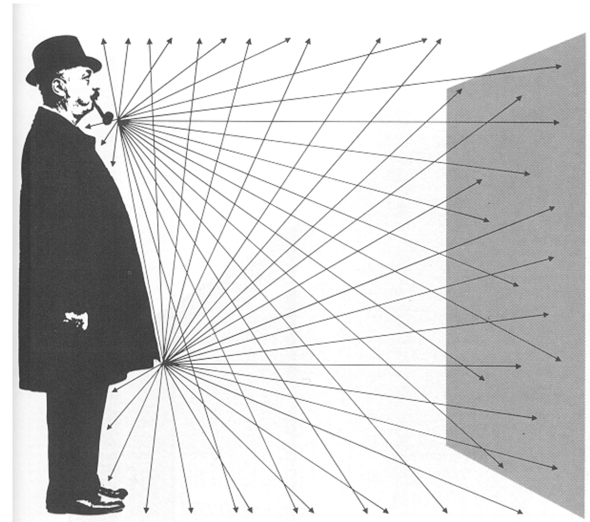
Radiometry and Photometry

Measuring spatial properties of light

- Radiant power
- Radiant intensity
- Irradiance
  - Inverse square law and cosine law
- Radiance
- Radiant exitance (radiosity)



From London and Upton

Goal is to perform lighting calculations in the physically correct way

Radiant Energy and Power

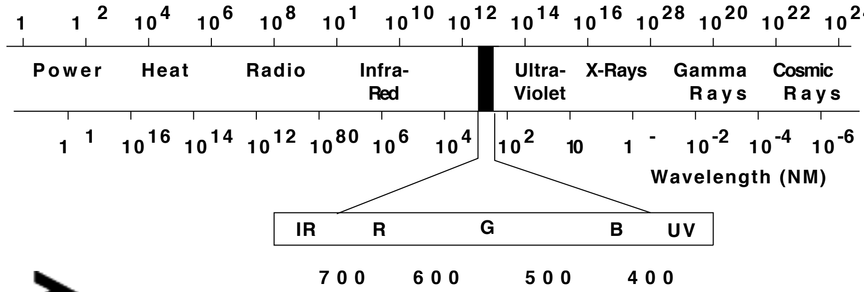
Power: Watts (radiometry)

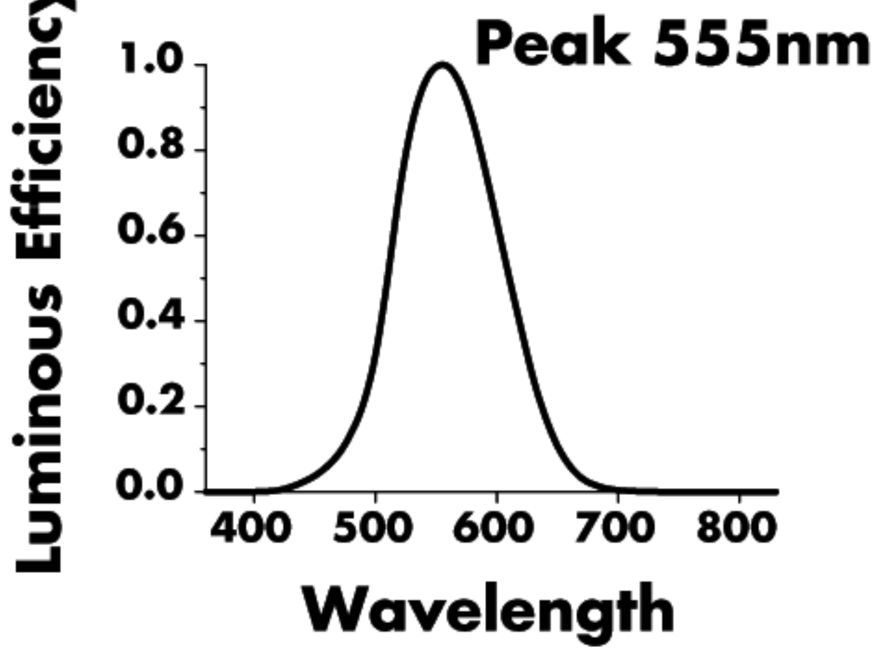
Φ vs. Lumens (photometry)

- **Spectral efficacy**
- **Energy efficiency**

Energy: Joules vs. Talbot

- **Exposure**
  - **Film response**
  - **Skin - sunburn**





Photometric luminance

$$Y = \int V(\lambda) L(\lambda) d\lambda$$

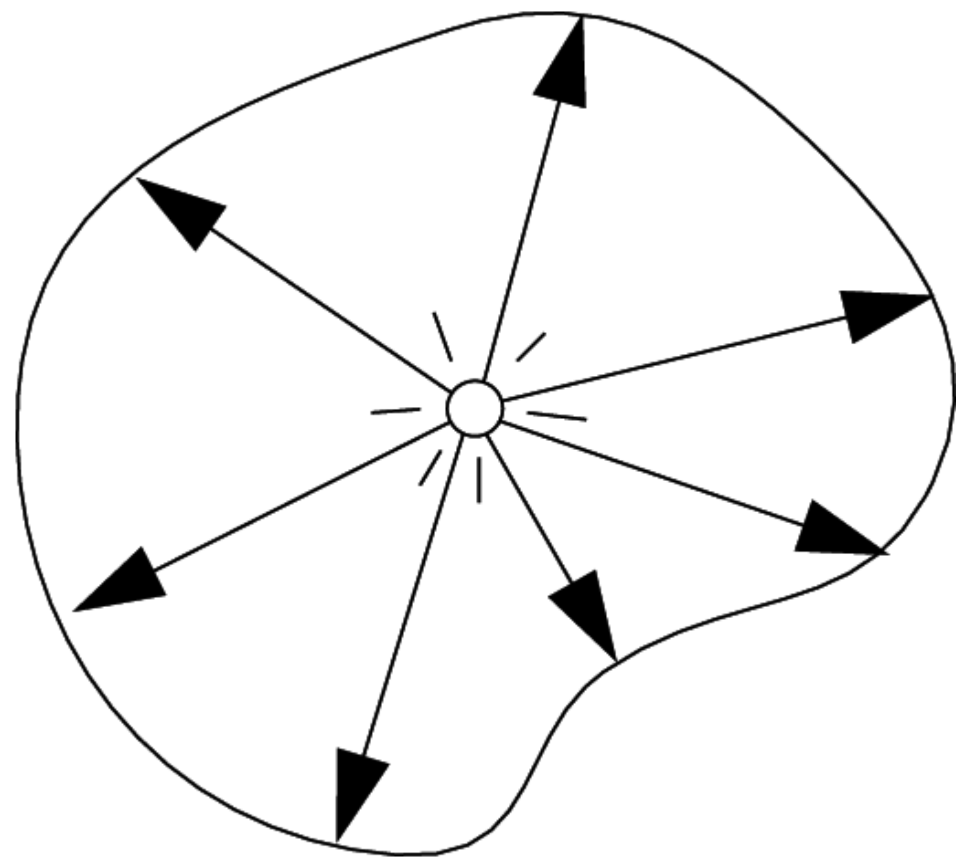
Radiant Intensity

Radiant Intensity

Definition: The radiant (luminous) intensity is the power per unit solid angle emanating from a point source.

