

Ray Tracing I: Basics

Today

- Basic algorithms
- Overview of pbrt
- Ray-surface intersection for single surface

Next lecture

- Techniques to accelerate ray tracing of large numbers of geometric primitives

Classic Ray Tracing

Greeks: Do light rays proceed from the eye to the light, or from the light to the eye?

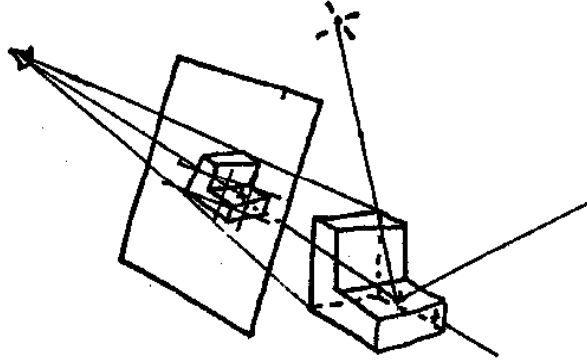
Three ideas about light

1. Light rays travel in straight lines (mostly)
2. Light rays do not interfere with each other if they cross (light is invisible!)
3. Light rays travel from the light sources to the eye (but the physics is invariant under path reversal - reciprocity).

Ray Tracing in Computer Graphics

Appel 1968 - Ray casting

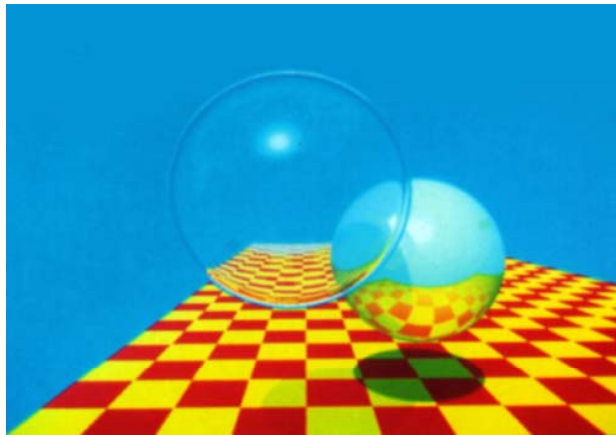
- 1. Generate an image by sending one ray per pixel**
- 2. Check for shadows by sending a ray to the light**



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Ray Tracing in Computer Graphics



Whitted 1979

Recursive ray tracing (reflection and refraction)

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Ray Tracing Video

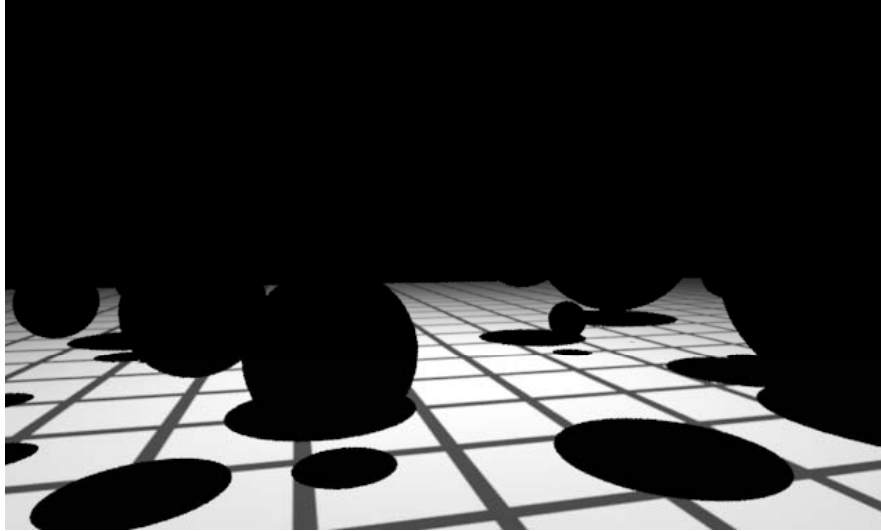
Spheres-over-plane.pbrt (g/m 10)



