Reflection Models

Last lecture

- Reflection models
- The reflection equation and the BRDF
- Ideal reflection, refraction and diffuse

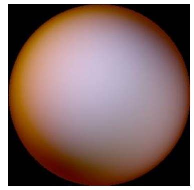
Today

- Phong model
- Microfacet models
- Torrance-Sparrow model
- Self-shadowing

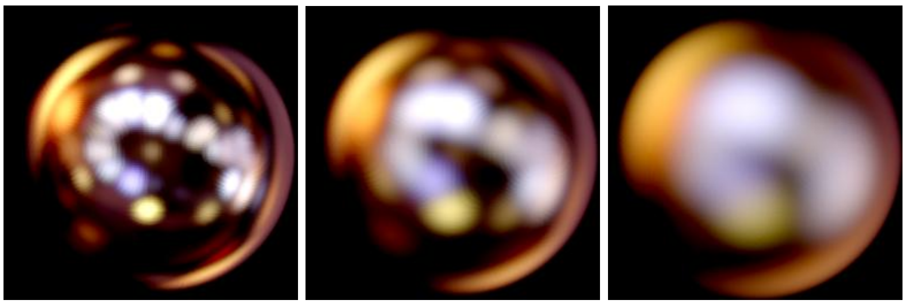
Phong Model

Glossy Surfaces

Mirror

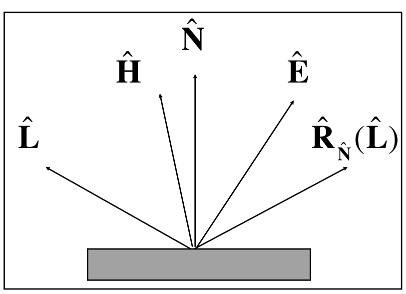


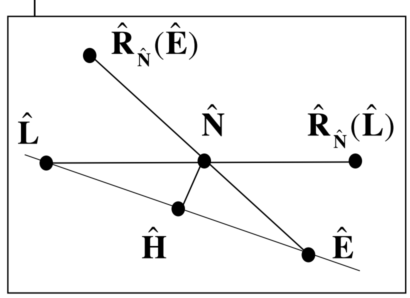
Diffuse



Reflection Geometry

$$\hat{\mathbf{H}} = \frac{\hat{\mathbf{L}} + \hat{\mathbf{E}}}{\left|\hat{\mathbf{L}} + \hat{\mathbf{E}}\right|}$$





$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$

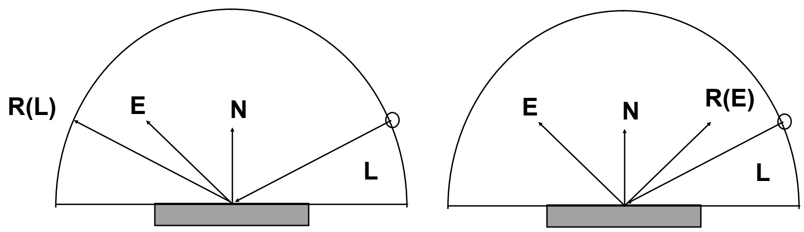
$$\cos\theta_r = \hat{\mathbf{E}} \bullet \hat{\mathbf{N}}$$

$$\cos\theta_s = \hat{\mathbf{E}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{L}}) = \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}) \bullet \hat{\mathbf{L}}$$

$$\cos\theta_g = \hat{\mathbf{E}} \bullet \hat{\mathbf{L}}$$

$$\cos\theta_{s'} = \hat{\mathbf{H}} \bullet \hat{\mathbf{N}}$$

Phong Model



$$\left(\hat{\mathbf{E}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{L}})\right)^s \qquad \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s$$

Reciprocity: $\left(\hat{\mathbf{E}} \bullet \mathbf{R}(\hat{\mathbf{L}})\right)^s = \left(\hat{\mathbf{L}} \bullet \mathbf{R}(\hat{\mathbf{E}})\right)^s$

Distributed light source!