$$I(\omega) \equiv \frac{d\Phi}{d\omega}$$

$$\left[\frac{W}{sr}\right]\left[\frac{lm}{sr} = cd = candela\right]$$



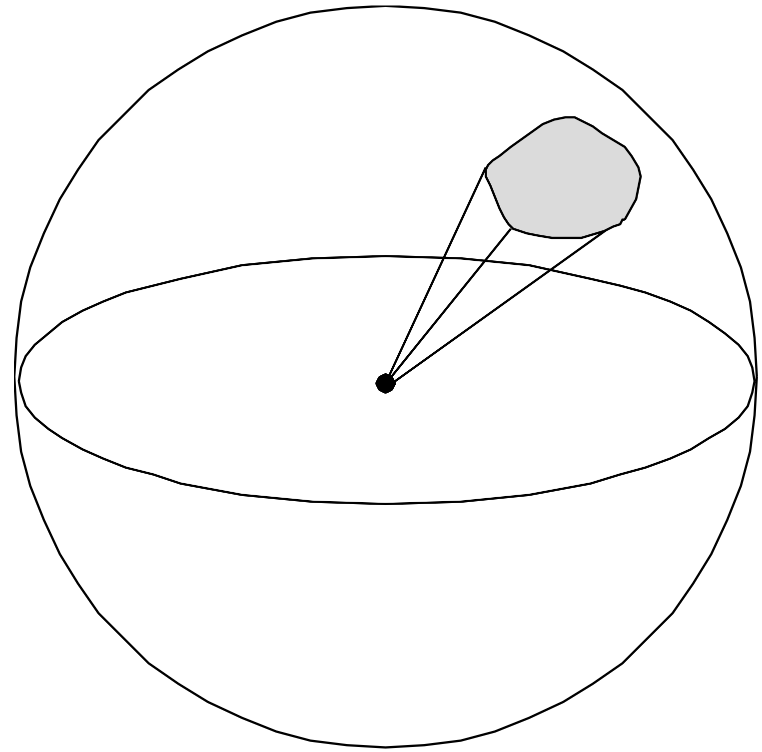
Angles and Solid Angles

Angle $$\theta = \frac{l}{r}$$

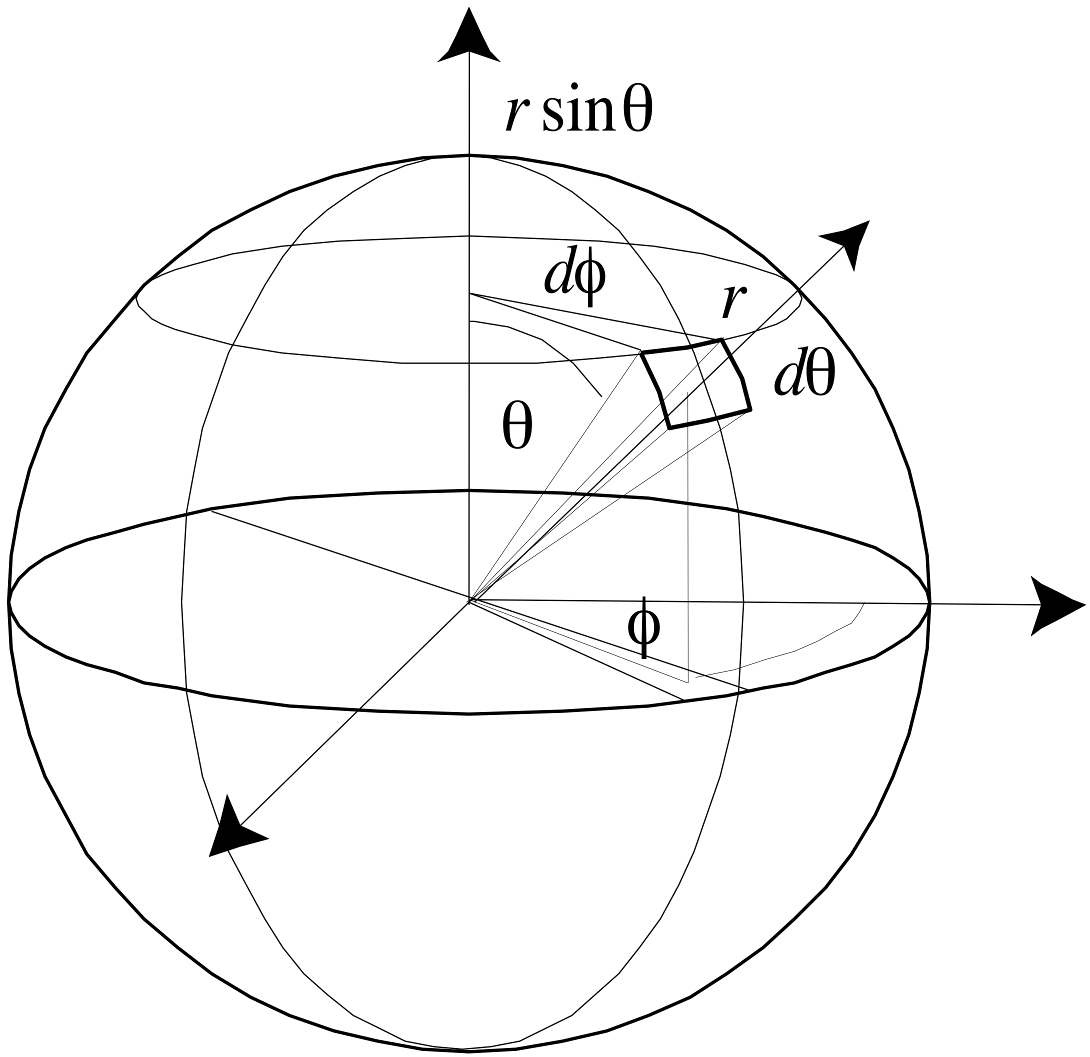
⇒ circle has 2 π radians

Solid angle $$\Omega = \frac{A}{R^2}$$

⇒ sphere has 4 π steradians

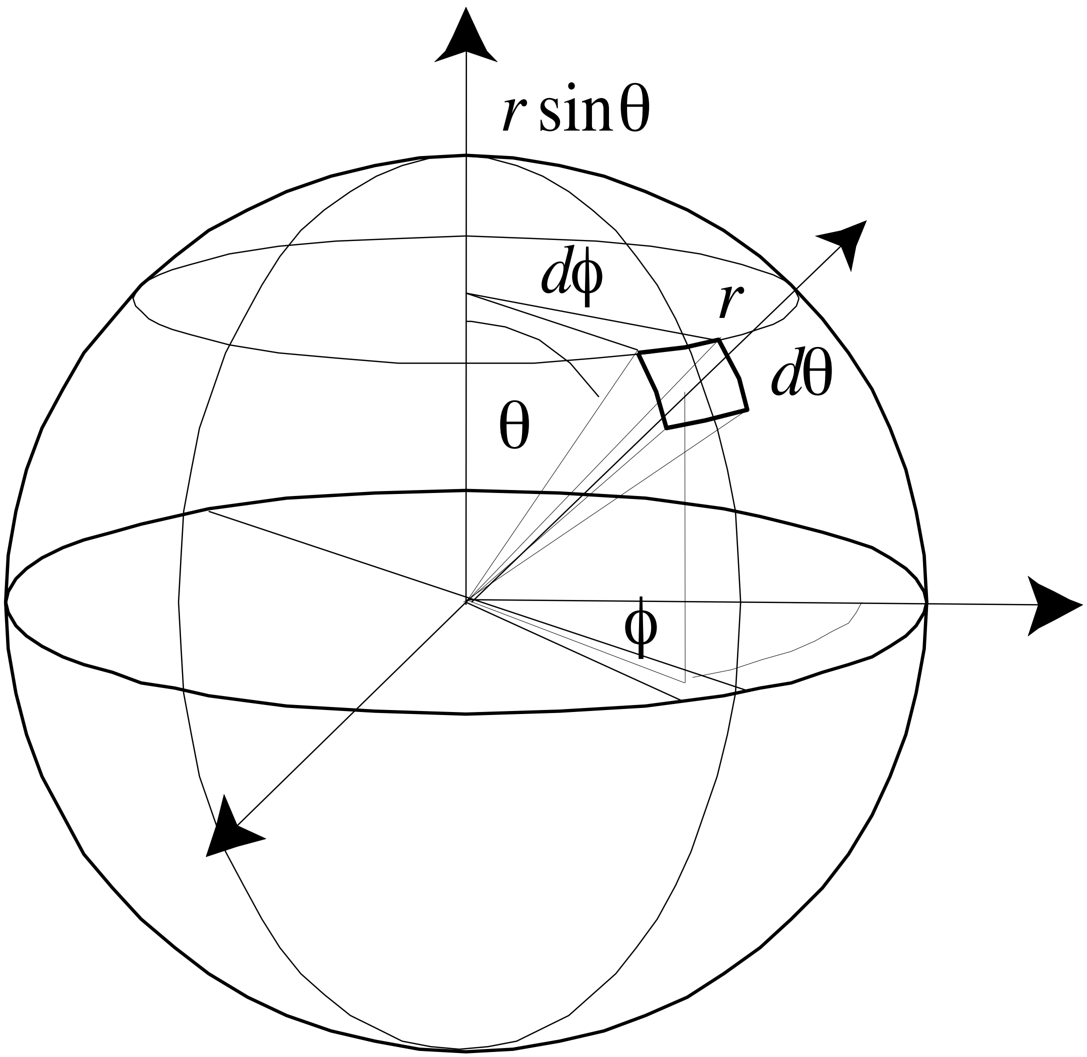


Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

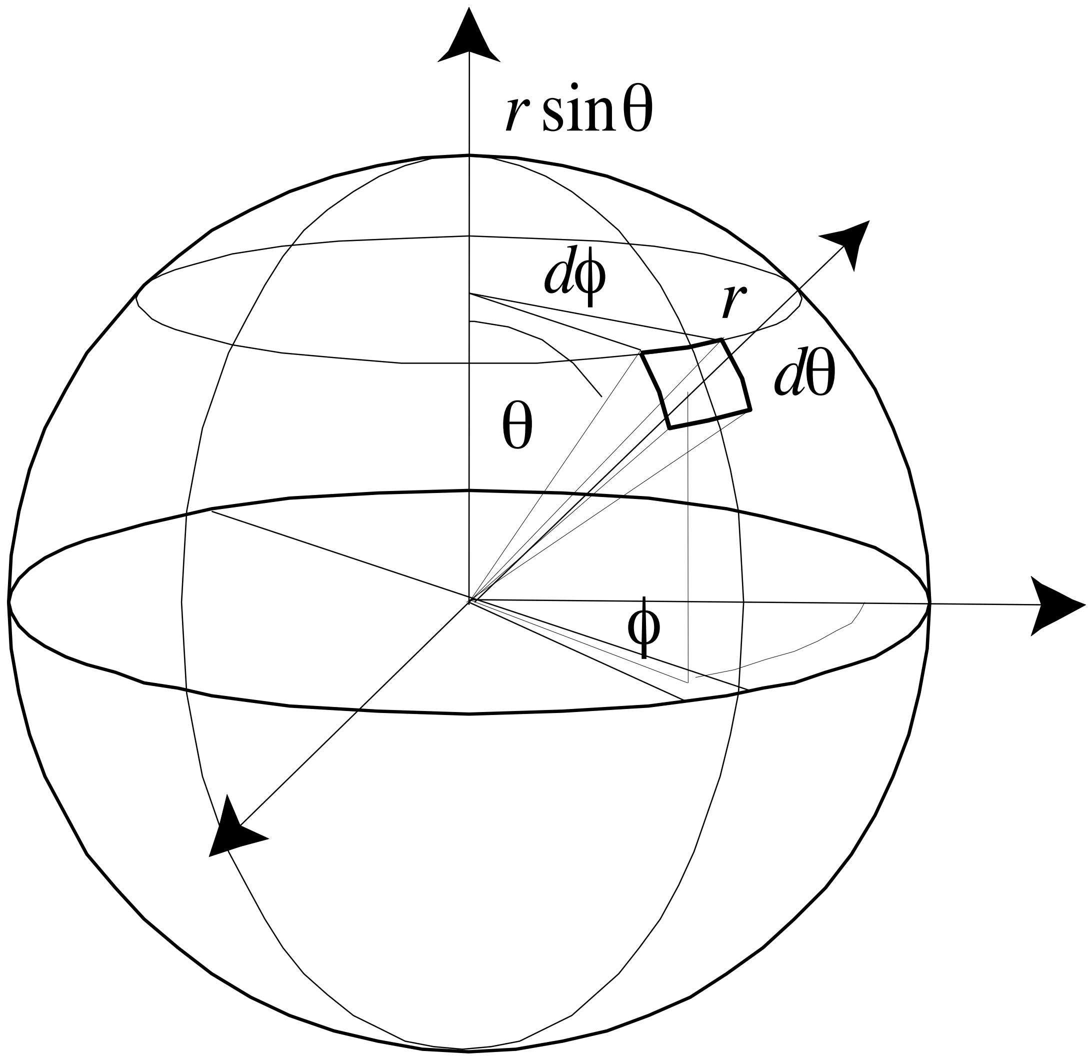
Differential Solid Angles



$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2\sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

