we observe that the reconfiguration procedure needed to maintain the MST is more complicated than that required by the $k$-neighbors graph, since it relies on global information. So, we can conclude that protocol $P_1$ is not suitable to be implemented in a mobile scenario; in other words, it is not *resilient to mobility*.

From the discussion above, it is clear that a mobility resilient topology control protocol should be based on a topology which can be computed locally, and which requires little maintenance in presence of mobility. Many of the topology control protocols presented in the literature meet this requirement. However, only some of them have been defined to explicitly deal with node mobility.

In [Li et al. 2001], an adaptation of the CBTC protocol to the case of mobile networks is discussed. It is shown that, if the topology ever stabilizes and the reconfiguration protocol is executed, then the network topology remains connected. The reconfiguration procedure is adapted to the case of $k$-connectivity in [Bahramgiri et al. 2002].

In [Rodoplu and Meng 1999], the authors discuss how their protocol can be adapted to the mobile scenario, and evaluate the protocol power consumption in presence of a mobility pattern which resembles the random direction model.

The MobileGrid [Liu and Li 2002] and LINT [Ramanathan and Rosales-Hain 2000] protocols, which are based on the $k$-neighbors graph, are explicitly designed to deal with node mobility. They are zero-overhead protocols, since the estimation of the number of neighbors is based on overhearing of data and control traffic. However, no explicit guarantee on network connectivity is given, and only simulation results are reported by the authors.

A more subtle effect of mobility on certain topology control protocols is due to the possibly non-uniform node spatial distribution generated by the mobility pattern. This fact should be considered in setting fundamental protocol parameters, such as the critical neighbor number in $k$-neighbors graph based protocols [Blough et al. 2002; Liu and Li 2002; Ramanathan and Rosales-Hain 2000]. In other words, it could be the case that the number of neighbors $k$ needed to obtain connectivity w.h.p. in presence of uniform node distribution is significantly different from the value $k_m$ needed when the node distribution is non-uniform, such as in presence of random waypoint mobility. Clearly, if nodes are expected to move with random waypoint-like mobility, $k_m$ must be used instead of $k$ in the protocol implementation.

## 7. OPEN ISSUES

Topology control has received increasing attention in the wireless ad hoc network community in these recent years, as witnessed by the considerable body of research on this field reported in this paper. However, several aspects related to topology control have not been carefully investigated yet. In this final section, we outline some of them, which we hope will motivate researchers to undertake additional studies on this field.

**TC for interference.** As stated in the introduction, topology control techniques have the potential to mitigate two important problems occurring in wireless ad hoc networks: node energy consumption and radio interference. Although the acknowledged advantages of TC are two-fold, current literature on this topic focused solely on reducing energy consumption. Only very recently some authors have investigated the topology control problem with the goal of reducing radio interference. In [Burkhart et al. 2004], Burkhart et al. show that reducing energy consumption and interference might be conflicting goals, and present centralized and distributed algorithms to build low-interference topologies. Moaveni-Nejad and Li [Moaveni-Nejad and Li 2005] consider several measures of radio interference in the communication graph, and propose algorithms for building optimal or near-optimal topologies according to these metrics. However, the studies presented in [Burkhart et al. 2004] and [Moaveni-Nejad and Li 2005] are only initial steps towards a thorough understanding of the inter-relationship between range assignment and level of interference generated in the network, and further research on this topic is needed.

**More realistic models.** The point graph model used to derive most of the results presented in this paper is an idealized model of a real ad hoc network. Although point graphs have proven useful to derive "qualitative" results, they can hardly be used to obtain the accurate quantitative information needed by the network designer. So, the need for a more realistic network model is urgent.

There are several ways in which the point graph model can be modified in order to be more realistic. For instance, we could define the occurrence of links between nodes in probabilistic rather than deterministic terms. A possible model could be the following: given nodes $u$ and $v$ at distance $\delta_{u,v}$, we have a link between $u$ and $v$ with probability 1 if $\delta_{u,v} \leq \overline{\delta}$, where $\overline{\delta}$ is an arbitrary constant, and with probability $p(\delta_{u,v}) < 1$ otherwise, where $p(\delta_{u,v})$ is an arbitrary decreasing function of the distance with values in $[0,1]$. This characterization of the occurrence of a wireless link is far more realistic than the 1/0 characterization used in the point graph model. For example, there could exist nodes $u, v, w$ with $\delta_{u,v} = \delta_{u,w} > \overline{\delta}$ such that link $(u,v)$ exists and link $(u,w)$ does not. Thus, the radio coverage area is in general not regular, as it is the case in real wireless networks. Radio link models similar to the one described above have been introduced in [Faragó 2002] and [Booth et al. 2003]. In particular, Booth et al. study network connectivity under this more realistic link model, and argue that the characterization of the critical range for connectivity based on the assumption of circular coverage area can be seen as a worst-case analysis, provided the (possibly irregular) area covered by the radio signal remains the same.

