Reflection Models I

Today

- Types of reflection models
- The BRDF and reflectance
- The reflection equation
- Ideal reflection and refraction
- Fresnel effect
- Ideal diffuse

Next lecture

- Glossy and specular reflection models
- Rough surfaces and microfacets

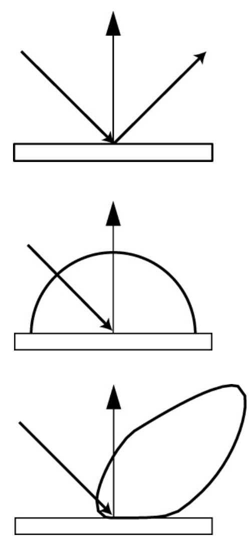
Reflection Models

Definition: Reflection is the process by which light incident on a surface interacts with the surface such that it leaves on the incident side without change in frequency.

Properties

- Spectra and Color
- Polarization
- Directional distribution

Types of Reflection Functions



Ideal Specular

- Reflection Law
- Mirror

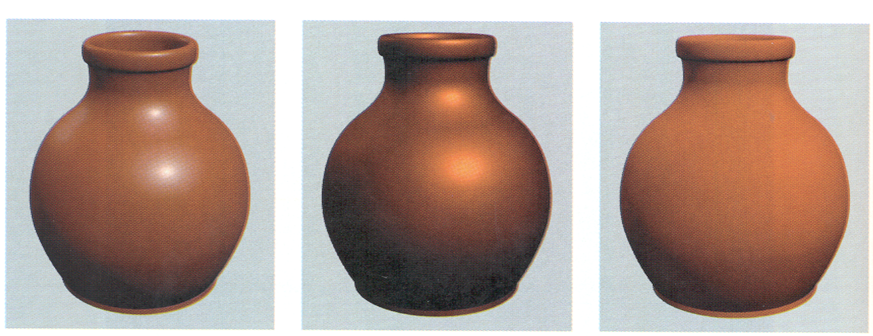
Ideal Diffuse

- Lambert's Law
- Matte

Specular

- Glossy
- Directional diffuse

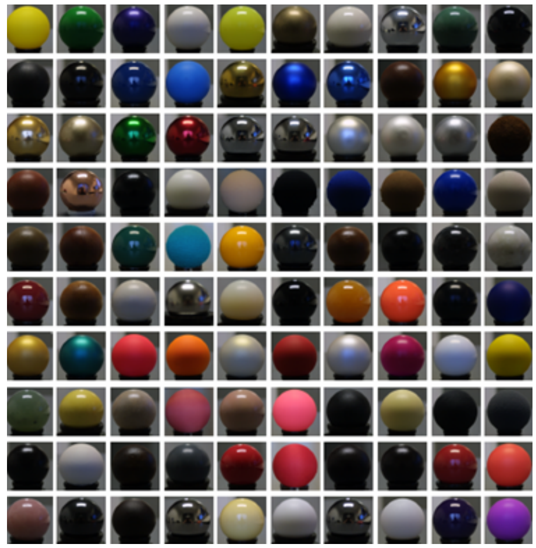
Materials



Plastic Metal Matte

From Apodaca and Gritz, *Advanced RenderMan*

Spheres [Matusik et al.]



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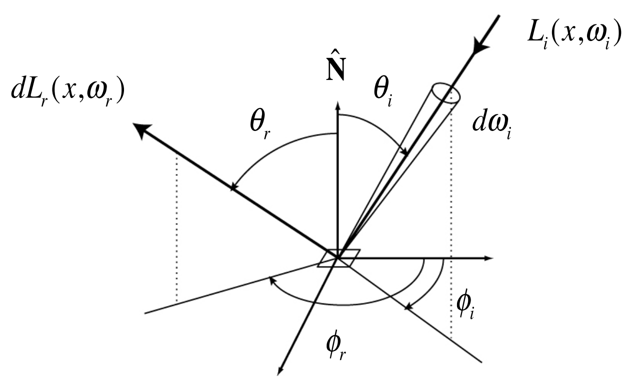
The BRDF