Differential Solid Angles



$$d\omega = \sin\theta \, d\theta \, d\phi$$

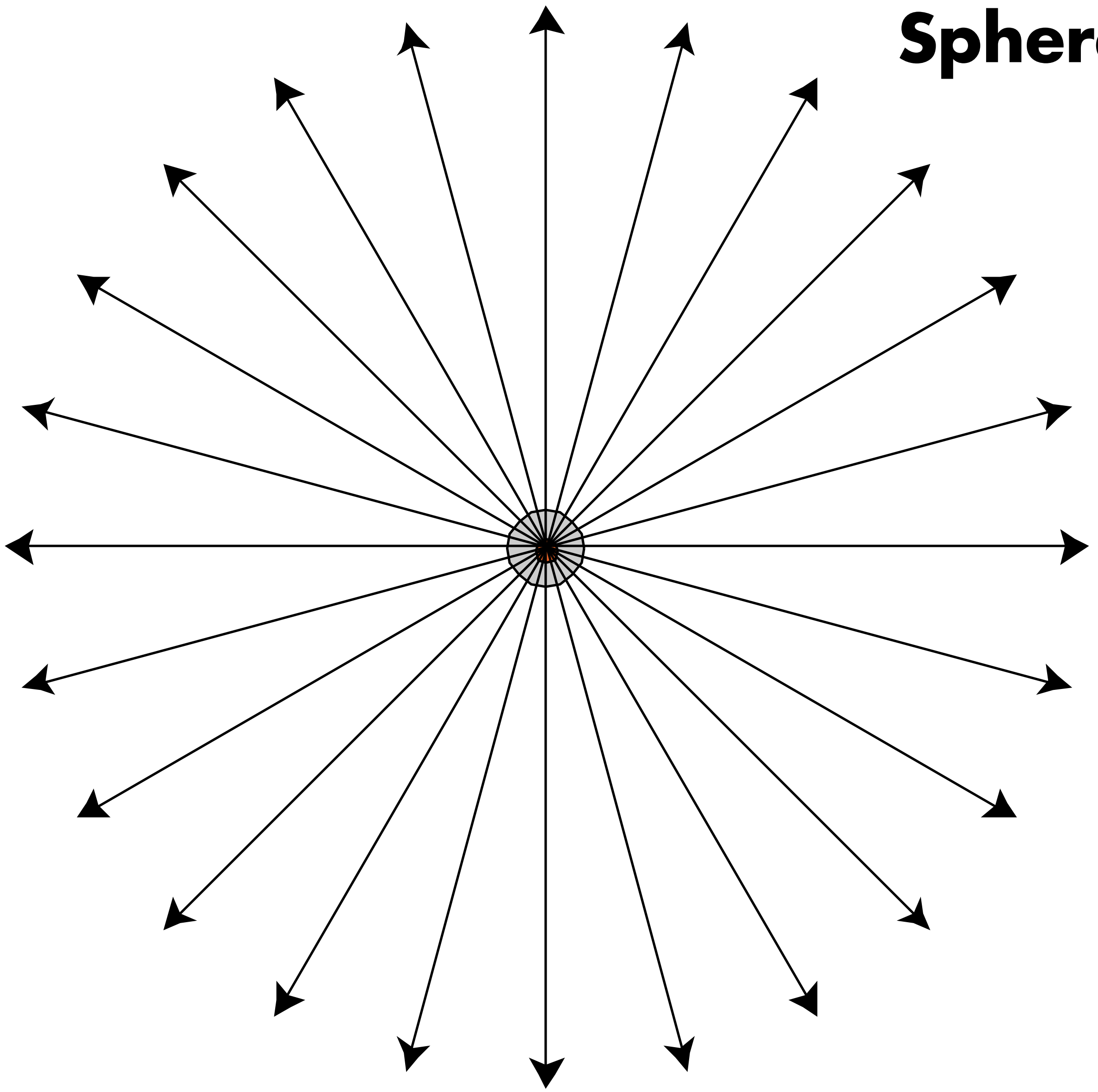
$$\Omega = \int_{S^2} d\omega$$

$$= \int_0^{\pi} \int_0^{2\pi} \sin\theta \, d\theta \, d\phi$$

$$= \int_{-1}^{1} \int_0^{2\pi} d\cos\theta \; d\phi$$

$$= 4\pi$$

Isotropic Point Source



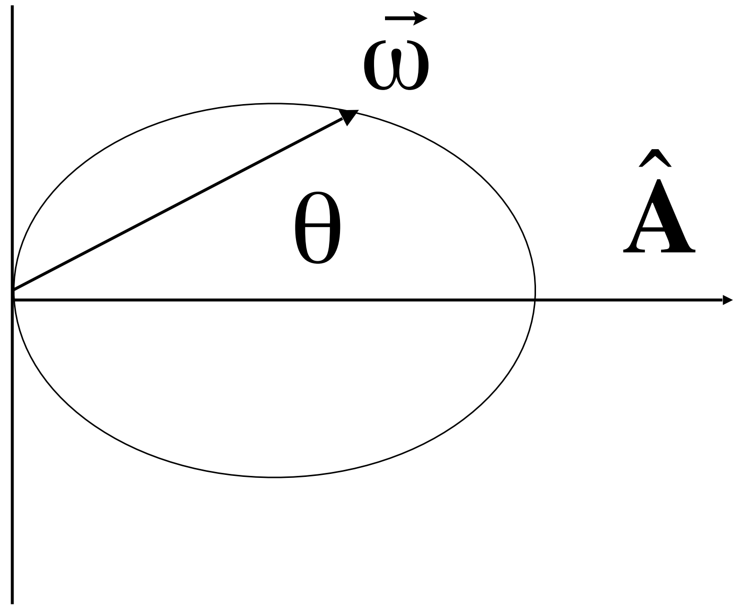
Sphere S^2

$$\Phi = \int_{S^2} I \, d\omega$$

$$= 4\pi I$$

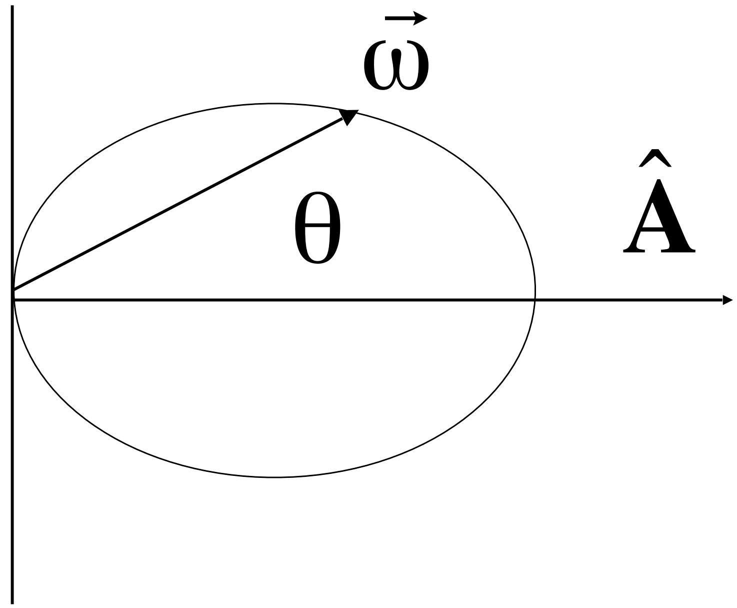
$$I = \frac{\Phi}{4\pi}$$

Warn's Spotlight



$$I(\omega) = \cos^s \theta = (\vec{\omega} \bullet \hat{\mathbf{A}})^s$$

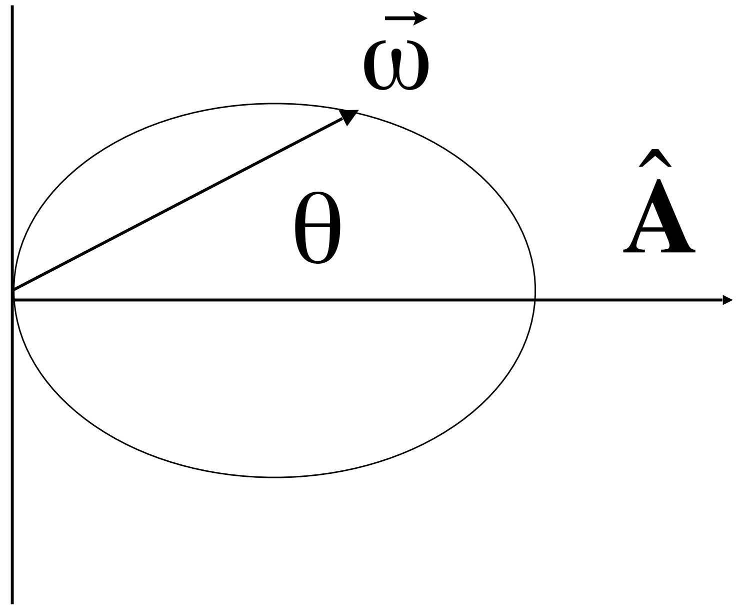
Warn's Spotlight



$$I(\omega) = \cos^s\theta = (\vec{\omega} \bullet \hat{\mathbf{A}})^s$$

$$\Phi = \int_0^{2\pi} \int_0^1 I(\omega)\, d\cos\theta\, d\varphi$$

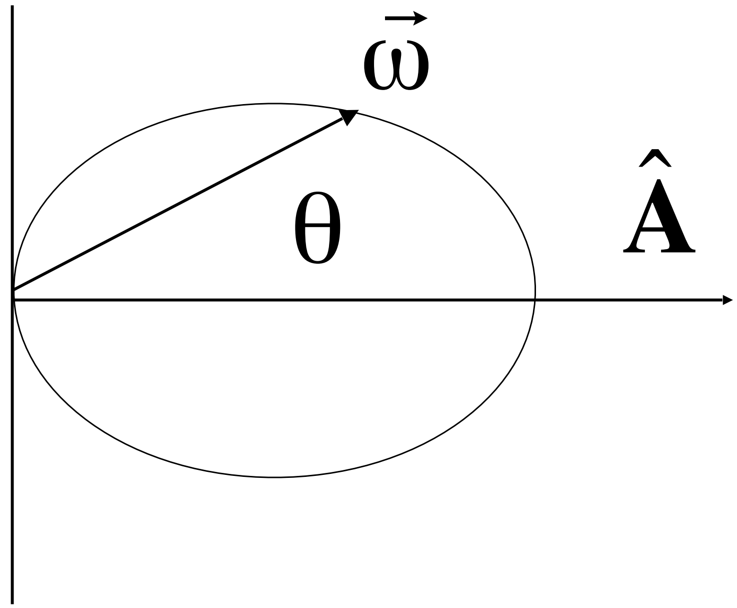
Warn's Spotlight



$$I(\omega) = \cos^s\theta = (\vec{\omega} \bullet \hat{\mathbf{A}})^s$$

$$\Phi = \int_0^{2\pi}\int_0^1 I(\omega)\, d\cos\theta\, d\varphi = 2\pi\int_0^1 \cos^s\theta\, d\cos\theta = \frac{2\pi}{s+1}$$

Warn's Spotlight



$$I(\omega) = \cos^s\theta = (\vec{\omega} \bullet \hat{\mathbf{A}})^s$$

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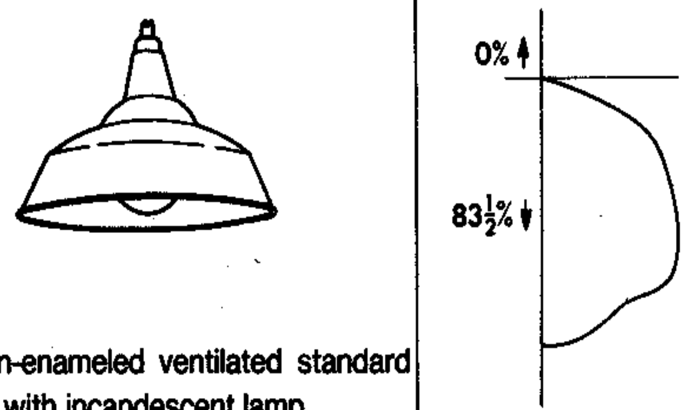
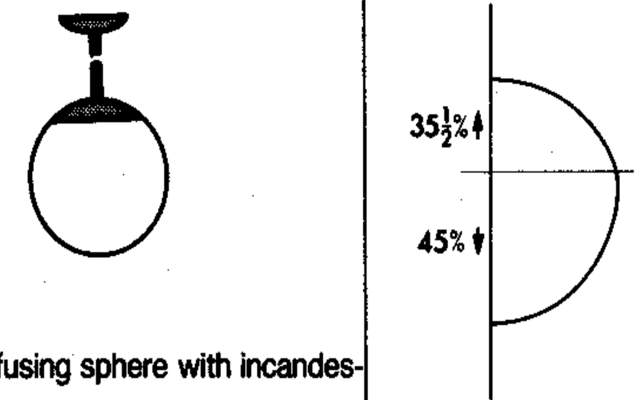
$$I(\omega) = \Phi\frac{s+1}{2\pi}\cos^s\theta$$

Light Source Goniometric Diagrams

3

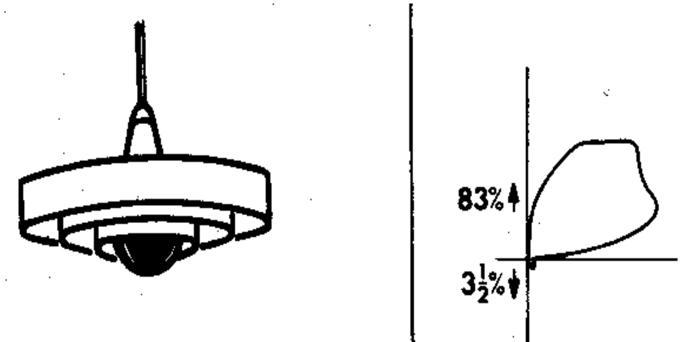


Porcelain-enameled ventilated standard dome with incandescent lamp

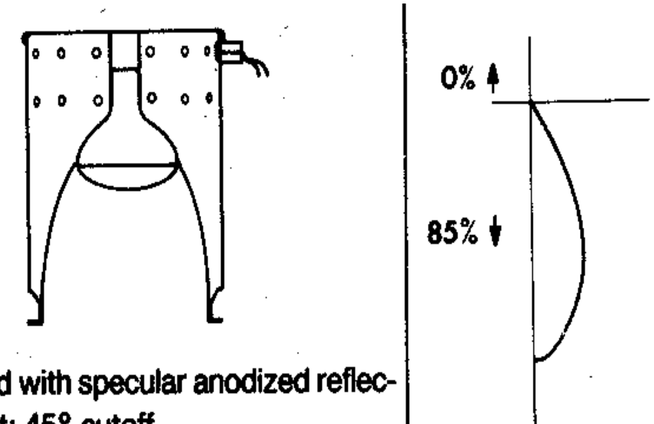


Pendant diffusing sphere with incandescent lamp

2



Concentric ring unit with incandescent silvered-bowl lamp



R-40 flood with specular anodized reflector skirt; 45° cutoff

Irradiance

Irradiance

Definition: The irradiance (illuminance) is the power per unit area incident on a surface.

$$E(x) \equiv \frac{d\Phi_i}{dA}$$

$$\left[\frac{W}{m^2}\right] \quad \left[\frac{lm}{m^2} = lux\right]$$

Sometimes referred to as the radiant (luminous) incidence.