Energy Normalization

Energy normalize Phong Model

$$\rho(H^2 \to \omega_r) = \int_{H^2(\hat{\mathbf{N}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s \cos\theta_i \, d\omega_i$$

$$\leq \int_{H^2(\hat{\mathbf{N}})} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s d\omega_i$$

$$\leq \int_{H^2(\mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}}))} \left(\hat{\mathbf{L}} \bullet \mathbf{R}_{\hat{\mathbf{N}}}(\hat{\mathbf{E}})\right)^s d\omega_R$$

$$= \int_{H^2} \cos^s \theta \, d\omega = \frac{2\pi}{s+1}$$

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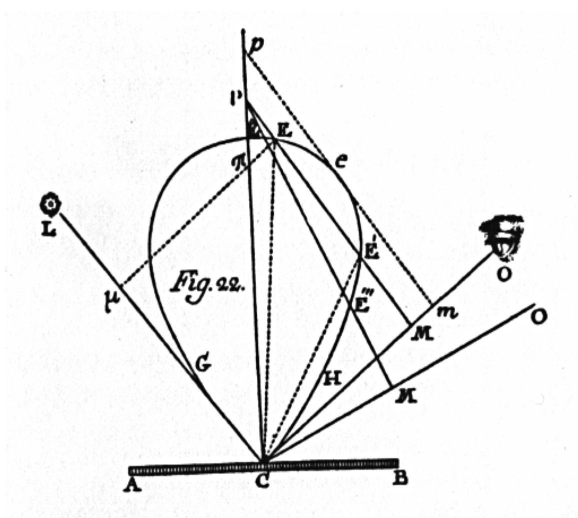
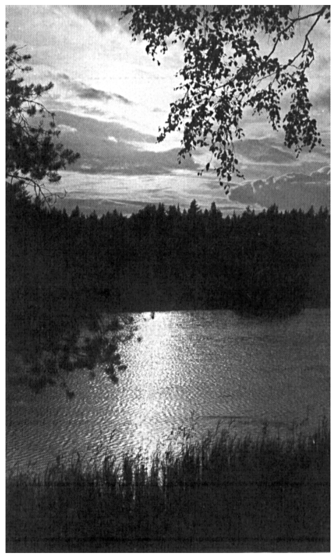
Microfacet Model

Bouguer's "little faces"