Bidirectional Reflectance-Distribution Function



$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i} \quad \left[\frac{1}{sr}\right]$$

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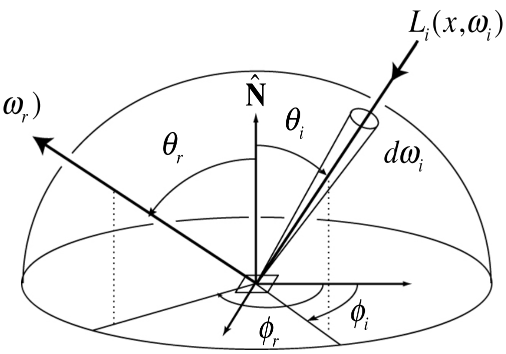
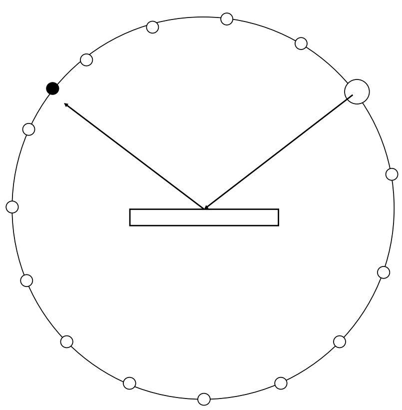
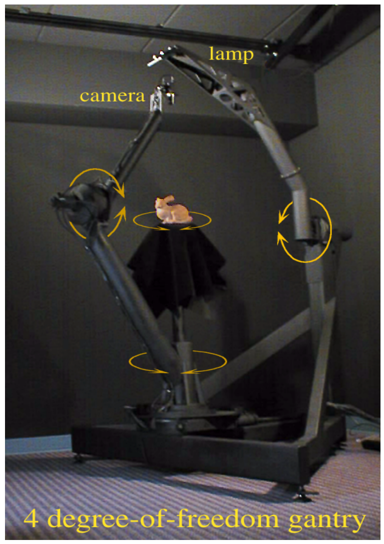
The Reflection Equation



$$L_r(x,\omega_r) = \int_{H^2} f_r(x,\omega_i \rightarrow \omega_r) L_i(x,\omega_i) \cos\theta_i \, d\omega_i$$

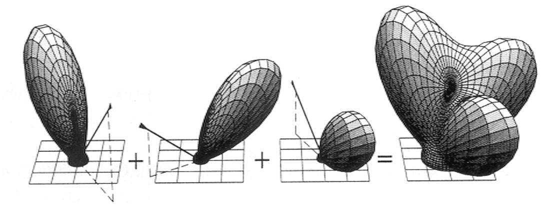
Gonioreflectometer





Properties of BRDF's

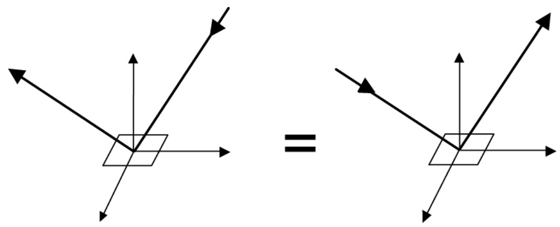
1. Linearity



From Sillion, Arvo, Westin, Greenberg

2. Reciprocity principle

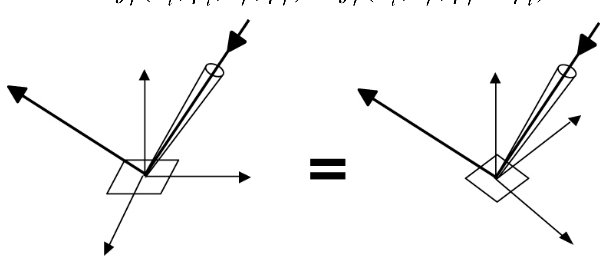
$$f_r(\omega_r \to \omega_i) = f_r(\omega_i \to \omega_r)$$