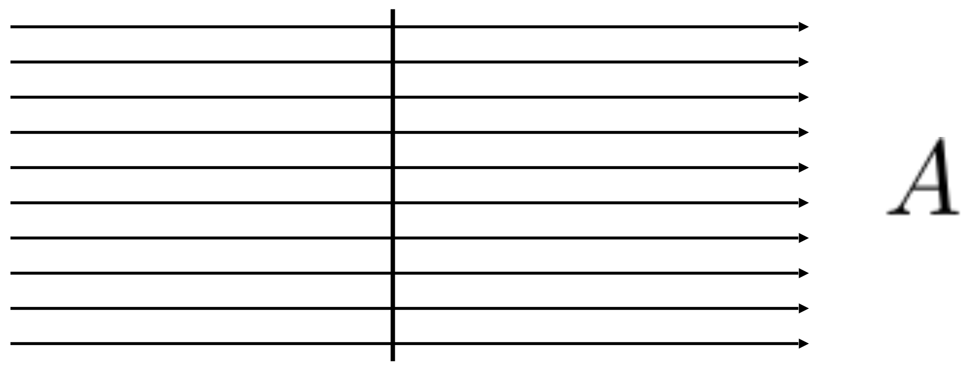
Typical Values of Illuminance [lm/m²]

Sunlight plus skylight	100,000 lux
Sunlight plus skylight (overcast)	10,000
Interior near window (daylight)	1,000
Artificial light (minimum)	100
Moonlight (full)	0.02
Starlight	0.0003