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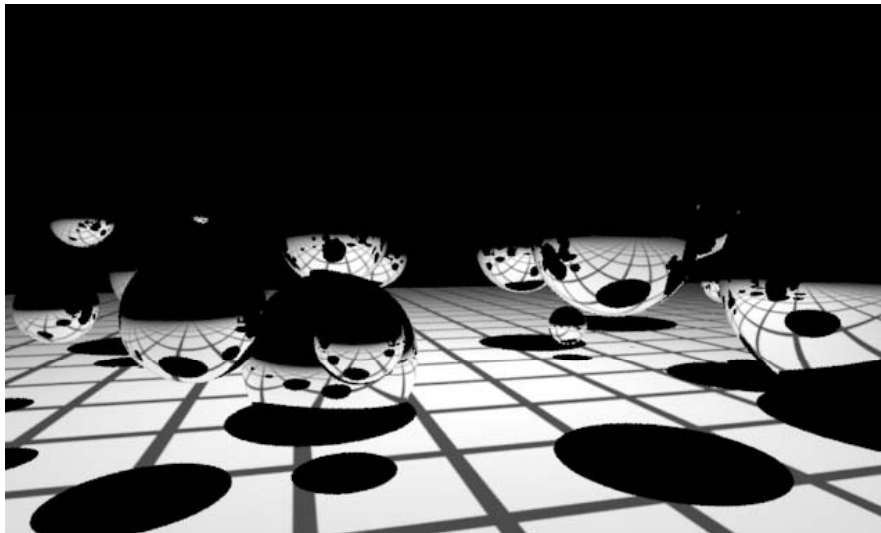
Spheres-over-plane.pbrt (mirror 0)



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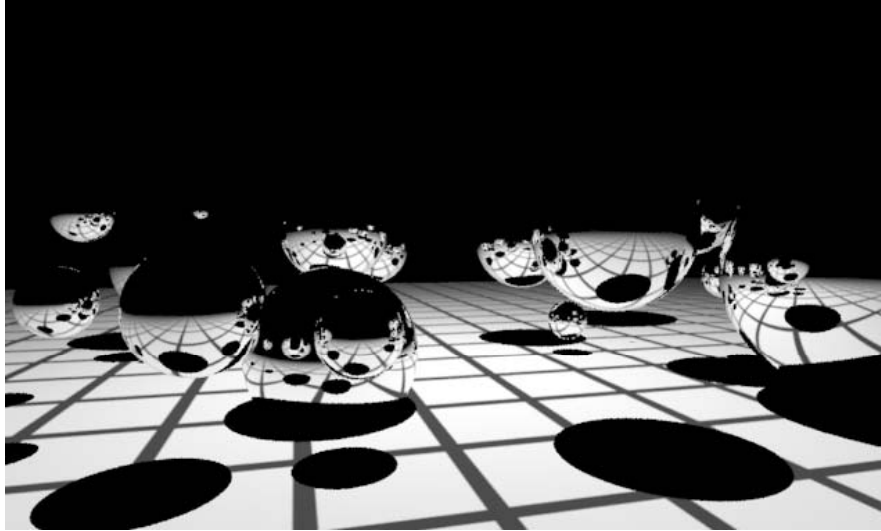
Spheres-over-plane.pbrt (mirror 1)



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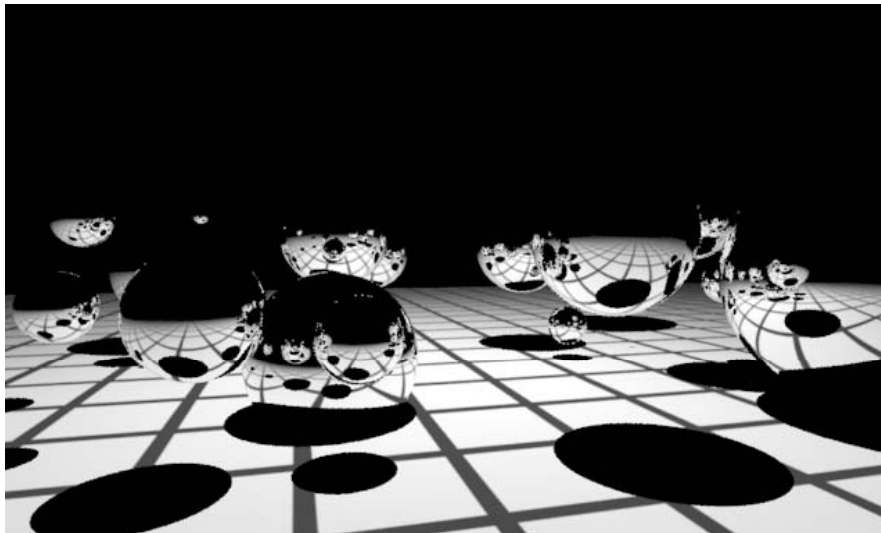
Spheres-over-plane.pbrt (mirror 2)