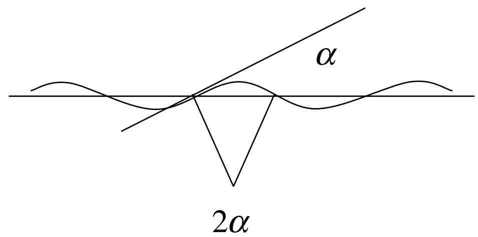


P. Bouguer, *Treatise on Optics*, 1760

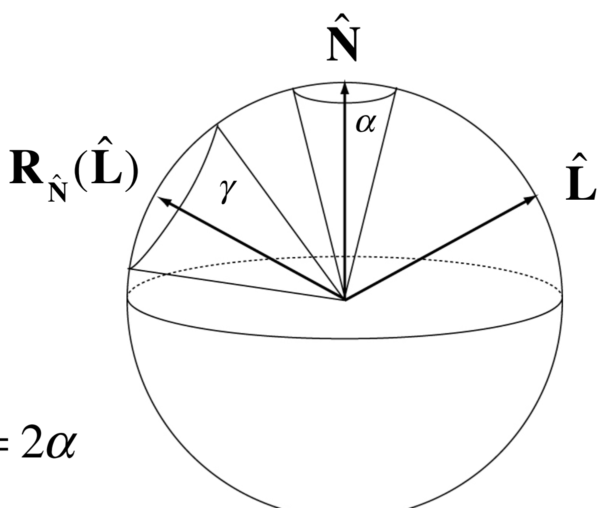
Reflection of the Sun from the Sea



Minnaert, *Light and Color in the Outdoors*, p. 28



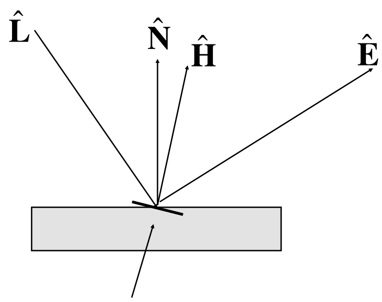
Analysis on the Sphere



$$\gamma = 2\alpha$$

$$\tan\psi = \tan\alpha\cos\theta$$

Microfacet Distributions



Microfacet

Total projected area

$$\int_{H^2} dA(\omega_h)\cos\theta_h\, d\omega_h = dA$$

Probability distribution

$$\int_{H^2} D(\omega_h)\cos\theta_h\, d\omega_h = 1$$

Area distribution $dA(\omega_h)$

Microfacet distribution $D(\omega_h) \equiv dA(\omega_h) / dA$

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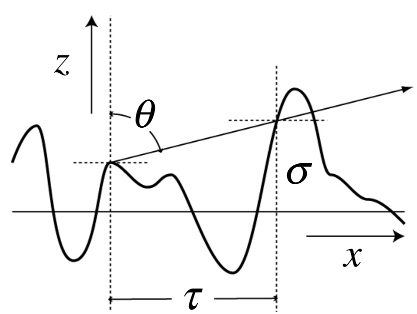
Gaussian Rough Surface

Gaussian distribution of heights

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z^2}{2\sigma^2}}$$



Gaussian distribution of slopes

$$D(\alpha) = \frac{1}{\sqrt{\pi} m^2 \cos^2 \alpha} e^{-\frac{\tan^2 \alpha}{m^2}} \qquad m = \frac{2\sigma}{\tau}$$

Beckmann distribution

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Microfacet Distribution Functions

Isotropic distributions $D(\omega_h) \Rightarrow D(\alpha)$

Characterize by half-angle β $D(\beta) = \frac{1}{2}$

Examples:

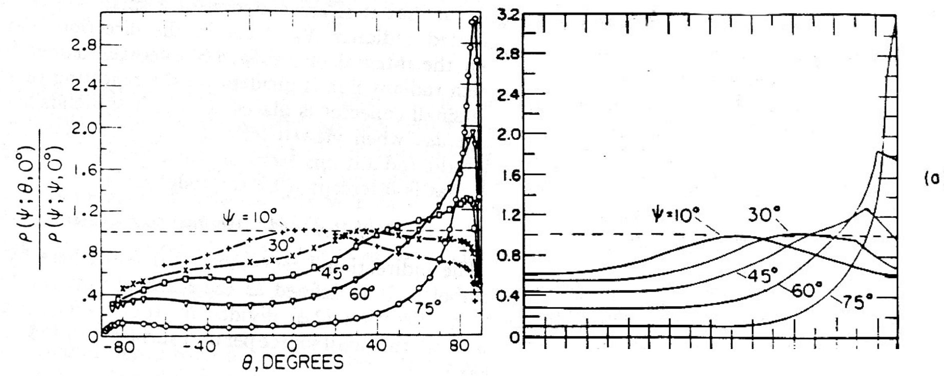
- **Blinn** $D_1(\alpha) = \cos^{c_1} \alpha$ $c_1 = \frac{\ln 2}{\ln \cos \beta}$
- **Torrance-Sparrow** $D_2(\alpha) = e^{-(c_2 \alpha)^2}$ $c_2 = \frac{\sqrt{2}}{\beta}$
- **Trowbridge-Reitz** $D_3(\alpha) = \frac{c_3^2}{(1-c_3^2)\cos^2 \alpha - 1}$