Beam Power in Terms of Irradiance

$$\Phi = EA$$

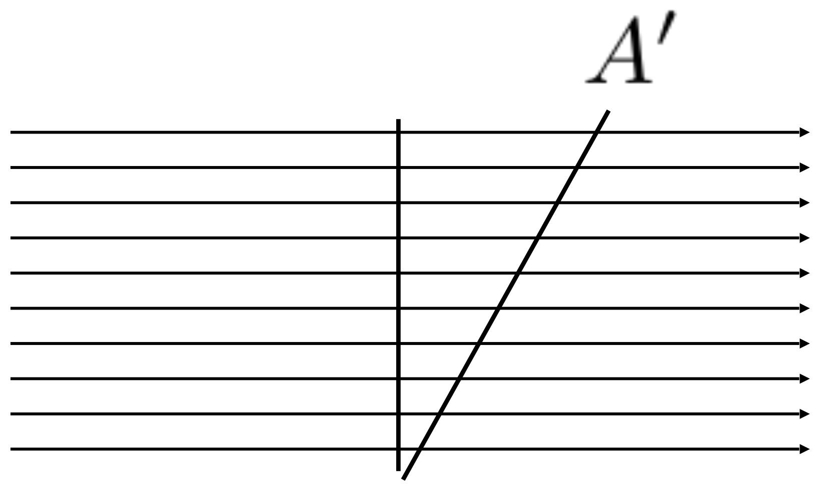
$$E = \frac{\Phi}{A}$$



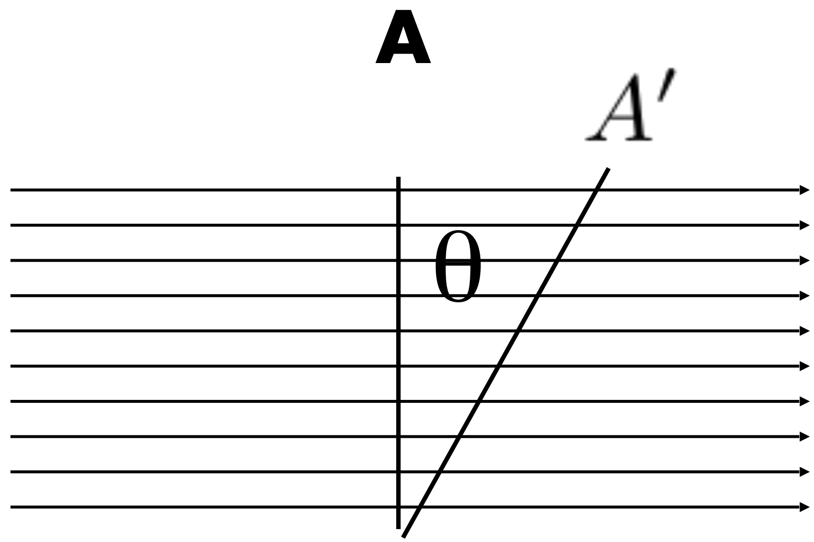
Beam Power Falling on the Surface

$$\Phi' = E' A'$$

$$E' = \frac{\Phi'}{A'}$$



Projected Area

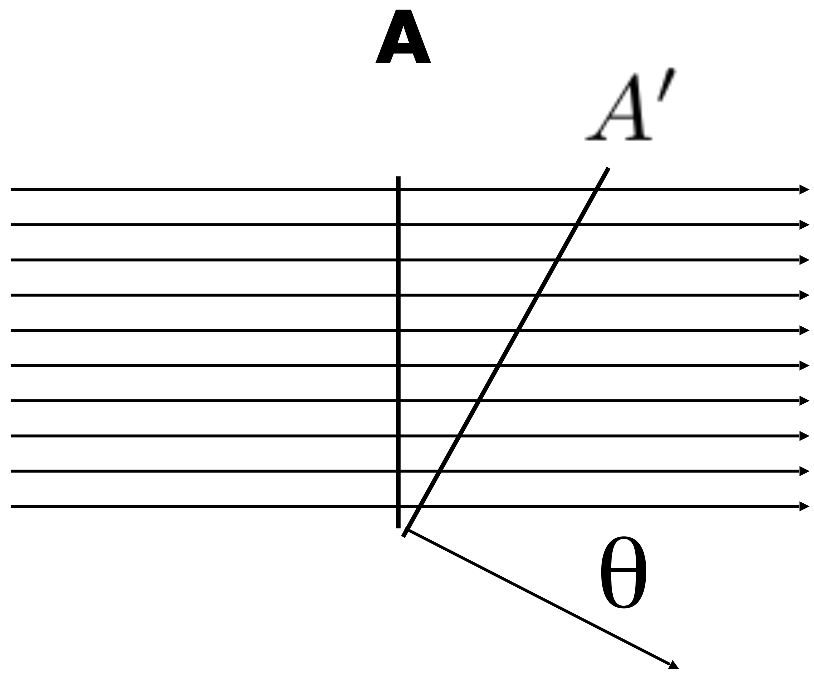


$$A = A' \cos\theta$$

Lambert's Cosine Law

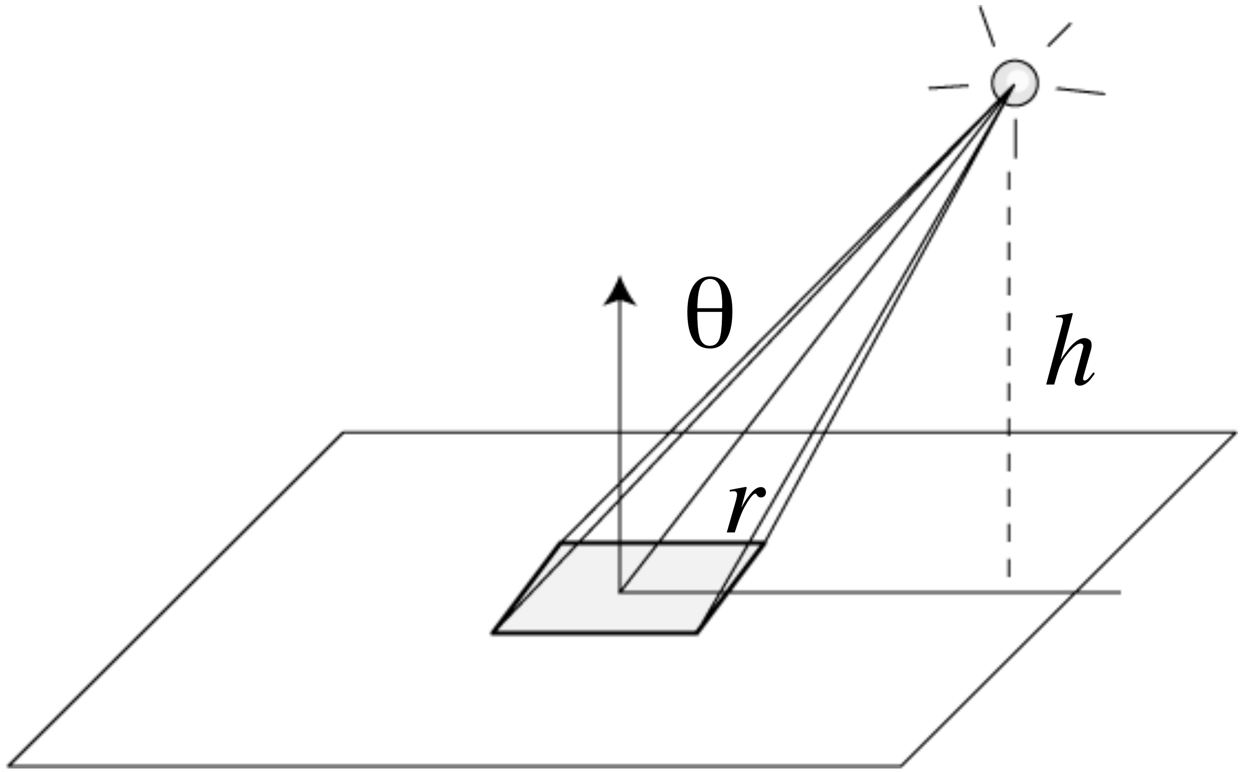
$$A = A' \cos\theta$$

$$\Phi = \Phi'$$



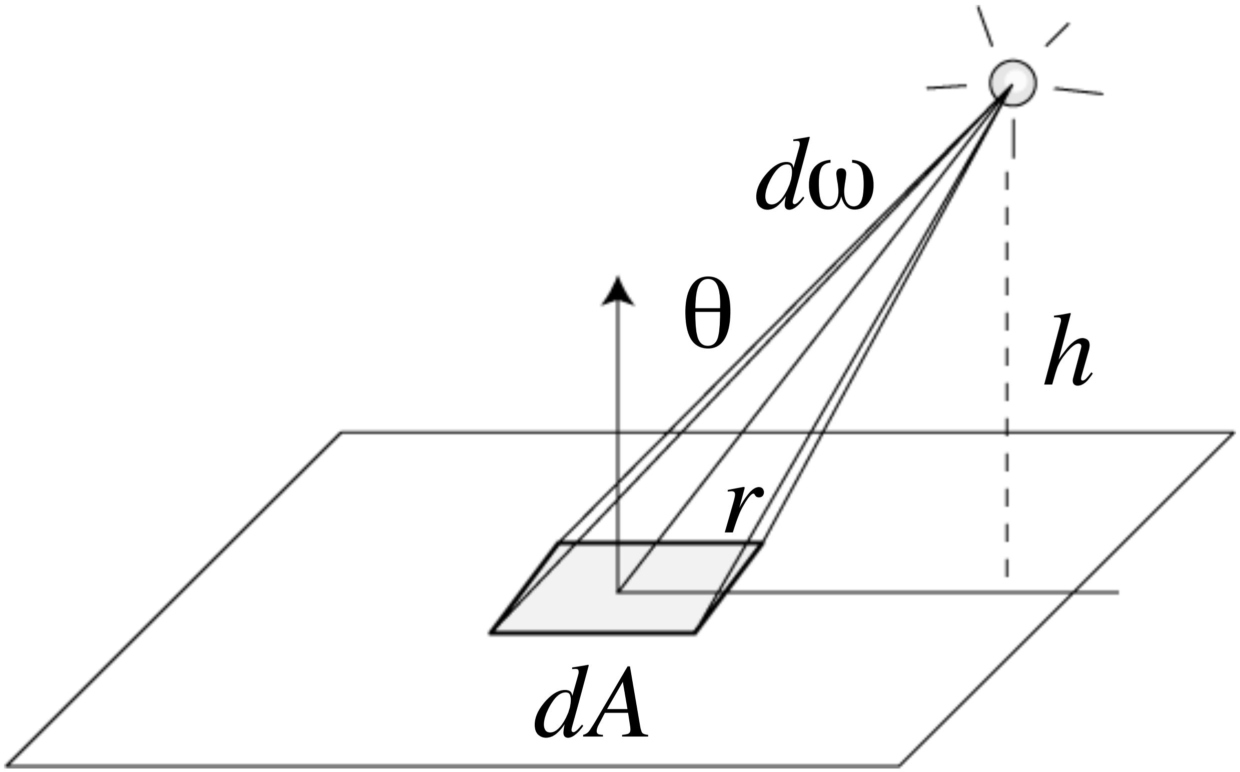
$$E' = \frac{\Phi'}{A'} = \frac{\Phi}{A}\cos\theta = E\cos\theta$$

Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

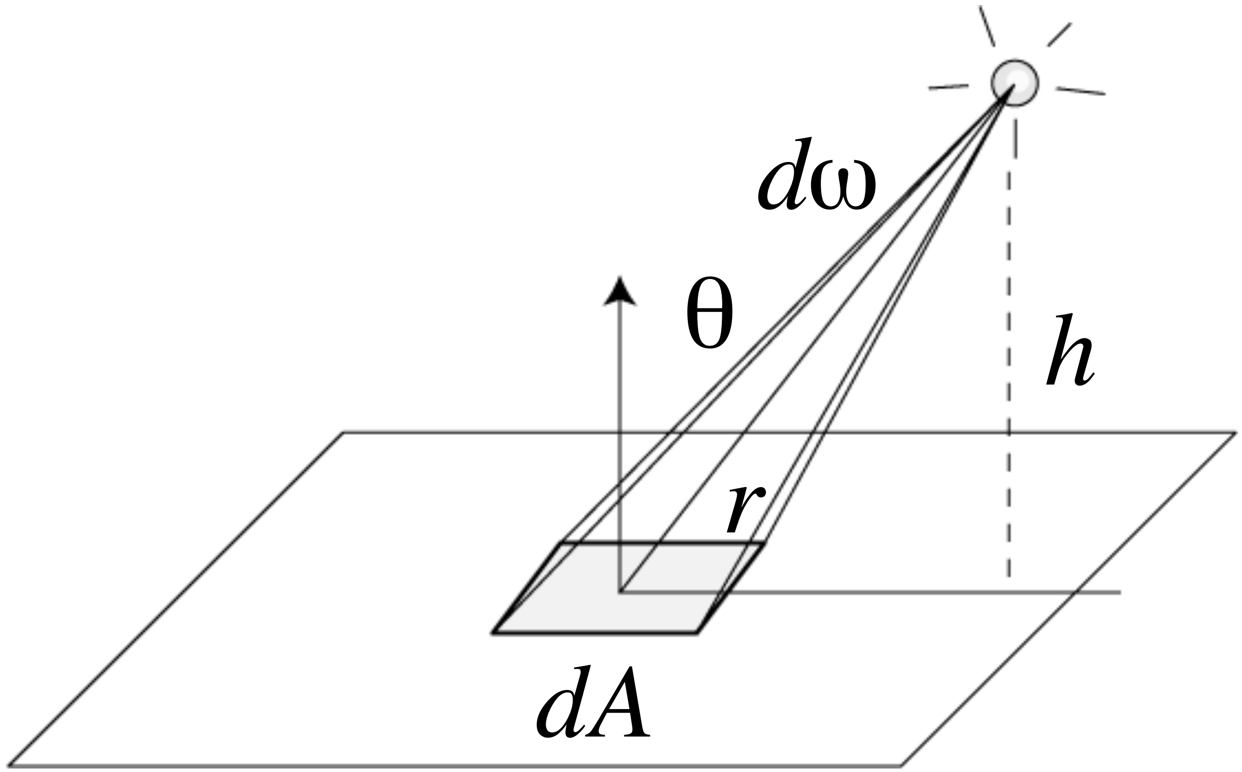
Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

$$d\Phi = I\, d\omega$$

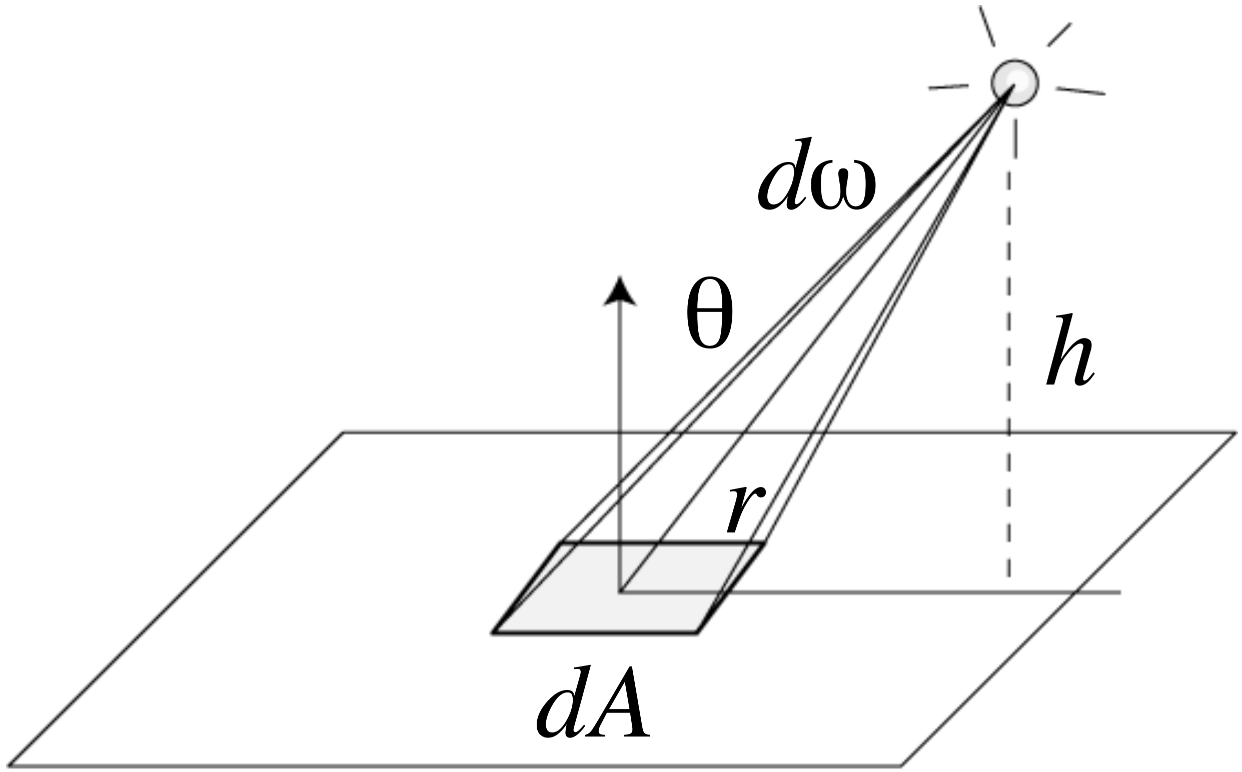
Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

$$d\omega = \frac{\cos\theta}{r^2} dA$$

Irradiance: Isotropic Point Source



$$I = \frac{\Phi}{4\pi}$$

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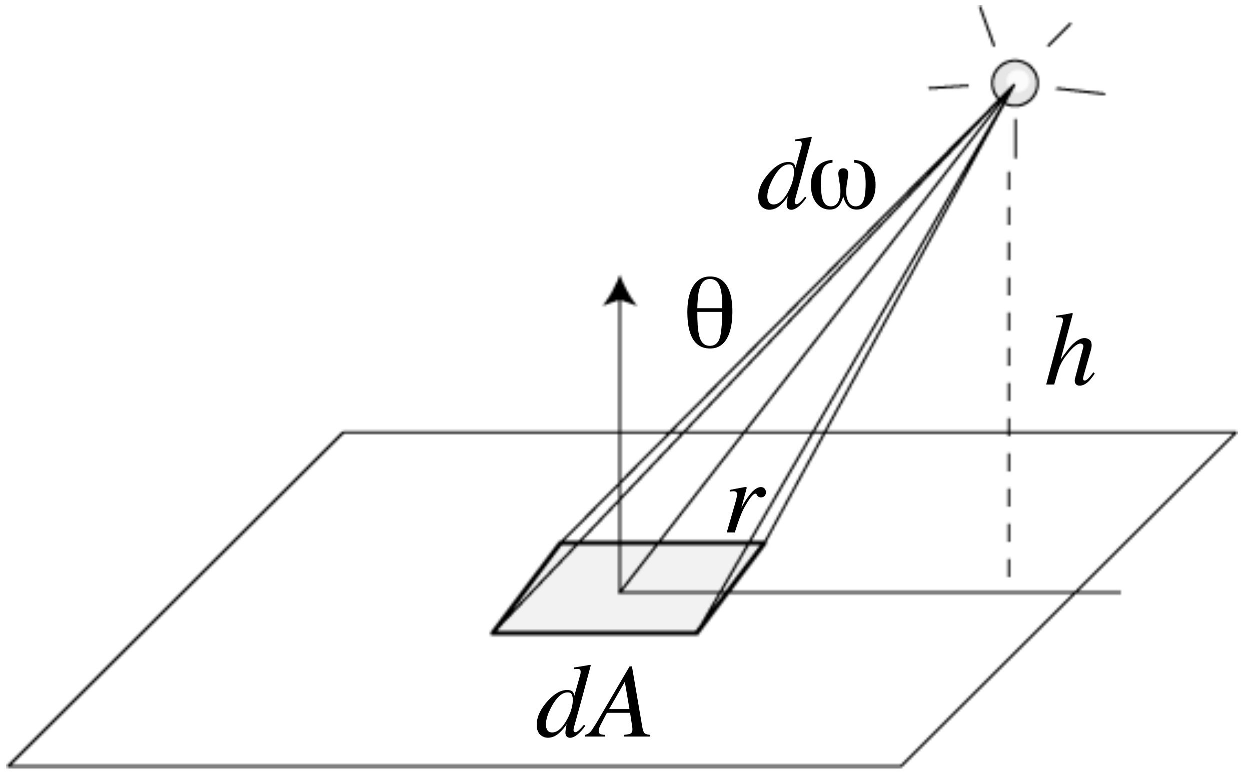
Irradiance: Isotropic Point Source