Another possibility to make the network model more realistic is to take into account interferences between nodes. For example, in [Dousse et al. 2003] a bi-directional link between nodes $u$ and $v$ exists if the signal to noise ratio at the receiver is larger than some threshold, where the noise is the sum of the contribution of interferences from all other nodes and of a background noise. The authors analyze the impact of such wireless link model on network connectivity.

Note that there is another major driver for more realistic network models, namely the usage of link-layer re-transmission protocols. In fact, it turns out that it usually pays off in term of minimal overall energy consumption in presence of re-transmissions to use connections at the boundary of the radio coverage area, where the packet loss probability is below 1 but greater than 0. This fact, which has been observed in [Seada et al. 2004], should be accounted for in the design of topology control mechanisms.

Although some research on the characterization of fundamental network properties with a more realistic link model has been recently done, further investigation in this direction is needed.

**More realistic node distribution.** A simplifying assumption commonly used in the analysis of ad hoc networks is that nodes are uniformly distributed in the deployment region. Although this assumption seems reasonable in some settings, it is quite unrealistic in many scenarios. For instance, as discussed above, this assumption does not hold when the nodes move according to the random waypoint model. Further, when nodes are dispersed from a moving vehicle, the assumption of uniform distribution is only a rough approximation of the actual node distribution. Thus, the analysis of network properties in presence of non-uniform node spatial distributions is another step forward in the direction of a more realistic characterization of ad hoc networks.

**More accurate analysis of mobile networks.** More work needs to be done to investigate the effect of mobility on topology control. In particular, the following issues shall be addressed:

– *is mobility beneficial or detrimental?* On one hand, we have seen that mobility causes an increased message overhead to restore the desired topology. On the other hand, mobility has the positive effect of balancing the node energy consumption: in stationary networks, if a node $u$ has twice the transmitting range of node $v$, it is likely to deplete its battery much faster than node $v$. In presence of mobility, nodes change the transmitting range dynamically, and a more balanced energy consumption is likely to occur. Since one of the ultimate goals of topology control is to extend network lifetime, the overall effect of mobility on network lifetime should be carefully investigated.

– *determination of the optimal frequency for reconfiguration.* As outlined in Section 6, there is a trade off between the message overhead caused by a topology control protocol and the "quality" of the topology generated. In general, to have a "high quality" topology (e.g., a connected topology), we should execute the reconfiguration protocol frequently. On the other hand, each execution of the reconfiguration protocol causes a significant message overhead. The careful investigation of this trade off would help in answering the previous issue.

**Group mobility.** In most of the mobility models considered in the literature (such as the random waypoint, random direction, and Brownian-like model), nodes move independently one of each other. However, in many realistic scenarios network nodes move in groups. This could be the case, for instance, of sensors dispersed in the ocean to monitor water temperature, which are moved by ocean flows. Or, the case of cars on a freeway, which exchange messages with the purpose of rapidly propagating information about traffic conditions. Thus, the impact of group mobility on topology control should be carefully investigated.

**Implementation of TC.** Despite the considerable body of research devoted to topology control presented in this paper, and the many theoretical and simulation-based evidences of the effectiveness of topology control techniques in reducing energy consumption and/or increasing network capacity, to date there is little *experimental* evidence that topology control can be actually used to these purposes. This is perhaps the main open issue in the field.

Note that the almost complete lack of experimental results about topology control techniques *is not* due to technological problems, as current wireless network cards (see, e.g., the CISCO Aironet 802.11 cards [Cisco 2004]) and wireless sensor nodes allow the transmit power to be dynamically adjusted.

## 8. ACKNOWLEDGEMENTS

The author wishes to thank the anonymous reviewers for the many suggestions that helped improving the presentation quality of the paper.