$$c_3 = \left(\frac{\cos^2 \beta - 1}{\cos^2 \beta - \sqrt{2}} \right)^{1/2}$$

Torrance-Sparrow Model

Torrance-Sparrow Comparison

Found an off-specular peak



Aluminum

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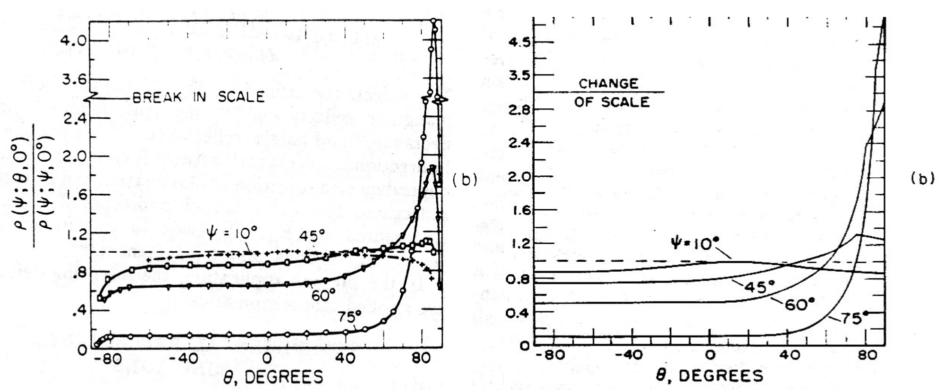
Torrance-Sparrow Comparison

Found an off-specular peak



Magnesium Oxide

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Explanation: Fresnel Term

$$f_r(\omega_i \to \omega_r) \approx F(\theta_i')D(\alpha)$$

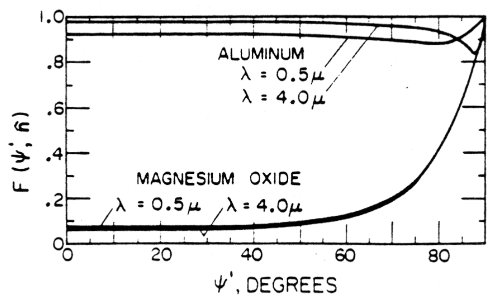
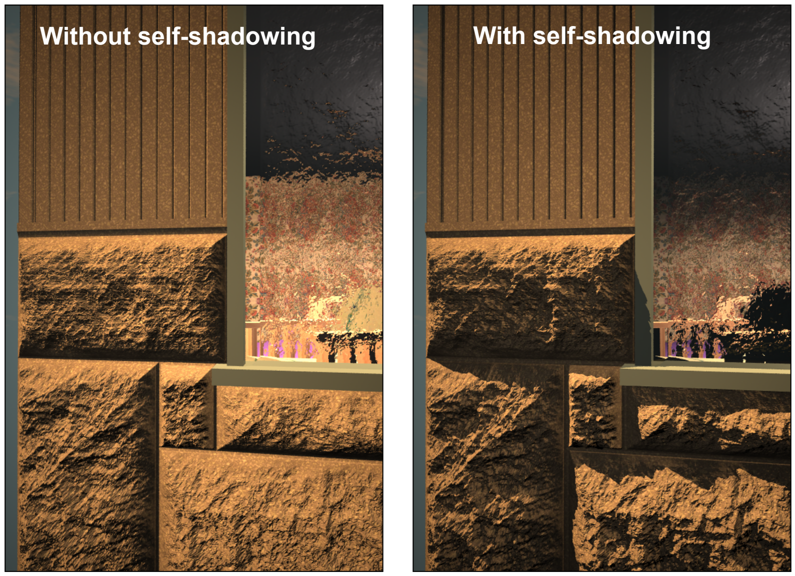


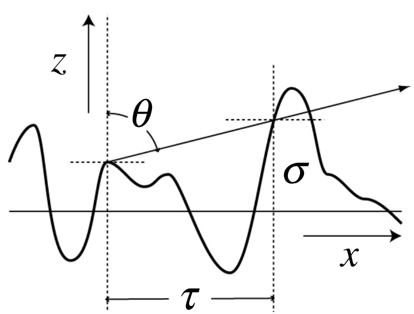
FIG. 6. Fresnel reflectance.