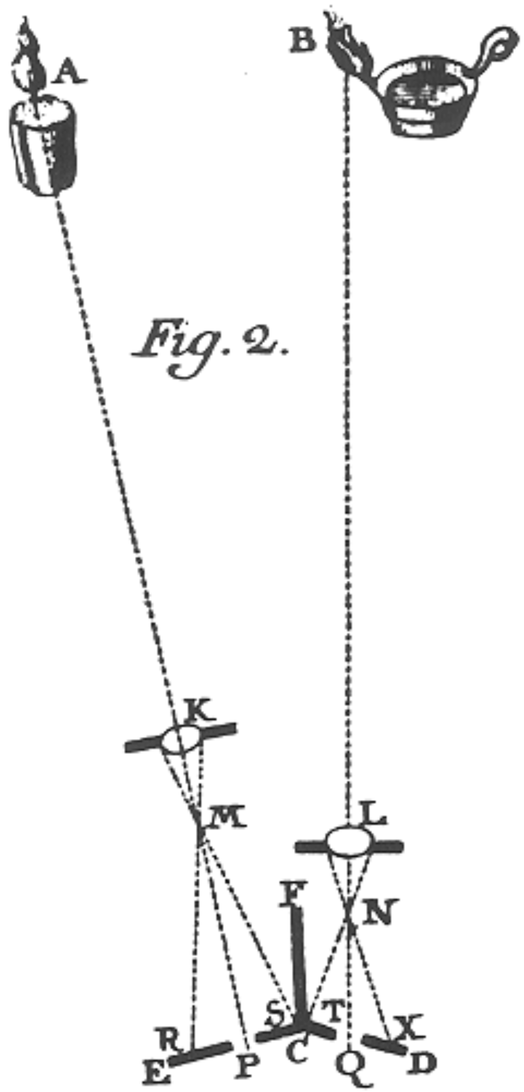


$$I = \frac{\Phi}{4\pi}$$

$$I\,d\omega = \frac{\Phi}{4\pi}\frac{\cos\theta}{r^2}dA = E\,dA$$

$$E = \frac{\Phi}{4\pi}\frac{\cos\theta}{r^2}$$

The Invention of Photometry



Bouguer's classic experiment

- Compare a light source and a candle
- Move until they both appear equally bright
- Intensity is proportional to ratio of distances squared

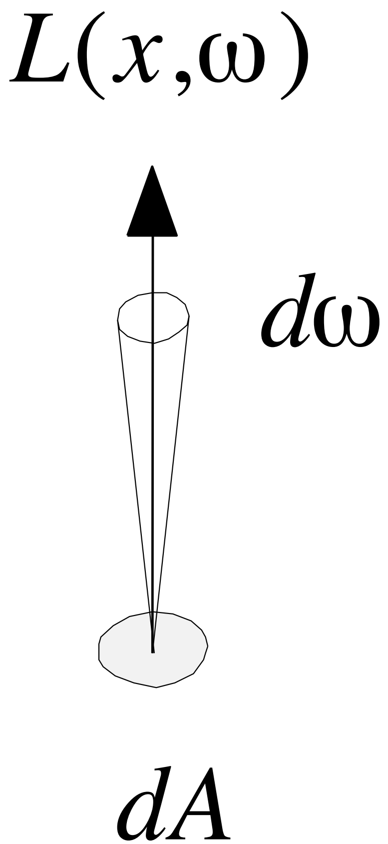
Definition of a candela

- Originally a "standard" candle
- Currently 550 nm laser with 1/683 W/sr
- 1 of 6 fundamental SI units

Radiance

Area Lights – Surface Radiance

Definition: The surface radiance (luminance) is the intensity per unit area leaving a surface



$$L(x,\omega) \equiv \frac{dI(x,\omega)}{dA}$$

$$= \frac{d^2\Phi(x,\omega)}{d\omega\, dA}$$

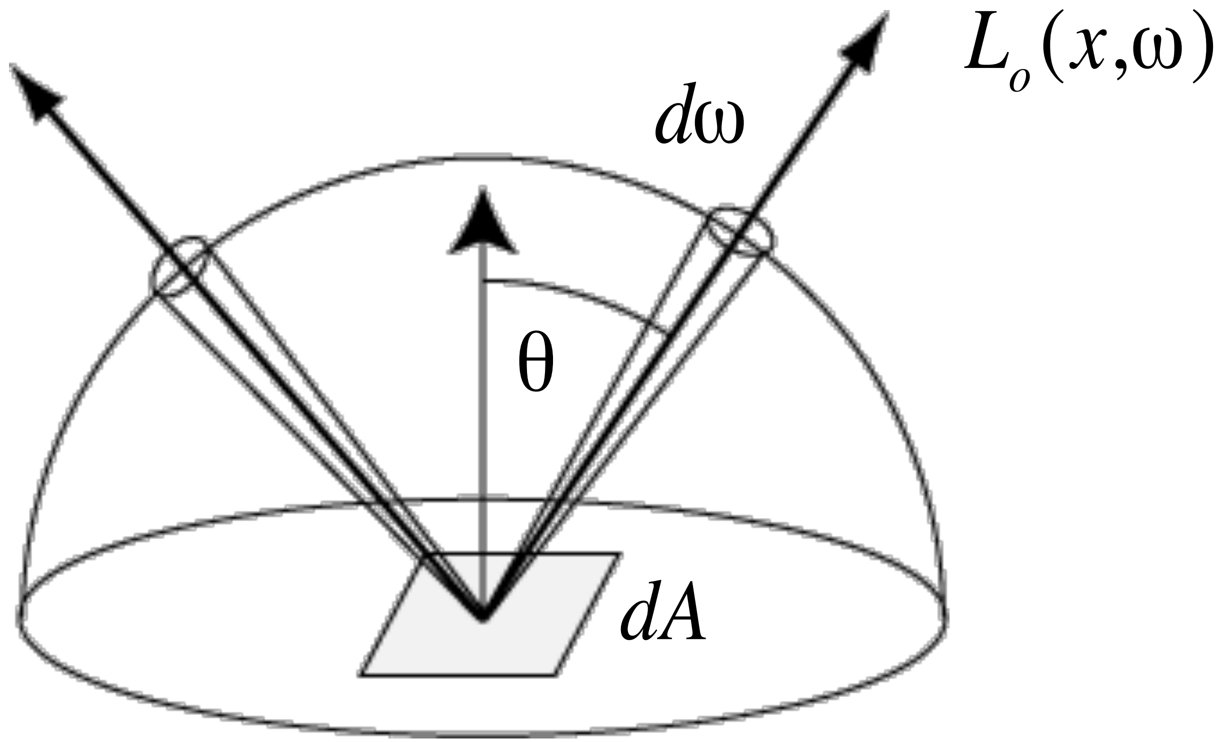
$$\left[\frac{W}{sr\, m^2}\right]\left[\frac{cd}{m^2} = \frac{lm}{sr\, m^2} = nit\right]$$

Typical Values of Luminance [cd/m^2]

Surface of the sun	**2,000,000,000 nit**
Sunlight clouds	**30,000**
Clear sky	**3,000**
Overcast sky	**300**
Moon	**0.03**

Directional Power Leaving a Surface

$$d^2\Phi_o(x,\omega) = L_o(x,\omega)\cos\theta\, dA\, d\omega$$



Same *dA* for all directions

Radiant Exitance

(Radiosity)

Radiant Exitance

Definition: The radiant (luminous) exitance is the energy per unit area leaving a surface.

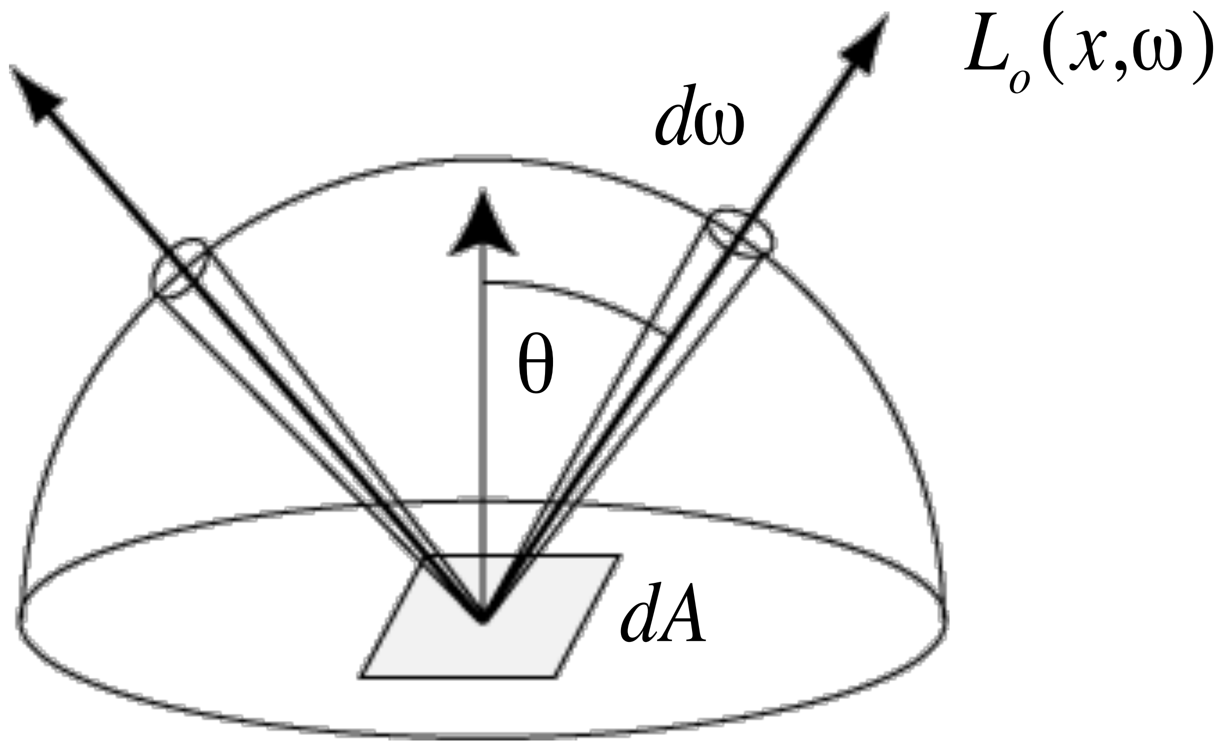
$$M(x) \equiv \frac{d\Phi_o}{dA}$$

$$\left[\frac{W}{m^2}\right] \quad \left[\frac{lm}{m^2} = lux\right]$$

In computer graphics, this quantity is usually referred to as the radiosity (*B*)