## REFERENCES

Aldous, D. and Steele, J. 1992. Asymptotics for euclidean minimal spanning trees on random points. *Probab. Theory Relat. Fields 92*, 247–258.

Althaus, E., Calinescu, G., Mandoiu, I., Prasad, S., Tchervenski, N., and Zelikovsky, A. 2003. Power efficient range assignment in ad hoc wireless networks. In *Proc. IEEE WCNC 03*.

Bahramgiri, M., Hajiaghayi, M., and Mirrokni, V. 2002. Fault-tolerant ad 3-dimensional distributed topology control algorithms in wireless multi-hop networks. In *Proc. IEEE Int. Conf. on Computer Communications and Networks*. 392–397.

Bao, L. and Garcia-Luna-Aceves, J. 2001. Channel access scheduling in ad hoc networks with unidirectional links. In *Proc. DIALM 01*. 9–18.

Basagni, S., Bruschi, D., and Chlamtac, I. 1999. A mobility-transparent deterministic broadcast mechanism for ad hoc networks. *IEEE Transactions on Networking 7,* 6, 799–807.

Bettstetter, C. 2001. Smooth is better than sharp: A random mobility model for simulation of wireless networks. In *Proc. ACM Workshop on Modeling, Analysis and Simulation of Wireless and Mobile Systems (MSWiM)*. 19–27.

Bettstetter, C. 2002a. On the connectivity of wireless multihop networks with homogeneous and inhomogeneous range assignment. In *Proc. 56th IEEE Vehicular Technology Conference (VTC-Fall)*. 1706–1710.

Bettstetter, C. 2002b. On the minimum node degree and connectivity of a wireless multihop network. In *Proc. ACM MobiHoc 02*. 80–91.

Bettstetter, C. and Krause, O. 2001. On border effects in modeling and simulation of wireless ad hoc networks. In *Proc. IEEE Int. Conf. on Mobile and Wireless Comm. Netw. (MWCN)*.

Bettstetter, C., Resta, G., and Santi, P. 2003. The node distribution of the random waypoint mobility model for wireless ad hoc networks. *IEEE Transactions on Mobile Computing 2,* 3, 257–269.

Blough, D., Leoncini, M., Resta, G., and Santi, P. 2002. On the symmetric range assignment problem in wireless ad hoc networks. In *Proc. IFIP Conf. on Theoretical Computer Science*. 71–82.

Blough, D., Leoncini, M., Resta, G., and Santi, P. 2003. The $k$-neighbors protocol for symmetric topology control in ad hoc networks. In *Proc. ACM MobiHoc 03*. 141–152.

Blough, D., Resta, G., and Santi, P. 2002. A statistical analysis of the long-run node spatial distribution in mobile ad hoc networks. In *Proc. ACM Workshop on Modeling, Analysis, and Simulation of Wireless and Mobile Systems (MSWiM)*. 30–37.

Bollobás, B. 1985. *Random Graphs*. Academic Press, London.

Booth, L., Bruck, J., Cook, M., and Franceschetti, M. 2003. Ad hoc wireless networks with noisy links. In *Proc. IEEE Int. Symposium on Information Theory (ISIT)*.

Borbash, S. and Jennings, E. 2002. Distributed topology control algorithm for multihop wireless networks. In *Proc. IEEE Int. Joint Conference on Neural Networks*. 355–360.

Bruck, J., Franceschetti, M., and Schulman, L. 2002. Microcellular systems, random walks, and wave propagation. In *Proc. IEEE Symposium on Antennas and Propagation*. 220–223.

Burkhart, M., Rickenbach, P. V., Wattenhofer, R., and Zollinger, A. 2004. Does topology control reduce interference? In *Proc. ACM MobiHoc 04*. 9–19.

Cagali, M., Hubaux, J., and Enz, C. 2002. Minimum-energy broadcast in all-wireless networks: Np-completeness and distribution issues. In *Proc. ACM Mobicom 02*. 172–182.

Calinescu, G., Mandoiu, I., and Zelikovsky, A. 2002. Symmetric connectivity with minimum power consumption in radio networks. In *Proc. IFIP Conf. on Theoretical Computer Science*. 119–130.

Calinescu, G. and Wan, P. 2003. Range assignment for high connectivity in wireless ad hoc networks. In *Proc. AdHoc-NoW*. 235–246.

