CS348n: Neural Representations and Generative Models for 3D Geometry



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02_14_CONDGEN_IMG 1

Project Proposal: One Page

- Title and participant names (up to three)
- Brief description of the project goal
 - Relation to class topics
 - Relation to other research projects of the participants (if any)
- Specific experiments or investigations to be conducted
- Evaluation metrics

Last Time: Conditional Shape Generation Based on 3D Data

Goal 1: Scan Completion

• Complete or re-generate shape from a single view scan



Symmetry & Priors: Each by Itself Has Difficulties

- Symmetry-based
 - Hard to predict from *partial* data.





• Hard to recover the *exact* shape.



[Shen et. al. 2012]

Complementary!

Completion Using Symmetry and Priors (Database)

• Combine both symmetry and database sources.





• Predict missing parts based on part relations.





Earlier efforts analyze **complete** shapes only

Approach

 Estimate part and symmetry structure from the *partial* scan data using data-driven *priors*.



Probabilistic Part Relations

- Part parameters
 - Local coordinates + Scale
- Pairwise relations
 - Gaussian distributions of *relative* pose, height, and scale





The Pipeline



Inference



Inference Time



Energy function



Comparison



Low

Goal 2: Composition-Based Modeling

Create a shape by assembling components of 3D models in a largescale repository.



Composition-Based Modeling

- Propose an iterative *assembly* system.
- Suggest *complementary* parts and their locations at each time.



Composition-Based Modeling

Interactive design interface

Automatic shape synthesis



Use Data Sets in the Wild

CAD data include *scene graphs*: Part geometry + Hierarchical structure





Observations

(+) Provides natural part segmentations.



Observations

(+) Provides natural part segmentations.

(-) Inconsistent and unlabeled.





Predict complementary parts using only geometric information



ComplementMe: Weakly-Supervised Component Suggestions



Retrieval and Embedding

Predict a multimodal probability distribution (Bishop 1994).



Neural Network Training



coordinates

Placement Network

- Sample a complement from the predicted distribution.
- Predict the location of the selected component.



Placement Network



Automatic Shape Synthesis

Add the maximum probability part iteratively.





Automatic Shape Synthesis



Automatic Shape Synthesis



Observation

The retrieval network discovers *interchangeable* parts.



Can discover semantic relationships among parts!

Limitations

Limitations

- Accumulating noise in iterations.
- Missing notion of termination.



Learn Relations Among Partial Shapes

Learn relations among *partial shapes*.

- Can complete an object with a single retrieval.
- Can discover group-to-group relations.



Relations Among Partial Shapes

Learn relations among *partial shapes*.

- Complementarity
- Interchangeability



Jointly encode both *complementary* and *interchangeable* relations in a *dual* embedding space.



Learning

Learn *interchangeability* from *complementarity*.

- Complementary pairs are created by splitting objects.
- No supervision for *interchangeability* is given.



Applications

• Shape analysis



• Shape completion



Complementary Shape Retrievals


Complementary Shape Retrievals



Complementary Shape Retrievals



Interchangeable Shape Retrievals



Interchangeable Shape Retrievals



Conditional Shape Generation Based on Image Data

3D Perception from a Single Image



Visual 3D Cues are Complicated

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Image 2 PointCloud

Fan, H., Su, H. and Guibas, L.J., 2017. A point set generation network for 3D object reconstruction from a single image. CVPR 2017

Point Cloud Synthesis from a Single Image



Input

Reconstructed 3D point cloud

Point Cloud Synthesis from a Single Image



Input

Reconstructed 3D point cloud

End-to-End Learning



Ground truth point cloud

Synthesize for Learning



End-to-End Learning



Point Cloud Distance Metrics

Given two sets of points, measure their discrepancy



Point Cloud Distance Metrics

Worst case: Hausdorff distance (HD)

 $d_{\text{HD}}(S_1, S_2) = \max\{\max_{x_i \in S_1} \min_{y_j \in S_2} \|x_i - y_j\|, \max_{y_j \in S_2} \min_{x_i \in S_1} \|x_i - y_j\|\}$

Average case: Chamfer distance (CD)

$$d_{CD}(S_1, S_2) = \frac{1}{n} \sum_{x \in S_1} \min_{y \in S_2} \|x - y\|_2^2 + \frac{1}{m} \sum_{y \in S_2} \min_{x \in S_1} \|x - y\|_2^2$$

Optimal case: Earth Mover's distance (EMD)

$$d_{EMD}(S_1, S_2) = \min_{\phi: S_1 \to S_2} \sum_{x \in S_1} \|x - \phi(x)\|_2$$

where $\phi: S_1 \to S_2$ is a bijection.