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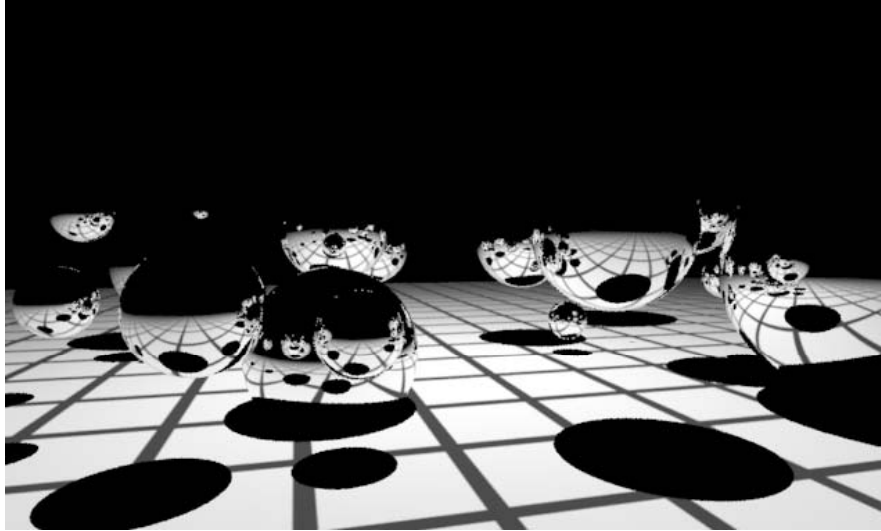
Spheres-over-plane.pbrt (mirror 5)



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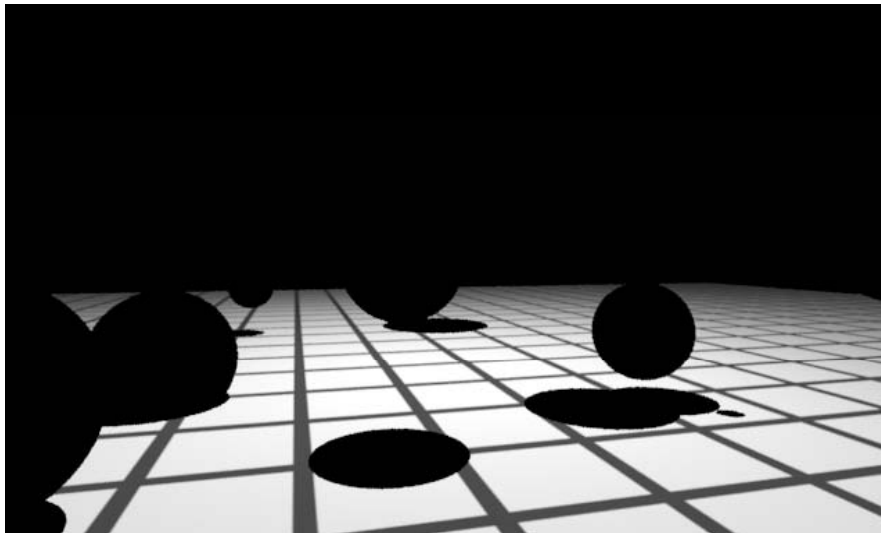
Spheres-over-plane.pbrt (mirror 10)