Properties of BRDF's

3. Isotropic vs. anisotropic

$$f_r(\theta_i,\varphi_i;\theta_r,\varphi_r) = f_r(\theta_i,\theta_r,\varphi_r-\varphi_i)$$



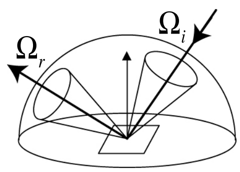
Reciprocity *and* isotropy

$$f_r(\theta_i,\theta_r,\varphi_r-\varphi_i) = f_r(\theta_r,\theta_i,\varphi_i-\varphi_r) = f_r(\theta_i,\theta_r,|\varphi_r-\varphi_i|)$$

4. Energy conservation

Energy Conservation

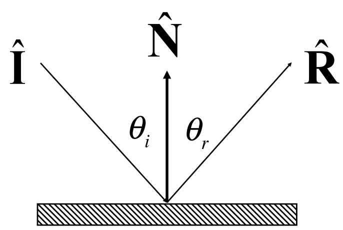
$$\frac{d\Phi_r}{d\Phi_i} = \frac{\int\limits_{\Omega_r} L_r(\omega_r)\cos\theta_r\, d\omega_r}{\int\limits_{\Omega_i} L_i(\omega_i)\cos\theta_i\, d\omega_i}$$



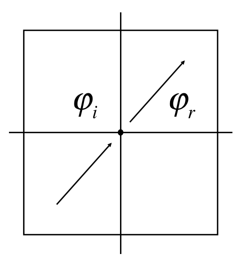
$$= \frac{\int\limits_{\Omega_r}\int\limits_{\Omega_i} f_r(\omega_i \to \omega_r) L_i(\omega_i)\cos\theta_i\, d\omega_i \cos\theta_r\, d\omega_r}{\int\limits_{\Omega_i} L_i(\omega_i)\cos\theta_i\, d\omega_i}$$

$$\le 1$$

Law of Reflection



$\theta_r = \theta_i$

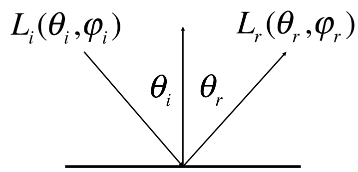


$\varphi_r = (\varphi_i + \pi) \bmod 2\pi$

$$\hat{\mathbf{R}} + (-\hat{\mathbf{I}}) = 2\cos\theta\,\hat{\mathbf{N}} = -2(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}})\hat{\mathbf{N}}$$

$$\hat{\mathbf{R}} = \hat{\mathbf{I}} - 2(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}})\hat{\mathbf{N}}$$

Ideal Reflection (Mirror)

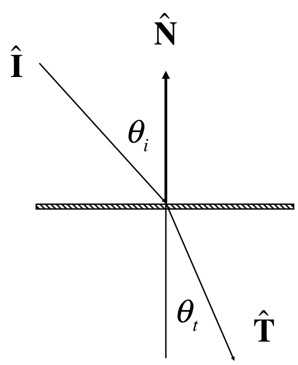


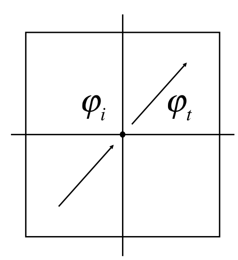
$$L_{r,m}(\theta_r,\varphi_r) = L_i(\theta_r,\varphi_r \pm \pi)$$

$$f_{r,m}(\theta_i,\varphi_i;\theta_r,\varphi_r) = \frac{\delta(\cos\theta_i - \cos\theta_r)}{\cos\theta_i}\delta(\varphi_i - \varphi_r \pm \pi)$$

$$\begin{aligned} L_{r,m}(\theta_r,\varphi_r) &= \int f_{r,m}(\theta_i,\varphi_i;\theta_r,\varphi_r) L_i(\theta_i,\varphi_i)\cos\theta_i \, d\cos\theta_i \, d\varphi_i \\ &= \int \frac{\delta(\cos\theta_i - \cos\theta_r)}{\cos\theta_i}\delta(\varphi_i - \varphi_r \pm \pi) L_i(\theta_i,\varphi_i)\cos\theta_i \, d\cos\theta_i \, d\varphi_i \\ &= L_i(\theta_r,\varphi_r \pm \pi) \end{aligned}$$