Solves the optimal transportation (bipartite matching) problem!



End-to-End Learning Architecture



End-to-End Learning Architecture



Natural Statistics of Object Geometry



- Many smooth local structures are common
 - e.g., planar patches, cylindrical patches
 - strong local correlation among point coordinates

Natural Statistics of Object Geometry

- Many local structures are common/shared
 - e.g., planar patches, cylindrical patches
 - strong local correlation among point coordinates
- But also some intricate local structures
 - some points have high variability neighborhoods



Two-Branch Architecture



Set union by array concatenation

Deconvolution Branch





- Deconvolution induces a smooth coordinate map
- Geometrically, learns a smooth parameterization

Fully Connected Branch



The Two Branches

blue: deconv branch - large, consistent, smooth structures
red: fully-connected branch - more intricate structures



Example Results



From Real Images



Ambiguity in Object Views

• A fundamental issue: inherent ambiguity in prediction



 By loss minimization, the network tends to predict a "mean shape" that averages out uncertainty

Distance Metrics Affect Mean Shapes



The mean shape carries characteristics of the distance metric

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Distance Metrics Affect Mean Shapes

The mean shape carries characteristics of the distance metric



EMD vs CD Predictions



Additional Views Reduce Uncertainty

Can we reduce prediction uncertainty by

factoring out the inherent ambiguity of groundtruth?



Input

Possible observations from a novel viewpoint

3D Voxel CNNs: Reconstruction from Views



3D-R2N2: A Unified Approach for Single and Multi-view 3D Object Reconstruction Christopher B. Choy, Danfei Xu, JunYoung Gwak, Kevin Chen, Silvio Savarese ECCV 2016

3D Voxel CNNs: Reconstruction



3D-R2N2: A Unified Approach for Single and Multi-view 3D Object Reconstruction **Christopher B. Choy, Danfei Xu, JunYoung Gwak, Kevin Chen, Silvio Savarese** *ECCV 2016*

Pose Estimation and View Aggregation



Canonical "Containers" for Object Categories





He Wang, Srinath Sridhar, Jingwei Huang, Julien Valentin, Shuran Song, Leonidas J. Guibas. *Normalized Object Coordinate Space for Category-Level 6D Object Pose and Size Estimation*. CVPR 2019.





Normalized Object Coordinate Spaces (NOCS)


Exploit ShapeNet Category Co-Alignments

*





Abstract away, pose, size, some intra-category variation



Normalized Object Coordinate Spaces (NOCS)



He Wang, Srinath Sridhar, Jingwei Huang, Julien Valentin, Shuran Song, Leonidas J. Guibas. *Normalized Object Coordinate Space for Category-Level 6D Object Pose and Size Estimation*. CVPR 2019.

NOCS Lifting Map



NOCS for 9D Object Pose and Size Estimation



Input: RGB-D Image + Category-Level CAD Model Repository

<u>Output</u>: 3 degrees Translation + 3 Rotation + 3 Size (9 DoF)

No object-specific CAD model

An Image-Centric Approach: Build on Mask-RCNN



A Mask-RCNN-based backbone

Category-level object pose can be defined for each category up to the limit of global symmetry in the category.





Context-Aware MixEd ReAlity (CAMERA) Dataset



CAMERA Dataset

- **300K** mixed reality images are generated
 - 275K training
 - 25K validation
- **31 scenes** captured from IKEA as real backgrounds
 - 27 scenes for training
 - 4 scene for validation
 - 553 images
- 6 object categories bottle, bowl, camera, can, laptop and mug
 - 1085 models, 184 for validation
- Distractor objects





Real Dataset

- 8K RGB-D frames
 - training/validation/testing
 - 4300/950/2750
- 18 different real scenes
 - training/validation/testing
 - 7/5/6
- 42 unique instances
 - 7 per category
 - training/validation/testing
 - 3/1/3

Plus COCO images without pose annotation



Qualitative Results: Real Data



Multi-View Aggregation as <u>Set Union</u>



Multi-View NOCS Aggregation



- Set union of
- No surface

Pix2Surf: Single-View Single-Chart



Lei, J., Sridhar, S., Guerrero, P., Sung, M., Mitra, N. and Guibas, L.J. Pix2surf: Learning parametric 3D surface models of objects from images. ECCV 2020.