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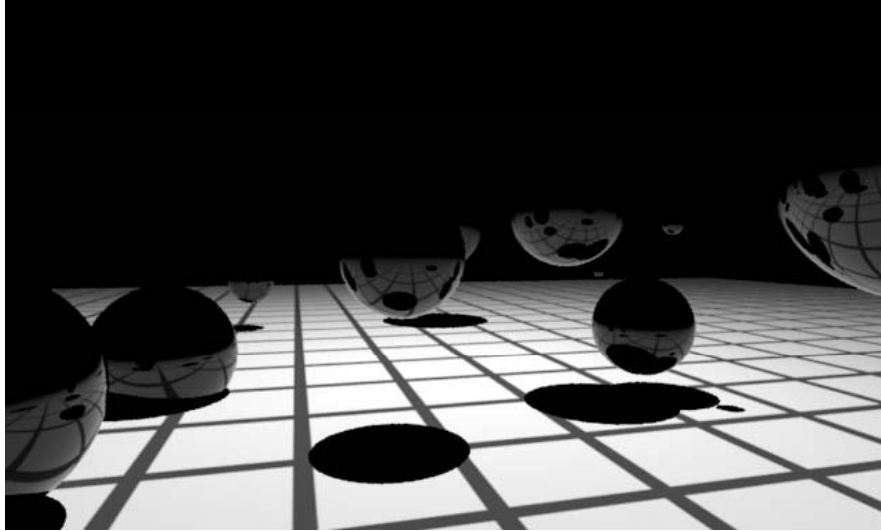
Spheres-over-plane.pbrt (glass 0)



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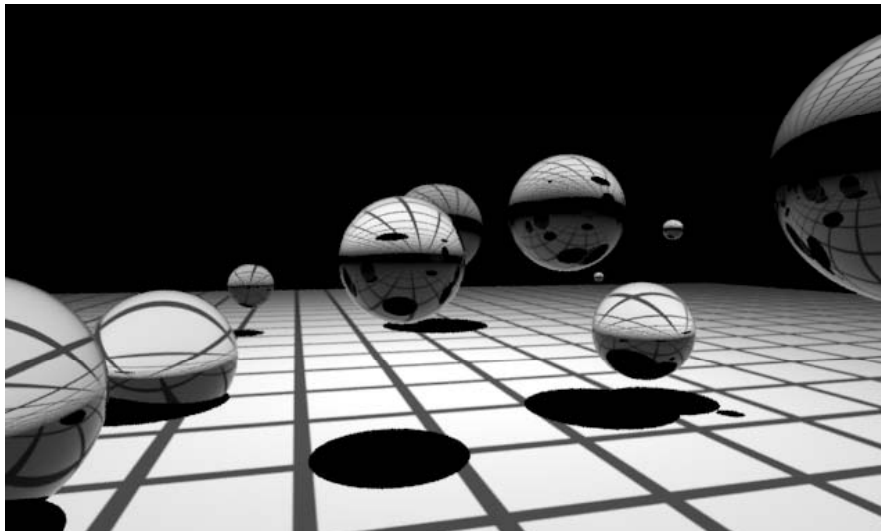
Spheres-over-plane.pbrt (glass 1)



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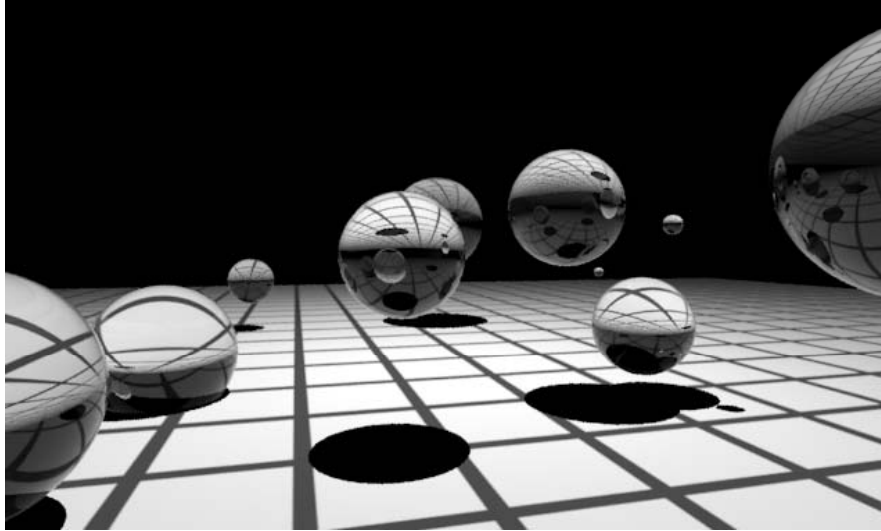
Spheres-over-plane.pbrt (glass 2)