Camp, T., Boleng, J., and Davies, V. 2002. A survey of mobility models for ad hoc network research. *Wireless Communication & Mobile Computing (WCMC) 2,* 5, 483–502.

Chlamtac, I. and Faragó, A. 1999. A new approach to the design and analysis of peer-to-peer mobile networks. *ACM/Baltzer Wireless Networks 5*, 149–156.

Cisco. 2004. Aironet data sheets. In *available at http://www.cisco.com/en/US/products/hw/ /wireless*.

Clementi, A., Ferreira, A., Penna, P., Perennes, S., and Silvestri, R. 2000. The minimum range assignment problem on linear radio networks. In *Proc. 8th European Symposium on Algorithms (ESA 2000)*. 143–154.

Clementi, A., Penna, P., and Silvestri, R. 1999. Hardness results for the power range assignment problem in packet radio networks. In *Proc. 2nd International Workshop on Ap-*

*proximation Algorithms for Combinatorial Optimization Problems (RANDOM/APPROX'99)*. 197–208.

CLEMENTI, A., PENNA, P., AND SILVESTRI, R. 2000. The power range assignment problem in radio networks on the plane. In *Proc. XVII Symposium on Theoretical Aspects of Computer Science (STACS 00)*. 651–660.

DETTE, H. AND HENZE, N. 1989. The limit distribution of the largest nearest neighbor link in the unit $d$-cube. *Journal of Applied Probability 26*, 67–80.

DIAZ, J., PENROSE, M., PETITA, J., AND SERNA, M. 2000. Convergence theorems for some layout measures on random lattice and random geometric graphs. *Combinatorics, Probability, and Computing 6*, 489–511.

DOUSSE, O., BACCELLI, F., AND THIRAN, P. 2003. Impact of interferences on connectivity in ad hoc networks. In *Proc. IEEE Infocom 03*. 1724–1733.

DOUSSE, O., THIRAN, P., AND HASLER, M. 2002. Connectivity in ad hoc and hybrid networks. In *Proc. IEEE Infocom 02*. 1079–1088.

ESTRIN, D., GOVINDAN, R., HEIDEMANN, J., AND KUMAR, S. 1999. Next century challenges: Scalable coordination in sensor networks. In *Proc. ACM Mobicom 99*. 263–270.

FARAGÓ, A. 2002. Scalable analysis and design of ad hoc networks via random graph theory. In *Proc. ACM DIAL-M 02*. 43–50.

GAO, J., GUIBAS, L., HERSHBERGER, J., ZHANG, L., AND ZHU, A. 2001. Geometric spanners for routing in mobile networks. In *Proc. ACM MobiHoc 01*. 45–55.

GERLA, M. AND TSAI, J. T.-C. 1995. Multicluster, mobile, multimedia radio networks. *ACM/Baltzer Wireless Networks 1*, 255–265.

GOODMAN, J. AND O'ROURKE, J. 1997. *Handbook of Discrete and Computational Geometry*. CRC Press, New York.

GROSSGLAUSER, M. AND TSE, D. 2001. Mobility increases the capacity of ad hoc wireless networks. In *Proc. IEEE Infocom 01*. 1360–1369.

GUPTA, P. AND KUMAR, P. 1998. Critical power for asymptotic connectivity in wireless networks. *Stochastic Analysis, Control, Optimization and Applications*, 547–566.

GUPTA, P. AND KUMAR, P. 2000. The capacity of wireless networks. *IEEE Trans. Information Theory 46,* 2, 388–404.

HEINZELMAN, W., KULIK, J., AND BALAKRISHNAN, H. 1999. Adaptive protocols for information dissemination in wireless sensor networks. In *Proc. ACM Mobicom 99*. 174–185.

HOLST, L. 1980. On multiple covering of a circle with random arcs. *Journal of Applied Probability 16*, 284–290.

HUANG, Z., SHEN, C., SRISATHAPORNPHAT, C., AND JAIKAEO, C. 2002. Topology control for ad hoc networks with directional antennas. In *Proc. IEEE Int. Conference on Computer Communications and Networks*. 16–21.

IEEE. 1999. Wireless lan medium access control and physical layer specifications. In *IEEE 802.11 Standard (IEEE Computer Society LAN MAN Standards Committee)*.