Pix2Surf: Single-View Single-Chart



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Pix2Surf: Multi-View Atlas



Multi-View consistency loss $L_c = \frac{1}{|P|} \sum_{(i,j)\in P} ||x_i - x_j||_2$

Multi-View Consistency Loss



Emergent Properties



Image 2 Shape from Unseen Classes

Learning to Reconstruct Shapes from Unseen Classes













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Formulation









DRC* [Tulsiani et al., CVPR '17] [Differentiable Ray Consistency]



AtlasNet* [Groueix et al., CVPR '18]

*Trained on cars, chairs, airplanes

Image I

Neural Network f_{θ}

Shape V

Training: $\operatorname{argmin}_{\theta} \operatorname{Loss}(f_{\theta}(I), V)$

Formulation









DRC* [Tulsiani et al., CVPR '17]

Image I

Neural Network f_{θ}

Shape V

Training: $\operatorname{argmin}_{\theta} \operatorname{Loss}(f_{\theta}(I), V)$

Directly regularize $f_{\theta}(I)$ by adding inductive biases

AtlasNet* [Groueix et al., CVPR '18]

*Trained on cars, chairs, airplanes

What is the proper inductive bias of $f_{\theta}(I)$?

Forward: image formation



Depth to Shape?





Image I

Neural Network f_{θ}



Depth **D**



Neural Network g_{θ}

Shape V

Neural network g_{θ} is over-parameterized: g_{θ} has to learn a deterministic mapping!

Projecting depth into 3D is a deterministic, fully differentiable process!

MarrNet [Wu et at., NIPS '17]

Depth to Shape?











Image I

Neural Network f_{θ}

Depth D

Neural Network g_{θ}

Shape V









Neural Network f_{θ}





Partial surface in 3D is very sparse, which makes it hard for g_{θ} to capture surface features.







Output Shape





Spherical map as a surrogate representation for surfaces in 3D















GenRe (view.) [this work]







GenRe (view.)

[this work]







AtlasNet (obj.) is 8% better in Chamfer distance than GenRe

Results: Generalizing to Unseen Classes





Training: cars, chairs, airplanes Testing: sofas




Training: cars, chairs, airplanes Testing: sofas



Training: cars, chairs, airplanes **Testing:** sofas





Partial/Single-View Spherical Map



Partial/Single-View Spherical Map

Full/Multi-View Spherical Map

Testing: bookcases, tables, loudspeakers





Training: cars, chairs, airplanes **Testing:** beds, benches



Training: cars, chairs, airplanes Testing: beds, benches

Image 2 Shape Representation Considerations

What is the best 3D Representation?



Voxel-based 3D Representations



Discretization of a 3D surface with a voxel grid:

- Can accurately capture shape details.
- The parametrization size is proportional to the reconstruction quality.
- Cannot yield smooth reconstructions.
- Cannot convey semantic information.

Point-based 3D Representations



Input Image

Neural Network

Pointcloud

Discretization of a 3D surface with 3D points:

- Can accurately capture shape details.
- The parametrization size is proportional to the reconstruction quality.
- Cannot yield smooth reconstructions.
- Cannot convey semantic information.
- Lacks surface connectivity and assumes a fixed number of points.

Mesh-based 3D Representations



Input Image

Neural Network

 \mathbf{Mesh}

Discretization of a 3D surface with vertices and faces:

- Can accurately capture shape details.
- Yields smooth reconstructions.
- Imposes a large parametrization size.
- Typically requires class-specific template topology.
- Cannot convey semantic information.

Primitive-based 3D Representations



Input Image

Neural Network

Primitives

Discretization of a 3D surface with parts:

- Can accurately capture shape details.
- Yields **smooth reconstructions.**
- Imposes a small parametrization size.
- Requires post-processing.
- Typically fails to reconstruct fine shape details.

Implicit-based 3D Representations



Input Image

Neural Network Implicit Surface

No Discretization:

- Can convey semantic information.
- Yields smooth reconstructions.
- Imposes a small parametrization size.
- Ensures inter-object coherence.
- Cannot convey semantic information.

Occupancy Networks: Learning 3D Reconstruction in Function Space

Lars Mescheder, Michael Oechsle, Michael Niemeyer, Sebastian Nowozin, Andreas Geiger

CVPR 2019

- Key Idea:
 - Do not represent the 3D shape explicitly
 - Consider the surface **implicitly**, as **the decision boundary of a non-linear classifier**, parametrized by the neural network:

$$\begin{array}{ccc} f_{\theta} : \mathbb{R}^{3} \times \mathcal{X} \to [0,1] \\ & \bigstar & \bigstar \\ & \text{3D} & \text{Condition} & \text{Occupancy} \\ & \text{Location} & (\text{eg, Image}) & \text{Probability} \end{array}$$

- Space partitioning:
 - Outside the surface: f(x) = 0
 - Inside the surface: f(x) = 1

- Key Idea:
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- Space partitioning:
 - Outside the surface: f(x) = 0
 - Inside the surface: f(x) = 1
 - Alternatively, we can use the level set of a signed distance function.