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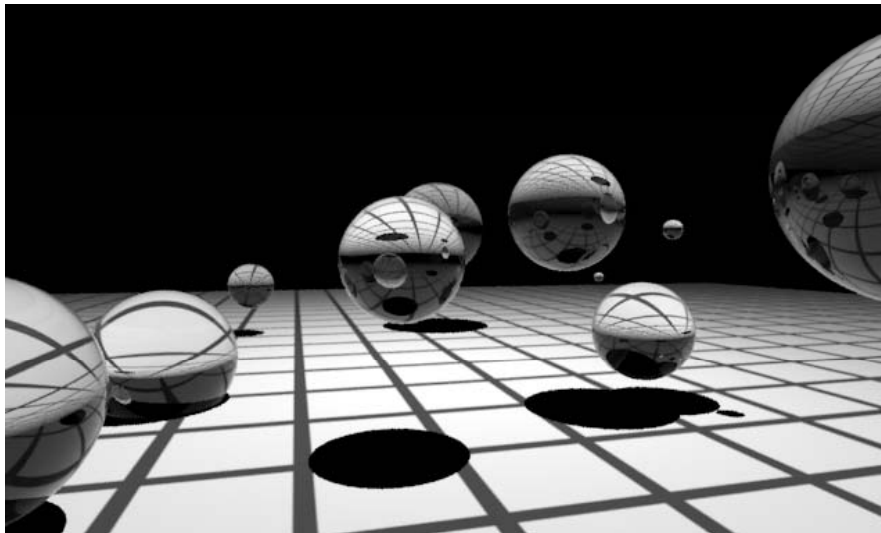
Spheres-over-plane.pbrt (glass 5)



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Spheres-over-plane.pbrt (glass 10)



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Spheres-over-plane.pbrt (glass 10)



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Spheres-over-plane.pbrt (g/m 10)



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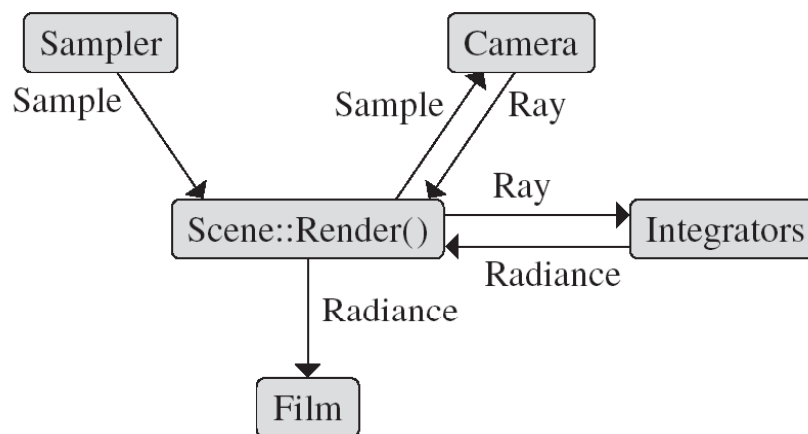
Table 1.1: Plug-ins. pbrt supports 13 types of plug-in objects that can be loaded at run time based on the contents of the scene description file. The system can be extended with new plug-ins, without needing to be recompiled itself.

Base class	Directory 📁	Section
Shape	shapes/	3.1
Primitive	accelerators/	4.1
Camera	cameras/	6.1
Film	film/	8.1
Filter	filters/	7.6
Sampler	samplers/	7.2
ToneMap	tonemaps/	8.4
Material	materials/	10.2
Texture	textures/	11.3
VolumeRegion	volumes/	12.3
Light	lights/	13.1
SurfaceIntegrator	integrators/	16
VolumeIntegrator	integrators/	17