Snell's Law



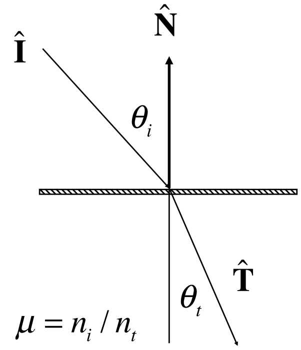


$$\varphi_t = \varphi_i \pm \pi$$

$$n_i \sin\theta_i = n_t \sin\theta_t$$

$$n_i \, \hat{\mathbf{N}} \times \hat{\mathbf{I}} = n_t \, \hat{\mathbf{N}} \times \hat{\mathbf{T}}$$

Law of Refraction



$\mu = n_i / n_t$

$$\hat{\mathbf{T}} = \mu \ \hat{\mathbf{I}} + \gamma \ \hat{\mathbf{N}}$$

$$\hat{\mathbf{T}}^2 = 1 = \mu^2 + \gamma^2 + 2\mu\gamma \ \hat{\mathbf{I}} \bullet \hat{\mathbf{N}}$$

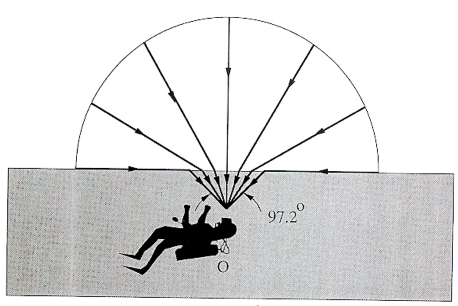
$$\begin{aligned} \gamma &= -\mu \ \hat{\mathbf{I}} \bullet \hat{\mathbf{N}} \pm \left\{ - \mu^2 \left(1 - \left(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}} \right)^2 \right) \right\}^{1/2} \\ &= \mu \ \cos\theta_i \pm \left\{ 1 - \mu^2 \sin^2 \theta_i \right\}^{1/2} \\ &= \mu \ \cos\theta_i \pm \cos\theta_t \quad \leftarrow \gamma = \mu - 1 \\ &= \mu \ \cos\theta_i - \cos\theta_t \end{aligned}$$

Total internal reflection:

$$1 - \mu^2 (1 - \left(\hat{\mathbf{I}} \bullet \hat{\mathbf{N}} \right)^2) < 0$$

Optical Manhole

Total internal reflection



$$n_w = \frac{4}{3}$$

From Livingston and Lynch

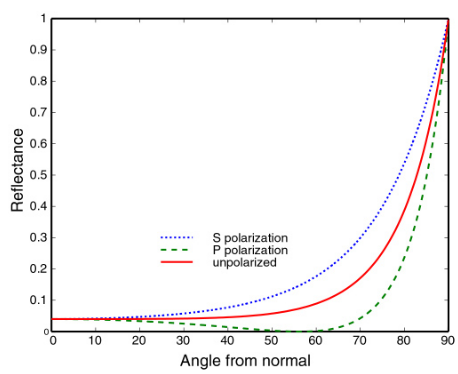
Experiment

Reflections from a shiny floor

From Lafortune, Foo, Torrance, Greenberg, SIGGRAPH 97

Fresnel Reflectance

Dielectric (N=1.5)