>0

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3D Locations





Occupancy Probability

The decision boundary of the classifier models the occupancy field.

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- Benefits:
 - Can generate 3D shapes of infinite high resolutions.
 - Can capture arbitrary topologies.

Network Architecture



Network Architecture



How can we train Occupancy Networks?

$$\mathcal{L}(\theta, \psi) = \sum_{j=1}^{K} \mathsf{BCE}(f_{\theta}(p_{ij}, z_i), o_{ij}) + KL\left[q_{\psi}(z | (p_{ij}, o_{ij})_{j=1:K}) \| p_0(z)\right]$$

 $\mathcal{L}(\theta, \psi) = \sum_{j=1}^{K} \underbrace{\mathsf{BCE}(f_{\theta}(p_{ij}, z_i), o_{ij})}_{j=1} + KL\left[q_{\psi}(z|(p_{ij}, o_{ij})_{j=1:K}) \parallel p_0(z)\right]$









Can we convert the implicit surface to mesh?

We need to extract the isosurface corresponding to the predicted implicit surface:



Can we convert the implicit surface to mesh?

We need to extract the isosurface corresponding to the predicted implicit surface:





• In a grid of points and find the points that are **occupied** and points that are **unoccupied**.



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- Mark the voxels between occupied and unoccupied points as voxels that require further investigation.



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- In a grid of points and find the points that are **occupied** and points that are **unoccupied**.
- Mark the voxels between occupied and unoccupied points as voxels that require further investigation.
- Further subdivide these voxels.
- Query these voxels again and find the occupied and unoccupied points.
Can we convert the implicit surface to mesh?

To extract an isosurface corresponding to a new observation given a trained occupancy network, use Multiresolution IsoSurface Extraction (MISE):



• Repeat this process N times until we reach the desired resolution.

Can we convert the implicit surface to mesh?

To extract an isosurface corresponding to a new observation given a trained occupancy network, use Multiresolution IsoSurface Extraction (MISE):



- Repeat this process N times until we reach the desired resolution.
- Extract triangular mesh using Marching Cubes.

How well does it work?



What about Generative Models?



DISN: Deep Implicit Surface Network for High-quality Singleview 3D Reconstruction

Weiyue Wang*, Qiangeng Xu*, Duygu Ceylan, Radomir Mech, Ulrich Neumann

NeurIPS 2019

Implicit 3D surfaces for recovering fine details

 Deep Implicit Surface Network (DISN) DISN predicts Signed Distance Function (SDF) for each 3D point. SDFs do not impose any constraints on the output topology and resolution.



SDF is a continuous function that maps a given 3D point $\mathbf{p} = (x, y, z) \in \mathbb{R}^3$ to a real value $s \in \mathbb{R} : s = SDF(\mathbf{p})$.

An isosurface $S_0 = \{p | SDF(p) = 0\}$ implicitly represents the underlying 3D shape.

Key Idea: Use both global and local features for capturing both the overall shape and the fine-grained details.



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• Loss: MSE between the transformed point cloud and the ground truth point cloud in the camera space.

$$L_{cam} = \frac{\sum_{\mathbf{p}_w \in PC_w} ||\mathbf{p}_G - (\mathbf{R}\mathbf{p}_w + \mathbf{t}))||_2^2}{\sum_{\mathbf{p}_w \in PC_w} 1}$$

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• The **rotation matrix R** and the **translation vector t** are directly predicted from the network.

• **Camera Pose Network:** Estimate the camera pose, the 6 DoF transformation from the camera coordinate to world coordinate.



Camera Pose Network

- **Camera Pose Network:** Estimate the camera pose, the 6 DoF transformation from the camera coordinate to world coordinate.
- Local Feature Extraction Network: Using the camera pose find a 3D point's 2D location on the image and extract local feature patches from multiple network layers.





Local Feature Extraction Network

Key Idea: Use both global and local features for capturing both the overall shape and the fine-grained details.



Optimization Objective

$$L_{SDF} = \sum_{\mathbf{p}} m |f(I, \mathbf{p}) - SDF^{I}(\mathbf{p})|,$$

$$m = \begin{cases} m_{1}, & \text{if } SDF^{I}(\mathbf{p}) < \delta, \\ m_{2}, & \text{otherwise}, \end{cases}$$

How well does it work?



How well does it work?