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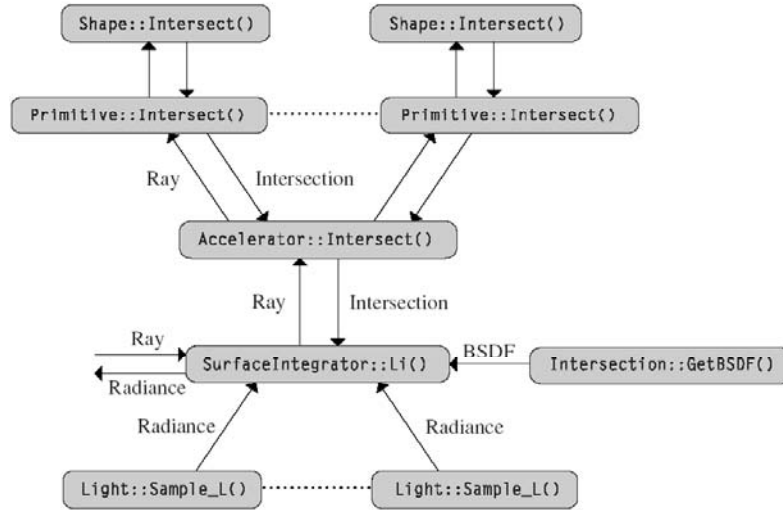
PBRT Architecture



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PBRT Architecture



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Ray-Surface Intersection

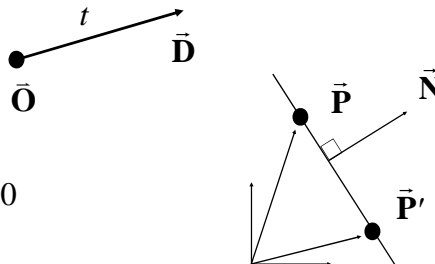
Ray-Plane Intersection

Ray: $\vec{P} = \vec{O} + t\vec{D}$

$$0 \leq t < \infty$$

Plane: $(\vec{P} - \vec{P}') \cdot \vec{N} = 0$

$$ax + by + cz + d = 0$$



Solve for intersection

Substitute ray equation into plane equation

$$(\vec{P} - \vec{P}') \cdot \vec{N} = (\vec{O} + t\vec{D} - \vec{P}') \cdot \vec{N} = 0$$

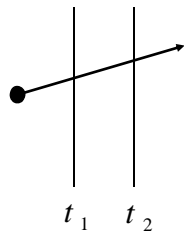
$$t = -\frac{(\vec{O} - \vec{P}') \cdot \vec{N}}{\vec{D} \cdot \vec{N}}$$

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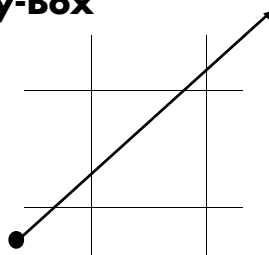
Ray-Polyhedra

Ray-Slab

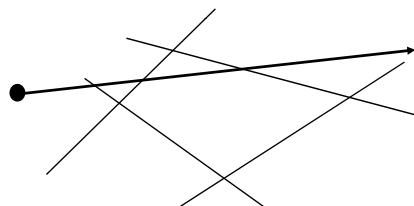


Note: Procedural geometry

Ray-Box



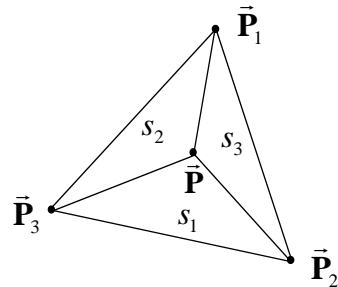
Ray-Convex Polyhedra



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Ray-Triangle Intersection 1



$$s_1 = \text{area}(\Delta P P_2 P_3)$$

$$s_2 = \text{area}(\Delta P P_3 P_1)$$

$$s_3 = \text{area}(\Delta P P_1 P_2)$$

Barycentric coordinates

$$\vec{P} = s_1 \vec{P}_1 + s_2 \vec{P}_2 + s_3 \vec{P}_3$$

Inside triangle criteria

$$0 \leq s_1 \leq 1$$

$$0 \leq s_2 \leq 1$$

$$0 \leq s_3 \leq 1$$

$$s_1 + s_2 + s_3 = 1$$

Ray-Triangle Intersection 2

$$\vec{P} = s_1 \vec{P}_1 + s_2 \vec{P}_2 + s_3 \vec{P}_3$$

$$\begin{bmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} \mathbf{P} \end{bmatrix}$$