Self-Shadowing

Problem: Conservation of Energy



Self-Shadowing Function



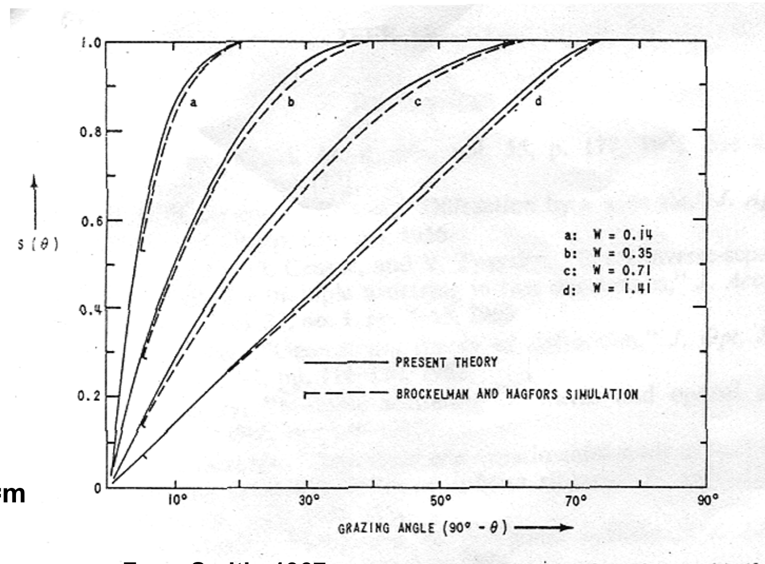
Probability of shadowing

$$S(\theta) = \frac{\left[1 - \frac{1}{2}\operatorname{erfc}\left(\mu/\sqrt{2}m\right)\right]}{1 + \Lambda(\mu)}$$

$$m = \frac{2\sigma}{\tau}$$

$$2\Lambda(\mu) = \left(\sqrt{\frac{2}{\pi}}\right)\frac{m}{\mu}e^{-\mu^2/2m^2} - \operatorname{erfc}\left(\mu/\sqrt{2}m\right)$$

Self-Shadowing Function



W=m

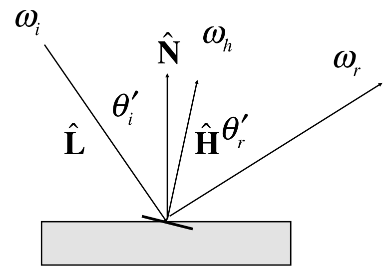
From Smith, 1967

Self-Consistency Condition

$$\int S(\theta)D(\alpha)\cos\theta' d\omega_\alpha = \cos\theta$$

The sum of the areas of the illuminated surface projected onto the plane normal to the direction of incidence is independent of the roughness of the surface, and equal to the projected area of the underlying mean plane.

Torrance-Sparrow Model



$$\cos\theta_i = \hat{\mathbf{L}} \bullet \hat{\mathbf{N}}$$
$$\cos\theta_i' = \hat{\mathbf{L}} \bullet \hat{\mathbf{H}}$$
$$d\omega_i' = d\omega_i$$

Prime indicates wrt H

$$d\Phi_h = L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, dA(\omega_h)$$
$$dA(\omega_h) = D(\omega_h)\, d\omega_h\, dA$$
$$d\Phi_h = L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA$$