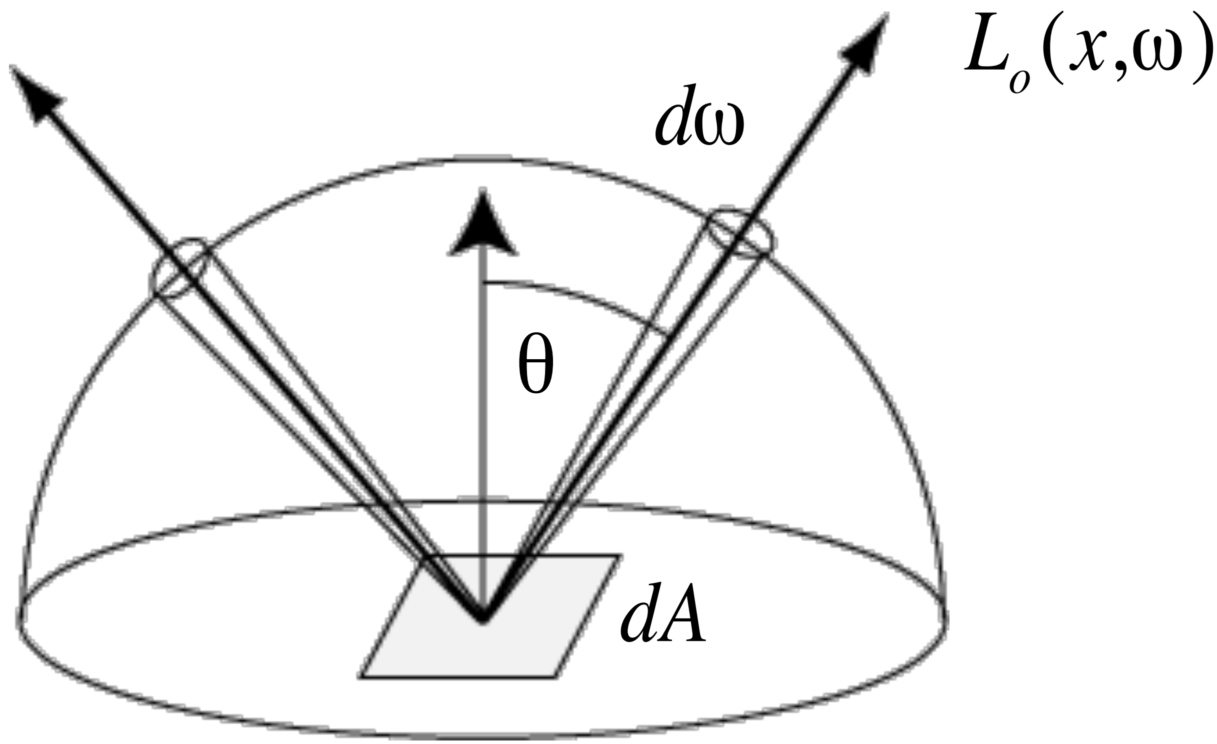
Area Light Source

$$d^2\Phi_o(x,\omega) = L_o(x,\omega)\cos\theta\, dA\, d\omega$$



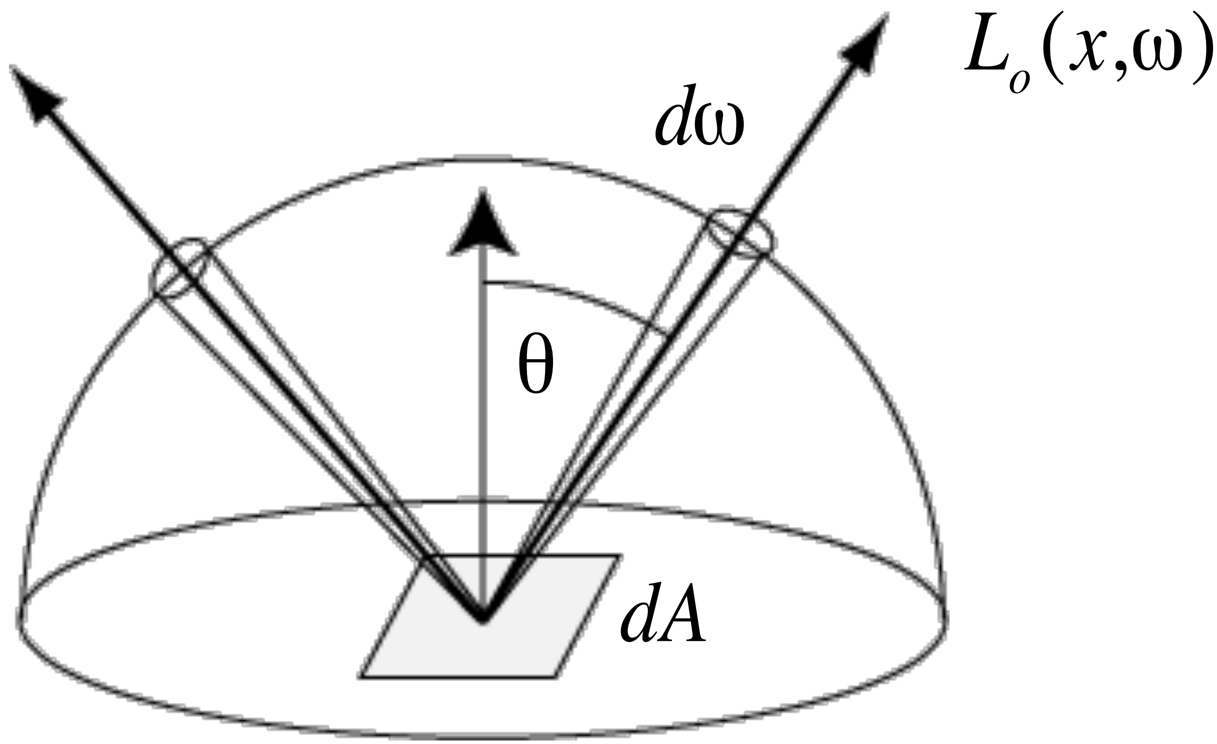
Area Light Source

$$dM(x,\omega) = \frac{d^2\Phi_o(x,\omega)}{dA} = L_o(x,\omega)\cos\theta\, d\omega$$



