Subdivision Surfaces

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Surface representations

- large class of surfaces
- interactive manipulation/display
- numerical modeling tasks
Representations

Desirable properties

- compatible with hardware and existing representations
- scalable: large data on small machines
- solid mathematical foundation
- suitable for animation and simulation
Representations

General philosophy

- same core representation for multiple purposes
  - transmission
  - rendering
  - simulation
  - editing
What does it offer?

- good editing semantics
- deep connection with wavelets
  - compression, solvers, speed/accuracy tradeoff, approximation properties
- builtin support for LOD display
- very efficient
Splined Surfaces

Advantages
- high level control (control points)
- compact representation
- multiresolution structure

Disadvantages
- difficult to maintain and manage
- naïve rendering of large models slow
Polygonal Meshes

Advantages

- very general
- direct hardware implementation

Disadvantages

- heavy weight representation
- good editing semantics difficult
- limited multiresolution structure
Subdivision Surfaces

Important modeling primitive

- smooth, arbitrary topology surface modeling
- generalizes spline patches
- covers range of representations from “pure” spline patches to “pure” meshes
- BUT: special connectivity (more on that later)
**Subdivision**

Smooth surfaces as the limit of a sequence of successive refinements
Subdivision

Topological rule

- refinement of abstract simplicial complex
- refine a triangular graph
**SUBDIVISION**

Geometric rule

- Geometry is a map defined on graph
- Extension rule

Even at level i

Odd at level i
INTERPOLATING

Keep old points, insert new ones

- affine combination of nearby points
Approximating

Insert new, smooth **new** and **old**
- generalizes spline patches
Subdivision

Established schemes

- Catmull-Clark
  - generalizes bi-cubic patches
- Loop
  - generalizes quartic box splines
- many others:
  - Doo-Sabin, Butterfly, Kobbelt, Peters/Reif
**Subdivision**

**Properties**

- affine invariance
- local definition
- compact support
Why Subdivision?

Advantages
- arbitrary topology, smooth surfaces
- support for compression and LOD
- suitable for wavelet-based numerical solvers

Scalability
- large datasets on small machines
ALGORITHMS

Properties

- exact evaluation
  - value, tangent planes, derivatives
  - moments
- little computation
- simple data structures
- integrates well with spline methods
Example: Loop Scheme

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Example: Loop scheme

For a “good” scheme, recursive application approximates a smooth surface
Control Points

- Vertices of the initial mesh define the surface
- Each vertex influences a finite part of the surface
TRIANGULAR AND QUADRILATERAL SUBDIVISION
Subdivision and splines

- Uniform splines can be computed using subdivision rules.
- Triangular spline.
For splines, the control mesh is regular
**Extraordinary Vertices**

**Triangular meshes**
- Regular vertices: valence 6
- Extraordinary vertices: valence ≠ 6

**Quad meshes**
- Regular vertices: valence 4
- Extraordinary vertices: valence ≠ 4
Constructing the rules

- Start with spline rules (or other smooth rules)
- Define rules for:

Extraordinary vertices

Boundaries

Creases etc.
Invariance under rotations and translations

Small support

Smoothness and Fairness

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AFFINE INVARIANCE

Subdivide

transform T

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Affine Invariance

the coefficients of any mask should sum up to

$$p = \sum a_i p_i$$

$$\sum a_i (p_i + t) = \left( \sum a_i \right) t + p$$
SUBDIVISION ZOO
Classification of schemes

Classification criteria

- type of refinement rule (primal or dual)
- type of mesh (triangular or quad or...)
- approximating or interpolating
Refinement rules

- **Primal (vertex insertion)**
- **Dual (corner cutting)**
Approximation and Interpolation

Advantages

- approximating schemes
  - based on splines, small support
- interpolating schemes
  - control points on surface
  - in-place implementation
Subdivision schemes

- Primal (vertex insertion)
  - Approximating: Catmull-Clark, Loop
  - Interpolating: Kobbelt, Butterfly

- Dual (corner cutting)
  - Approximating: Doo-Sabin, Midedge
  - Interpolating: ?
Boundaries and Creases

- special rules on and near the boundary
- boundary independent of the interior

Boundaries:

1/8  3/4  1/8

Creases:

1/8  3/4  1/8
Loop scheme, boundaries and creases

Hoppe et al

Our rules

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**Modified Butterfly Scheme**

- Triangular meshes, interpolating: only one rule
- Needs larger support

Regular mask
extraordinary vertices

- coefficients derived to ensure good eigenvalues and eigenvectors

\[ s_j = \frac{1}{K} \left( \frac{1}{4} + \cos \frac{2j\pi}{K} + \frac{1}{2} \cos \frac{4j\pi}{K} \right) \]

\[ K > 4 \]
Catmull-Clark Scheme

- quadrilateral, approximating
- tensor-product bicubic splines

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**Catmull-Clark Scheme**

Reduction to a quadrilateral mesh

- do one step of subdivision with special rules;
  all polygons become quads

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Catmull-Clark scheme

Extraordinary vertices

\[ \gamma = \frac{1}{4K} \]

\[ \beta = \frac{3}{2K} \]
Catmull-Clark Scheme

Boundaries and creases

- cubic spline (same as Loop!)
- need to fix rules for C1-continuity
**Kobbelt Scheme**

- Quadrilateral, interpolating
- Tensor product 4-point scheme

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Kobbelt scheme

extraordinary vertices

- computation in two steps
  - edge vertices
  - face vertices

- two ways to compute
- results are the same
Doo-Sabin scheme
- dual scheme, quadrilateral
- extends tensor-product biquadratic splines
Doo-Sabin scheme

- after one step, all valences = 4
- rule for extraordinary polygons:

\[
\begin{align*}
\alpha_0 &= \frac{1 + 5K}{4} \\
\alpha_i &= \frac{1}{K} \left( 3 + 2 \cos \frac{2i\pi}{K} \right) \\
\text{for } i &= 1 \ldots K - 1
\end{align*}
\]
**Midedge Scheme**

- dual scheme, quadrilateral
- extends 4-directional box spline
Summary

- Primal (vertex insertion)
  - Approximating: Catmull-Clark
  - Interpolating: Loop

- Dual (corner cutting)
  - Approximating: Kobbelt
  - Interpolating: Butterfly
  - Doo-Sabin, Midedge
Limitations of Subdivision

- no C2 with small support
- decrease of smoothness with valence
- ripples
- no direct control of fairness
Limitations of stationary subdivision

Problems with curvature continuity

- The only practical C2 schemes (Umlauf) have “flat spots” at extraordinary vertices.
- “True” C2 has to have very large support.
- Lack of C2-continuity = nonsmooth normal changes.
- Visible for high valences.
**LIMITATIONS OF SUBDIVISION**

- Decrease of smoothness with valence; possible to fix

Loop

Modified Loop
Limitations of Subdivision

- Ripples

Loop

Modified Loop

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Limitations of Subdivision

- Uneven structure of the mesh
Extensions

Umlauf
- nonstationary; C2 surfaces; small size of flat spot

Sederberg et al
- allows one to integrate NURBS with subdivision

Variational subdivision: Kobbelt, Warren
- slower, more complex; higher quality surfaces