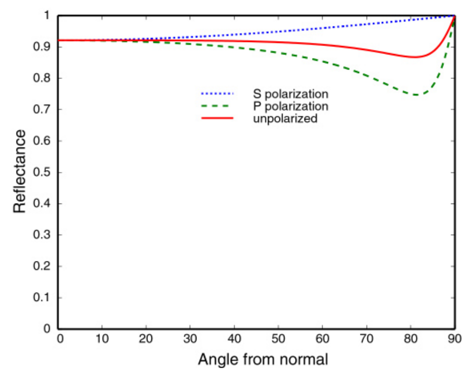


Glass n=1.5 F(0)=0.04
Diamond n=2.4 F(0)=0.15

Schlick Approximation $F(\theta) = F(0) + (1 - F(0))(1 - \cos\theta)^5$

Fresnel Reflectance

Metal (Aluminum)



Gold F(0)=0.82
Silver F(0)=0.95

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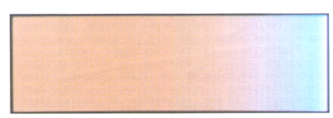
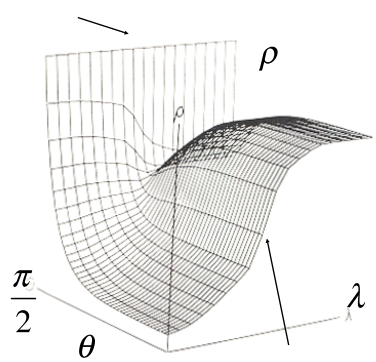
Cook-Torrance Model for Metals

Reflectance of Copper as a function of wavelength and angle of incidence



Measured Reflectance

Light spectra



Copper spectra

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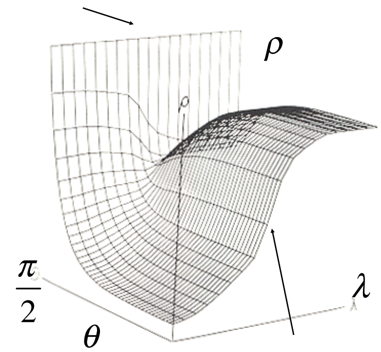
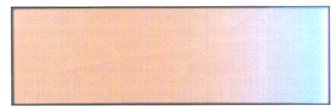
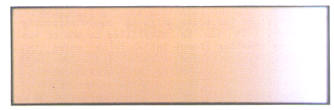
Cook-Torrance Model for Metals

Reflectance of Copper as a function of wavelength and angle of incidence





Measured Reflectance



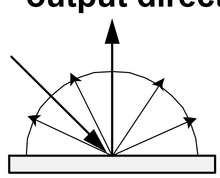
Approximated Reflectance

Cook-Torrance approximation

$$R = R(0) + R(\pi/2)\left[\frac{F(\theta) - F(0)}{F(\pi/2) - F(0)}\right]$$

Ideal Diffuse Reflection

Assume light is equally likely to be reflected in any output direction



$$f_{r,d} = c$$

$$L_{r,d}(\omega_r) = \int f_{r,d} L_i(\omega_i)\cos\theta_i \, d\omega_i$$
$$= f_{r,d}\int L_i(\omega_i)\cos\theta_i \, d\omega_i$$
$$= f_{r,d} E$$

$$M = \int L_r(\omega_r)\cos\theta_r \, d\omega_r = L_r \int \cos\theta_r \, d\omega_r = \pi L_r$$

$$\rho_d = \frac{M}{E} = \frac{\pi L_r}{E} = \frac{\pi f_{r,d} E}{E} = \pi f_{r,d} \quad \Rightarrow \quad f_{r,d} = \frac{\rho_d}{\pi}$$

Lambert's Cosine Law $M = \rho_d E = \rho_d E_s \cos\theta_s$

“Diffuse” Reflection

Theoretical

- **Bouguer - Special micro-facet distribution**
- **Seeliger - Subsurface reflection**
- **Multiple surface or subsurface reflections**

Experimental

- **Pressed magnesium oxide powder**
- **Almost never valid at high angles of incidence**

Paint manufactures attempt to create ideal diffuse