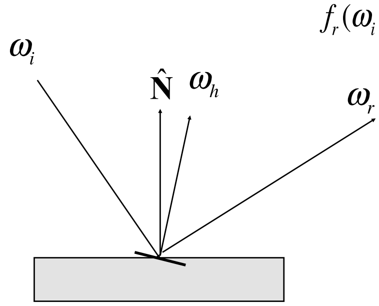
$$d\Phi_r = dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA$$

$$d\Phi_r = d\Phi_h$$

$$\therefore dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA$$
$$= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA$$

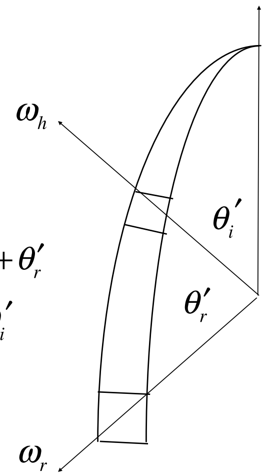
Torrance-Sparrow Model

$$dL_r(\omega_i \to \omega_r)\cos\theta_r\, d\omega_r\, dA$$
$$= L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA$$



$$f_r(\omega_i \to \omega_r) \equiv \frac{dL_r(\omega_i \to \omega_r)}{dE_i(\omega_i)}$$
$$= \frac{L_i(\omega_i)\cos\theta_i'\, d\omega_i'\, D(\omega_h)\, d\omega_h\, dA}{\left(\cos\theta_r d\omega_r dA\right)\left(L_i(\omega_i)\cos\theta_i\, d\omega_i\right)}$$
$$= \frac{D(\omega_h)}{\cos\theta_i \cos\theta_r}\cos\theta_i' \frac{d\omega_h}{d\omega_r}$$
$$= \frac{D(\omega_h)}{4\cos\theta_i \cos\theta_r}$$

Solid Angle Distributions



$$\theta_r' = \theta_i'$$

$$\theta_r = \theta_i' + \theta_r'$$

$$= 2\theta_i'$$

$$\varphi_r = \varphi_i$$

$$\begin{aligned} d\omega_r &= \sin\theta_r \, d\theta_r \, d\varphi_r \\ &= (\sin 2\theta_i')2d\theta_i' \, d\varphi_i \\ &= (2\sin\theta_i' \cos\theta_i')2d\theta_i' \, d\varphi_i \\ &= 4\cos\theta_i' \sin\theta_i' d\theta_i' \, d\varphi_i \\ &= 4\cos\theta_i' \, d\omega_h \end{aligned}$$

$$\frac{d\omega_h}{d\omega_r} = \frac{1}{4\cos\theta_i'}$$

Torrance-Sparrow Theory

$$f_r(\omega_i \to \omega_r) = \frac{F(\theta_i')S(\theta_i)S(\theta_r)D(\alpha)}{4\cos\theta_i \cos\theta_r}$$

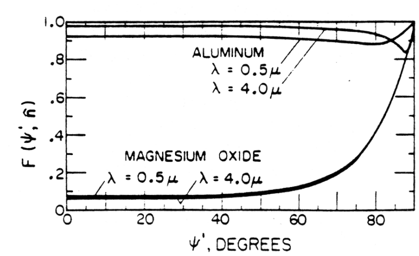


FIG. 6. Fresnel reflectance.

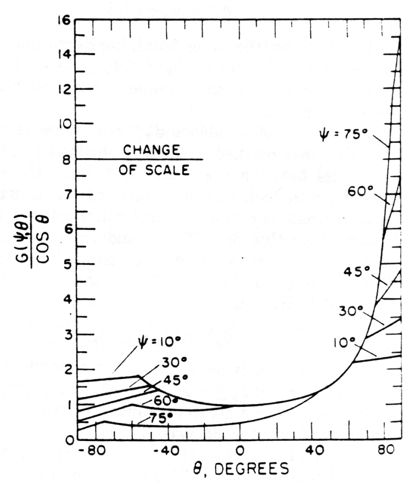


FIG. 7. The factor $G(\psi,\theta)/\cos\theta$ in the plane of incidence for various incidence angles ψ.