JANSON, S., KNUTH, D., LUCZAK, T., AND PITTEL, B. 1993. The birth of the giant component. *Random Structures and Algorithms 4,* 3, 233– 359.

JOHNSON, D. AND MALTZ, D. 1996. Dynamic source routing in ad hoc wireless networks. *Mobile Computing, Kluwer Academic Publishers*, 153–181.

KHAN, J., KATZ, R., AND PISTER, K. 2000. Emerging challenges: Mobile networking for smart dust. *Journal of Communication and Networks 2,* 3, 186–196.

KIM, D., TOH, C., AND CHOI, Y. 2001. On supporting link asymmetry in mobile ad hoc networks. In *Proc. IEEE Globecom 01*. 2798–2803.

KIROUSIS, L., KRANAKIS, E., KRIZANC, D., AND PELC, A. 2000. Power consumption in packet radio networks. *Theoretical Computer Science 243*, 289–305.

KO, Y. AND VAIDYA, N. 1998. Location-aided routing (lar) in mobile ad hoc networks. In *Proc. ACM Mobicom 98*. 66–75.

KOLCHIN, V., SEVAST'YANOV, B., AND CHISTYAKOV, V. 1978. *Random Allocations*. V.H. Winston and Sons, Washington D.C.

LI, J., BLAKE, C., COUTO, D. D., LEE, H. I., AND MORRIS, R. 2001. Capacity of ad hoc wireless networks. In *Proc. ACM Mobicom 01*. 61–69.

LI, L., HALPERN, J., BAHL, P., WANG, Y., AND WATTENHOFER, R. 2001. Analysis of a cone-based distributed topology control algorithm for wireless multi-hop networks. In *Proc. ACM PODC 01*. 264–273.

LI, N. AND HOU, J. 2004. Flss: a fault-tolerant topology control algorithm for wireless networks. In *Proc. ACM Mobicom 04*. 275–286.

LI, N., HOU, J., AND SHA, L. 2003. Design and analysis of an mst-based topology control algorithm. In *Proc. IEEE Infocom 03*. 1702–1712.

LI, X. AND WAN, P. 2001. Constructing minimum energy mobile wireless networks. In *Proc. ACM Mobihoc 01*. 283–286.

LI, X., WAN, P., WANG, Y., AND FRIEDER, O. 2002. Sparse power efficient topology for wireless networks. In *Proc. IEEE Hawaii Int. Conference on System Sciences (HICSS)*.

LI, X., WANG, Y., AND SONG, W. 2004. Applications of k-local mst for topology control and broadcasting in wireless ad hoc networks. *IEEE Transactions on Parallel and Distributed Systems 15,* 12, 1057–1069.

LIANG, W. 2002. Constructing minimum-energy broadcast trees in wireless ad hoc networks. In *Proc. ACM Mobihoc 02*. 112–122.

LIU, J. AND LI, B. 2002. Mobilegrid: Capacity-aware topology control in mobile ad hoc networks. In *Proc. IEEE Int. Conference on Computer Communications and Networks*. 570–574.

LOYD, E., LIU, R., MARATHE, M., RAMANATHAN, R., AND RAVI, S. 2002. Algorithmic aspects of topology control problems for ad hoc networks. In *Proc. ACM Mobihoc 02*. 123–134.

MAINWARING, A., POLASTRE, J., SZEWCZYK, R., CULLER, D., AND ANDERSON, J. 2002. Wireless sensor networks for habitat monitoring. In *Proc. ACM WSNA 02*. 88–97.

MARINA, M. AND DAS, S. 2002. Routing performance in the presence of unidirectional links in multihop wireless networks. In *Proc. ACM Mobihoc 02*. 12–23.

MEESTER, R. AND ROY, R. 1996. *Continuum Percolation*. Cambridge University Press, Cambridge, MA.

MICHAIL, A. AND EPHREMIDES, A. 2003. Energy-efficient routing for connection-oriented traffic in wireless ad hoc networks. *Mobile Networks and Applications 8,* 5, 517–533.

MOAVENI-NEJAD, K. AND LI, X. 2005. Low-interference topology control for wireless ad hoc networks. In *Ad Hoc and Sensor Networks: an International Journal (to appear)*.

MURTHY, S. AND GARCIA-LUNA-ACEVES, J. 1996. An efficient routing protocol for wireless networks. *Mobile Networks and Applications 1,* 2, 183–197.