Ray-Triangle Intersection 3

$$s_1 = \frac{\begin{vmatrix} \mathbf{P} & \mathbf{P}_2 & \mathbf{P}_3 \\ \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 \end{vmatrix}}{\begin{vmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 \end{vmatrix}} = \frac{\mathbf{P} \cdot \mathbf{P}_2 \times \mathbf{P}_3}{\Delta} \propto \mathbf{P} \cdot \mathbf{P}_2 \times \mathbf{P}_3$$

$$s_2 = \frac{\begin{vmatrix} \mathbf{P}_1 & \mathbf{P} & \mathbf{P}_3 \\ \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 \end{vmatrix}}{\begin{vmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 \end{vmatrix}} = \frac{\mathbf{P} \cdot \mathbf{P}_3 \times \mathbf{P}_1}{\Delta} \propto \mathbf{P} \cdot \mathbf{P}_3 \times \mathbf{P}_1$$

$$s_3 = \frac{\begin{vmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P} \\ \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 \end{vmatrix}}{\begin{vmatrix} \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 \end{vmatrix}} = \frac{\mathbf{P} \cdot \mathbf{P}_1 \times \mathbf{P}_2}{\Delta} \propto \mathbf{P} \cdot \mathbf{P}_1 \times \mathbf{P}_2$$

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} \mathbf{P}_2 \times \mathbf{P}_3 \\ \mathbf{P}_3 \times \mathbf{P}_1 \\ \mathbf{P}_1 \times \mathbf{P}_2 \end{bmatrix} [\mathbf{P}]$$

↑
Precompute

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Triple Product

$$\vec{\mathbf{P}}_1 \cdot \vec{\mathbf{P}}_2 \times \vec{\mathbf{P}}_3 = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

$$\text{volume}(\mathbf{P}_1 \mathbf{P}_2 \mathbf{P}_3) \propto \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

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Moller-Trumbore Algorithm

$$\vec{O} + t\vec{D} = (1 - b_1 - b_2)\vec{P}_0 + b_1\vec{P}_1 + b_2\vec{P}_2$$

$$\begin{bmatrix} t \\ b_1 \\ b_2 \end{bmatrix} = \frac{1}{\vec{S}_1 \cdot \vec{E}_1} \begin{bmatrix} \vec{S}_2 \cdot \vec{E}_2 \\ \vec{S}_1 \cdot \vec{S} \\ \vec{S}_2 \cdot \vec{D} \end{bmatrix}$$

Where:

$$\vec{E}_1 = \vec{P}_1 - \vec{P}_0$$

$$\vec{E}_2 = \vec{P}_2 - \vec{P}_0$$

$$\vec{S} = \vec{O} - \vec{P}_0$$

$$\vec{S}_1 = \vec{D} \times \vec{E}_2$$

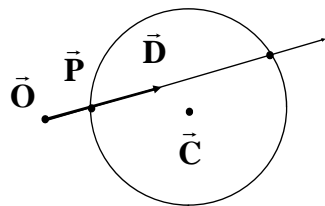
$$\vec{S}_2 = \vec{S} \times \vec{E}_1$$

Cost = (1 div, 27 mul, 17 add)

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Ray-Sphere Intersection



Ray: $\vec{P} = \vec{O} + t\vec{D}$

Sphere: $(\vec{P} - \vec{C})^2 - R^2 = 0$

$$(\vec{O} + t\vec{D} - \vec{C})^2 - R^2 = 0$$

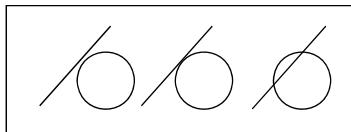
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$at^2 + bt + c = 0$$

$$a = \vec{D}^2$$

$$b = 2(\vec{O} - \vec{C}) \cdot \vec{D}$$

$$c = (\vec{O} - \vec{C})^2 - R^2$$



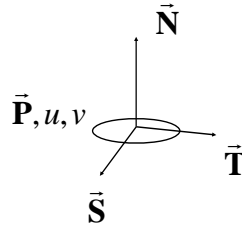
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Geometric Methods: Normals

e.g. Sphere

$$\vec{N} = \vec{P} - \vec{C}$$



$$x = \sin \theta \cos \phi$$

$$y = \sin \theta \sin \phi$$

$$z = \cos \theta$$

$$\vec{P} = (x, y, z)$$

$$\frac{\partial \vec{P}}{\partial \theta} = (\cos \theta \cos \phi, \cos \theta \sin \phi, -\sin \theta)$$

$$\frac{\partial \vec{P}}{\partial \phi} = (-\sin \theta \sin \phi, \sin \theta \cos \phi, 0)$$