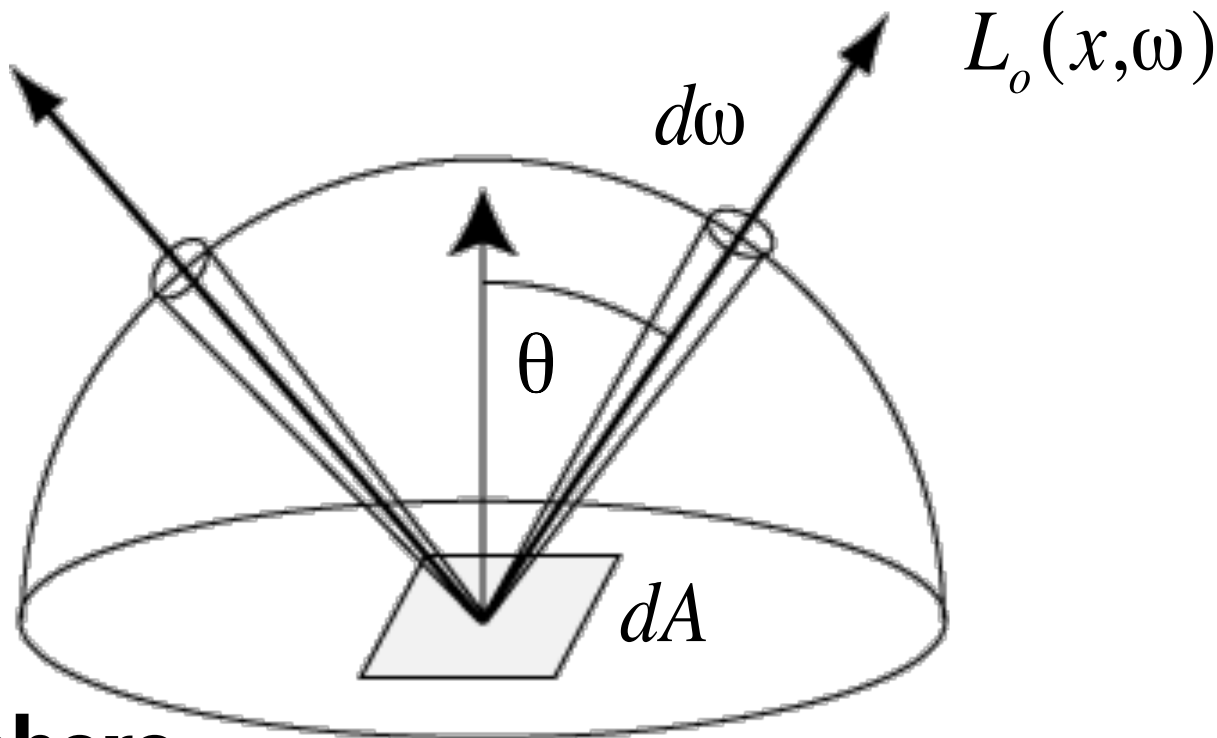
Area Light Source

$$dM(x,\omega) = L_o(x,\omega)\cos\theta \, d\omega$$



Area Light Source

$$M = \int_{H^2} dM(x,\omega) = \int_{H^2} L_o(x,\omega)\cos\theta \, d\omega$$



H^2 **Hemisphere**

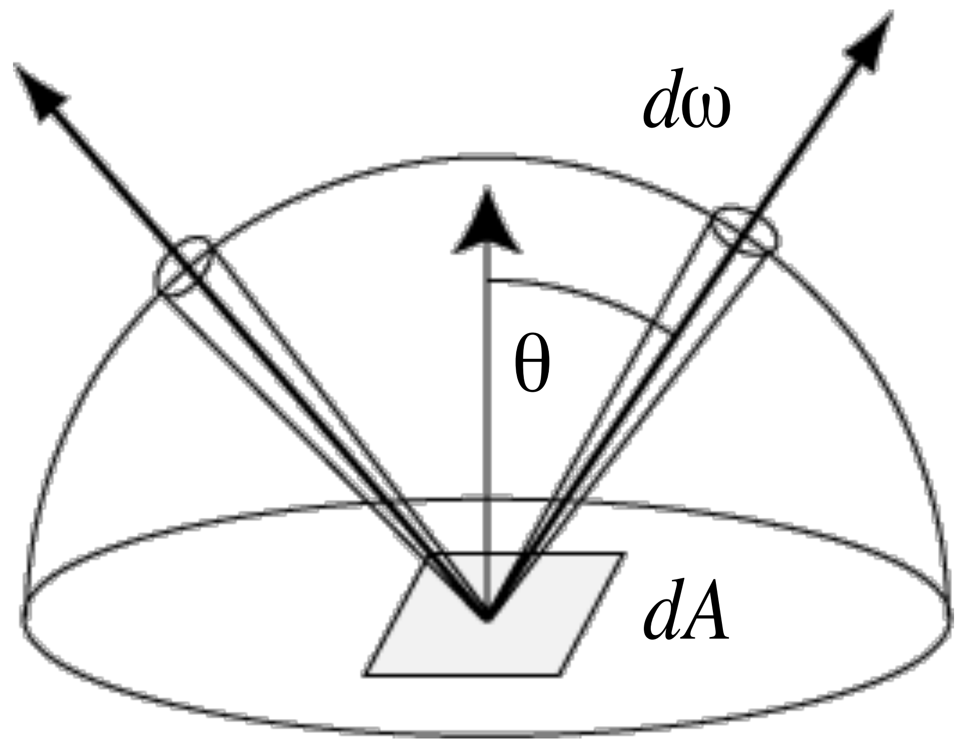
Uniform Diffuse Emitter

$$M = \int_{H^2} L_o \cos\theta \, d\omega$$

$$= L_o \int_{H^2} \cos\theta \, d\omega$$

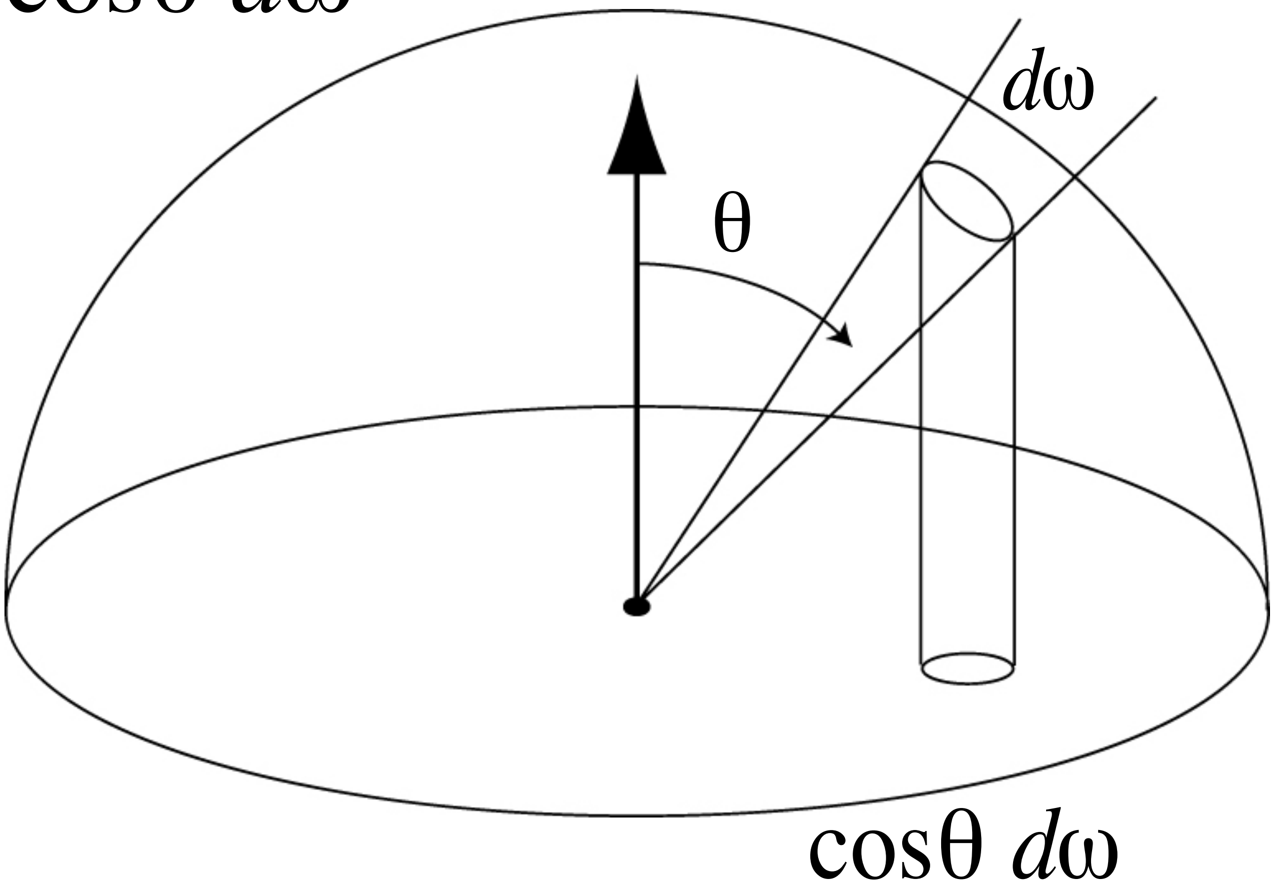
Uniform means L_o is not a function of direction

$$L_o(x,\omega) = L_o$$



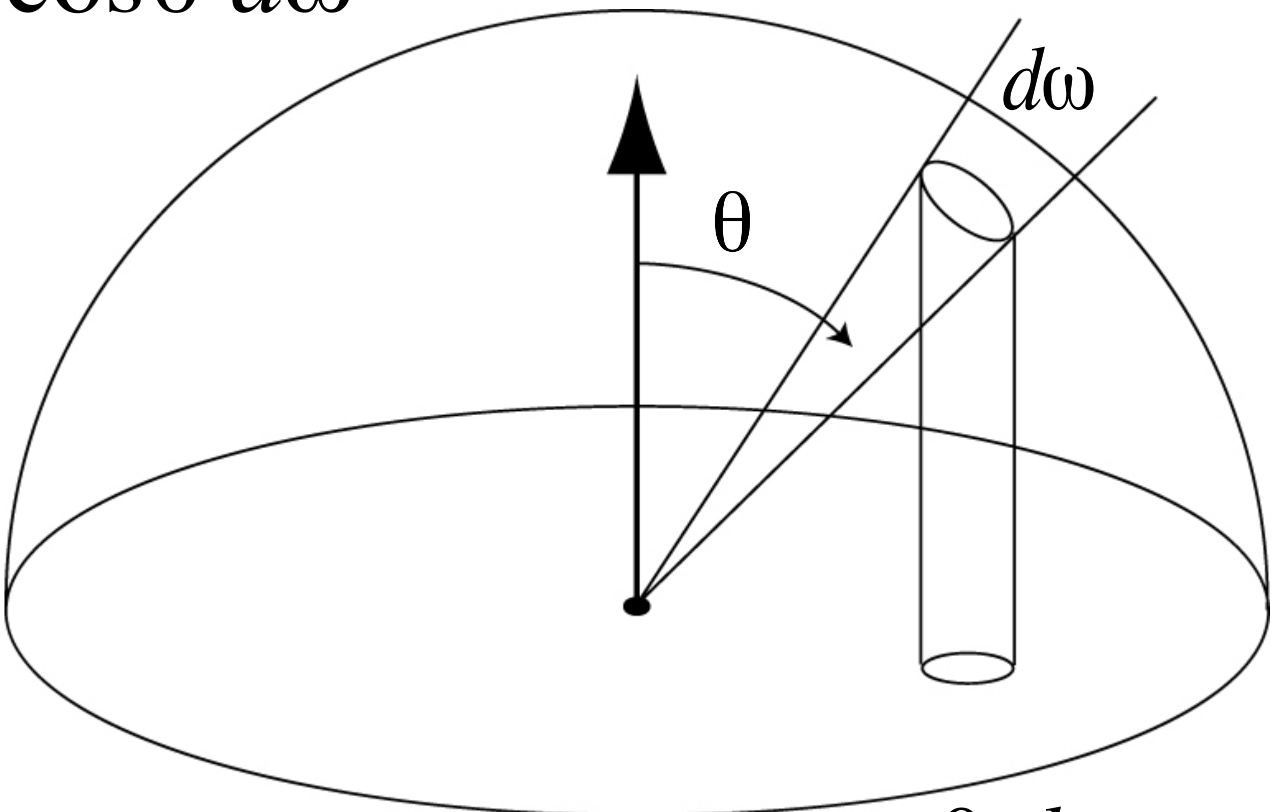
Projected Solid Angle

$$\tilde{\Omega} \equiv \int_{\Omega} \cos\theta \, d\omega$$



Projected Solid Angle

$$\tilde{\Omega} \equiv \int_{\Omega} \cos\theta \, d\omega$$



$$\tilde{\Omega} = \int_{H^2} \cos\theta \, d\omega = \pi$$

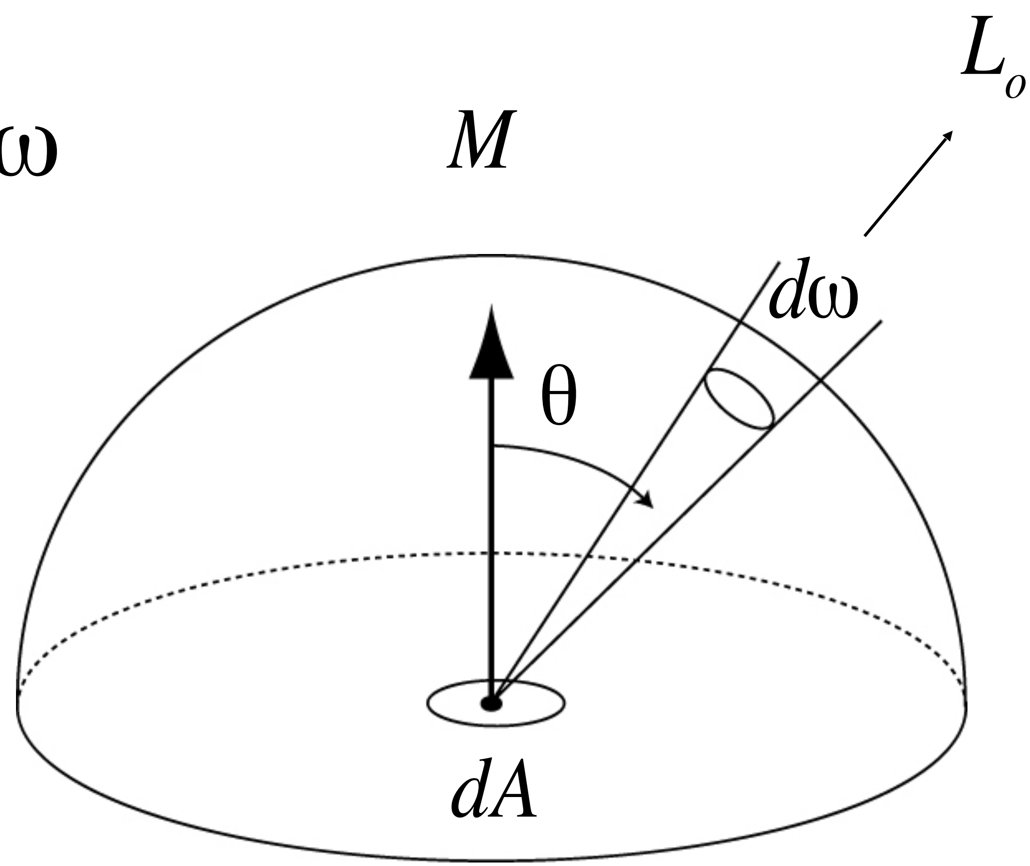
Uniform Diffuse Emitter

$$M = \int_{H^2} L_o \cos\theta \, d\omega$$

$$= L_o \int_{H^2} \cos\theta \, d\omega$$

$$= \pi L_o$$

$$L_o = \frac{M}{\pi}$$



Radiometry and Photometry

Summary

Radiometric and Photometric Terms

Physics	Radiometry	Photometry
Energy	Radiant Energy	Luminous Energy
Flux (Power)	Radiant Power	Luminous Power
Flux Density	Irradiance Radiosity	Illuminance Luminosity
Angular Flux Density	Radiance	Luminance
Intensity	Radiant Intensity	Luminous Intensity

Photometric Units

Photometry	Units		
	MKS	CGS	British
Luminous Energy	Talbot		
Luminous Power	Lumen		
Illuminance Luminosity	Lux	Phot	Footcandle
Luminance	Nit Apostilb, Blondel	Stilb Lambert	Footlambert
Luminous Intensity	Candela (Candle, Candlepower, Carcel, Hefner)		

"Thus one nit is one lux per steradian is one candela per square meter is one lumen per square meter per steradian. Got it?", James Kajiya