NARAYANASWAMY, S., KAWADIA, V., SREENIVAS, R., AND KUMAR, P. 2002. Power control in ad hoc networks: Theory, architecture, algorithm and implementation of the compow protocol. In *Proc. European Wireless 2002*. 156–162.

PALMER, E. 1985. *Graphical Evolution*. John Wiley and Sons, New York.

PANCHAPAKESAN, P. AND MANJUNATH, D. 2001. On the transmission range in dense ad hoc radio networks. In *Proc. IEEE SPCOM 2001*.

PAPADIMITRIOU, I. AND GEORGIADIS, L. 2004. Energy-aware broadcast trees in wireless networks. *Mobile Networks and Applications 9,* 6, 567–581.

PEARLMAN, M., HAAS, Z., AND MANVELL, B. 2000. Using multi-hop acknowledgements to discover and reliably communicate over unidirectional links in ad hoc networks. In *Proc. Wireless Communications and Networking Conference (WCNC)*. 532–537.

PENROSE, M. 1997. The longest edge of the random minimal spanning tree. *The Annals of Applied Probability 7,* 2, 340–361.

PENROSE, M. 1998. Extremes for the minimal spanning tree on normally distributed points. *Advances in Applied Probability 30*, 628–639.

PENROSE, M. 1999a. On $k$-connectivity for a geometric random graph. *Random Structures and Algorithms 15,* 2, 145–164.

PENROSE, M. 1999b. A strong law for the largest nearest-neighbour link between random points. *Journal of London Mathematical Society 60,* 2, 951–960.

PENROSE, M. 1999c. A strong law for the longest edge of the minimal spanning tree. *The Annals of Probability 27,* 1, 246–260.

PHILIPS, T., PANWAR, S., AND TANTAWI, A. 1989. Connectivity properties of a packet radio network model. *IEEE Trans. Information Theory 35,* 5, 1044–1047.

PIRET, P. 1991. On the connectivity of radio networks. *IEEE Trans. Information Theory 37,* 5, 1490–1492.

POTTIE, G. AND KAISER, W. 2000. Wireless integrated network sensors. *Communications of the ACM 43,* 5, 51–58.

PRAKASH, R. 2001. A routing algorithm for wireless ad hoc networks with unidirectional links. *ACM/Kluwer Wireless Networks 7,* 6, 617–625.

RAJARAMAN, R. 2002. Topology control and routing in ad hoc networks: A survey. *SIGACT News 33,* 2, 60–73.

RAMANATHAN, R. AND ROSALES-HAIN, R. 2000. Topology control of multihop wireless networks using transmit power adjustment. In *Proc. IEEE Infocom 00*. 404–413.

RAMASUBRAMANIAN, V., CHANDRA, R., AND MOSSE, D. 2002. Providing a bidirectional abstraction for unidirectional ad hoc networks. In *Proc. IEEE Infocom 02*. 1258–1267.

RAPPAPORT, T. 2002. *Wireless Communications: Principles and Practice, Second Edition*. Prentice Hall, Upper Saddle River, NJ.

RODOPLU, V. AND MENG, T. 1999. Minimum energy mobile wireless networks. *IEEE Journal Selected Areas in Comm. 17,* 8, 1333–1344.

ROYER, E., MELLIAR-SMITH, P., AND MOSER, L. 2001. An analysis of the optimum node density for ad hoc mobile networks. In *Proc. IEEE International Conference on Communications*. 857–861.

SADLER, C., ZHANG, P., MARTONOSI, M., AND LYON, S. 2004. Hardware design experiences in zebranet. In *Proc. ACM SenSys 04*. 227–238.

SANCHEZ, M., MANZONI, P., AND HAAS, Z. 1999. Determination of critical transmitting range in ad hoc networks. In *Proc. Multiaccess, Mobility and Teletraffic for Wireless Communications Conference*.

SANTI, P. 2005. The critical transmitting range for connectivity in mobile ad hoc networks. *IEEE Transactions on Mobile Computing 4,* 3, 310–317.

SANTI, P. AND BLOUGH, D. 2002. An evaluation of connectivity in mobile wireless ad hoc networks. In *Proc. IEEE DSN 2002*. 89–98.

SANTI, P. AND BLOUGH, D. 2003. The critical transmitting range for connectivity in sparse wireless ad hoc networks. *IEEE Transactions on Mobile Computing 2,* 1, 25–39.