What about generative models?



PIFu: Pixel-Aligned Implicit Function for High-Resolution Clothed Human Digitization

Shunsuke Saito*, Zeng Huang*, Ryota Natsume*, Shigeo Morishima,

Angjoo Kanazawa, Hao Li

ICCV 2019

Key Idea: The **pixel-aligned implicit function** consists of a fully convolutional **image encoder** $g(\cdot)$ and a **continuous implicit function** $f(\cdot)$ represented by multi-layer perceptrons (MLPs), where the surface is defined as a level set of

$$f(F(x), z(X)) = s : s \in \mathbb{R}$$

where for a 3D point X, $x = \pi(X)$ is its 2D projection, z(X) is the **depth value in the camera coordinate space**, F(x) = g(I(x)) is **the image feature at x**.



















What about texture?













Optimization Objective

• Surface Reconstruction:

$$\mathcal{L}_{V} = \frac{1}{n} \sum_{i=1}^{n} |f_{v}(F_{V}(x_{i}), z(X_{i})) - f_{v}^{*}(X_{i})|^{2}$$

Groundtruth surface: 1 if X is inside the mesh, 0 otherwise.

Optimization Objective

• Surface Reconstruction:

$$\mathcal{L}_{V} = \frac{1}{n} \sum_{i=1}^{n} |f_{v}(F_{V}(x_{i}), z(X_{i})) - f_{v}^{*}(X_{i})|^{2}$$

Groundtruth surface: 1 if X is inside the mesh, 0 otherwise.

• Texture Reconstruction:

$$\mathcal{L}_{C} = \frac{1}{n} \sum_{i=1}^{n} |f_{c}(F_{C}(x_{i}), z(X_{i})) - C(X_{i})|$$

Groundtruth RGB value on the surface point

Inference



 $\begin{array}{l} \text{n-view inputs} \\ (n \geq 1) \end{array}$

Inference



• PiFU maps the input image into a continuous occupancy field.
Inference



- PiFU maps the input image into a continuous occupancy field.
- Using Marching Cubes we can recover the surface of the object.

Inference



- PiFU maps the input image into a continuous occupancy field.
- Using Marching Cubes we can recover the surface of the object.
- For every point in the reconstructed surface, we use Text-PiFu to estimate its corresponding colour.

How well does it work?



How well does it work?



Texture Fields: Learning Texture Representations in Function Space

Michael Oechsle, Lars Mescheder, Michael Niemeyer,

Thilo Strauss, Andreas Geiger

ICCV 2019

A similar idea: Texture Fields



A similar idea: Texture Fields



Texture Fields as Generative Models



What about Generative Models?



Occupancy Networks:









The decision boundary of the classifier models the occupancy field.

Occupancy Probability

3D Locations



Camera Pose Estimation Network

• PiFU:



• Texture Fields:



Learning to Infer Implicit Surfaces without 3D Supervision

Shichen Liu, Shunsuke Saito, Weikai Chen (B), Hao Li

NeurIPS 2019

Infer Implicit Surfaces without 3D Supervision

• How can we learn to **infer implicit surfaces solely from images**, without any 3D supervision?



Infer Implicit Surfaces without 3D Supervision

• How can we learn to **infer implicit surfaces solely from images**, without any 3D supervision?



How do we define our loss function?



(a) 3D anchor points

• Sample a sparse set of 3D points.



- Sample a sparse set of 3D points.
- For each 3D point compute its occupancy value. Each point is assigned a support region to enable ray point intersection.



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$$\mathcal{L}_{sil} = \frac{1}{N_r} \sum_{i=1}^{N_r} \sum_{k=1}^{N_K} \|\psi(\pi_k, \mathbf{x}_i) - S_k(\mathbf{x}_i)\|^2$$

Geometric Regularization on Implicit Surfaces

$$\mathcal{L}_{geo} = \frac{1}{N_p} \sum_{j=1}^{N_p} W(\phi(\mathbf{s}_j)) \frac{\sum_{l=1}^{6} W(\phi(\mathbf{q}_j^l)) \|\mathbf{n}(\mathbf{s}_j) - \mathbf{n}(\mathbf{q}_j^l)\|_p^p}{\sum_{l=1}^{6} W(\phi(\mathbf{q}_j^l))}$$
$$W(x) = \mathbb{I}(|x - 0.5| < \epsilon)$$



$$\mathbf{n}(\mathbf{p}_j) = \frac{\delta\phi}{\delta\mathbf{p}_j} / \left| \frac{\delta\phi}{\delta\mathbf{p}_j} \right|$$

How well does it work?