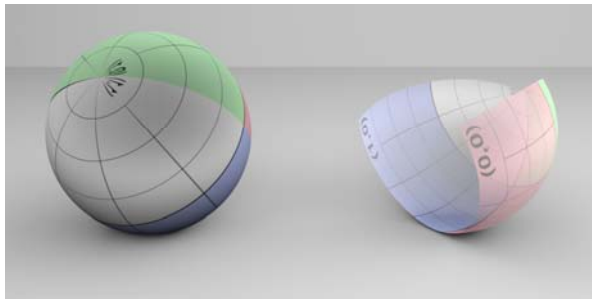
$$\vec{N} = \frac{\partial \vec{P}}{\partial \theta} \times \frac{\partial \vec{P}}{\partial \phi}$$

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Geometric Methods: Parameters

e.g. Sphere



$$x = \sin \theta \cos \phi$$

$$y = \sin \theta \sin \phi$$

$$z = \cos \theta$$

$$\theta = \theta_{\min} + v(\theta_{\max} - \theta_{\min})$$

$$\phi = u\phi_{\max}$$

$$\phi = \tan^{-1}(x, y)$$

$$\theta = \cos^{-1} z$$

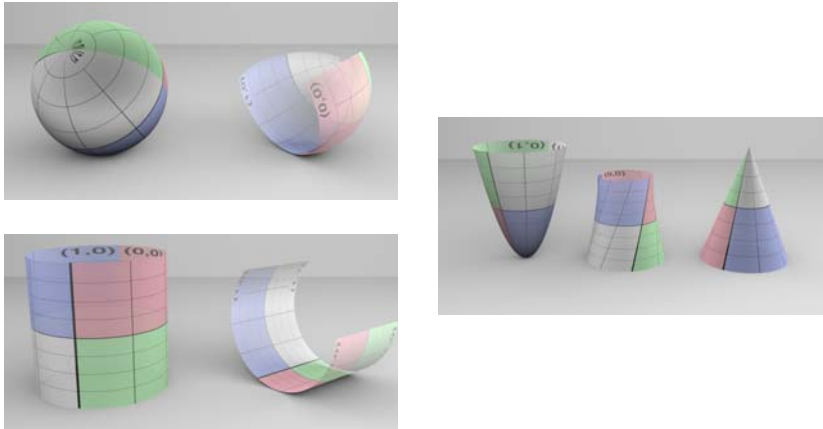
$$v = (\theta - \theta_{\min}) / (\theta_{\max} - \theta_{\min})$$

$$u = \phi / \phi_{\max}$$

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Quadrics



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Ray-Implicit Surface Intersection

$$f(x, y, z) = 0$$

$$x = x_0 + x_1 t$$

$$y = y_0 + y_1 t$$

$$z = z_0 + z_1 t$$

$$f^*(t) = 0$$

1. Substitute ray equation
2. Find *positive, real* roots

Univariate root finding

- Newton's method
- *Regula-falsi*
- Interval methods
- Heuristics

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Ray-Algebraic Surface Intersection

$$p_n(x, y, z) = 0$$

$$x = x_0 + x_1 t$$

$$y = y_0 + y_1 t$$

$$z = z_0 + z_1 t$$

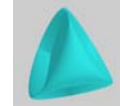
$$p_n^*(t) = 0$$

Degree n

Linear: Plane

Quadratic: Spheres, ...

Quartic: Tori



Polynomial root finding

- Quadratic, cubic, quartic
- Bezier/Bernoulli basis
- Descartes' rule of signs
- Sturm sequences

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History

Polygons	Appel '68
Quadratics, CSG	Goldstein & Nagel '71
Tori	Roth '82
Bicubic patches	Whitted '80, Kajiya '82
Superquadrics	Edwards & Barr '83
Algebraic surfaces	Hanrahan '82
Swept surfaces	Kajiya '83, van Wijk '84
Fractals	Kajiya '83
Height fields	Coquillart & Gangnet '84, Musgrave '88
Deformations	Barr '86
Subdivision surfs.	Kobbelt, Daubert, Siedel, '98

P. Hanrahan, A survey of ray-surface intersection algorithms

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