SANTI, P., BLOUGH, D., AND VAINSTEIN, F. 2001. A probabilistic analysis for the range assignment problem in ad hoc networks. In *Proc. ACM Mobihoc 01*. 212–220.

SCHWIEBERT, L., GUPTA, S., AND WEINMANN, J. 2001. Research challenges in wireless networks of biomedical sensors. In *Proc. ACM Mobicom 01*. 151–165.

SEADA, K., ZUNIGA, M., HELMY, A., AND KRISHNAMACHARI, B. 2004. Energy-efficient forwarding strategies for geographic routing in lossy wireless sensor networks. In *Proc. ACM SenSys 04*. 108–121.

SEN, A. AND HUSON, M. 1996. A new model for scheduling packet radio networks. In *Proc. IEEE Infocom 96*. 1116–1124.

SONG, W., WANG, Y., LI, X., AND FRIEDER, O. 2004. Localized algorithms for energy efficient topology in wireless ad hoc networks. In *Proc. ACM MobiHoc*. 98–108.

SRIVASTAVA, M., MUNTZ, R., AND POTKONJAK, M. 2001. Smart kindergarten: Sensor-based wireless networks for smart developmental problem-solving environments. In *Proc. ACM Mobicom 01*. 132–138.

STEELE, J. 1988. Growth rates of euclidean minimal spanning trees with power weighted edges. *Annals of Probability 16*, 1767–1787.

STEERE, D., BAPTISTA, A., MCNAMEE, D., PU, C., AND WALPOLE, J. 2000. Research challenges in environmental observation and forecasting systems. In *Proc. ACM Mobicom 00*. 292–299.

SZEWCZYK, R., MAINWAIRING, A., POLASTRE, J., AND CULLER, D. 2004. An analysis of a large scale habitat monitoring application. In *Proc. ACM SenSys 04*. 214–226.

WAN, P., CALINESCU, G., LI, X., AND FRIEDER, O. 2002. Minimum energy broadcasting in static ad hoc wireless networks. *ACM/Kluwer Wireless Networks 8,* 6, 607–617.

WAN, P. AND YI, C. 2004. Asymptotical critical transmission radius and critical neighbor number for k-connectivity in wireless ad hoc networks. In *Proc. ACM MobiHoc 04*. 1–8.

WANG, W., LI, X., MOAVENINEJAD, K., WANG, Y., AND SONG, W. 2003. The spanning ratio of $\beta$-skeletons.

WANG, Y., LI, X., AND FRIEDER, O. 2002. Distributed spanners with bounded degree for wireless ad hoc networks. *International Journal of Foundations of Computer Science (to appear)*.

WATTENHOFER, R., LI, L., BAHL, P., AND WANG, Y. 2001. Distributed topology control for power efficient operation in multihop wireless ad hoc networks. In *Proc. IEEE Infocom 01*. 1388–1397.

WATTENHOFER, R. AND ZOLLINGER, A. 2004. Xtc: A practical topology control algorithm for ad hoc networks. In *4th International Workshop on Algorithms for Wireless, Mobile, Ad Hoc and Sensor Networks (WMAN)*.

WIESELTHIER, J., NGUYEN, G., AND EPHREMIDES, A. 2000. On the construction of energy-efficient broadcast and multicast trees in wireless networks. In *Proc. IEEE Infocom 00*. 585–594.

XUE, F. AND KUMAR, P. 2004. The number of neighbors needed for connectivity of wireless networks. *Wireless Networks 10,* 2, 169–181.

YI, C. AND WAN, P. 2005. Asymptotic critical transmission ranges for connectivity in wireless ad hoc networks with bernoulli nodes. In *Proc. IEEE Wireless Communications and Networking Conference (WCNC) (to appear)*.

YI, C., WAN, P., LI, X., AND FRIEDER, O. 2003. Asymptotic distribution of the number of isolated nodes in wireless ad hoc networks with bernoulli nodes. In *Proc. IEEE Wireless Communications and Networking Conference (WCNC)*. 1585–1590.

YUKICH, J. 2000. Asymptotics for weighted minimal spanning trees on random points. *Stochastic Processes and their Appl. 85*, 123–128.

Received April 2003; revised December 2004; accepted (with minor revisions) May 2005.