Robust Distributed Network Localization with Noisy Range Measurements

Author: David Moore,
John Leonard,
Daniela Rus,
Seth Teller

Speaker: Cheng-Tao Chu

Outline

- Problem definition
- Features of the algorithm
- Proposed Algorithm
- Background Knowledge
- Proof of Correctness
- Computational Complexity
- Experiment
- Conclusion

Problem definition

- Given a planar graph with edges of known length, recover the Euclidean position of each vertex up to a global rotation and translation.
- Difficulties:
  - Insufficient data to uniquely compute the positions
  - Noisy measurement
  - Lack of absolute reference points
  - Scalability over the size of the network

Problem definition – cont.

- Let $G = (V, E, \omega)$ be an incomplete undirected edge-weighted graph. G is said to be r-embeddable if there exists a mapping $\phi : V \rightarrow R^r$ such that for every edge $(v_i, v_j) \in E \Rightarrow \left\| \phi(v_i) - \phi(v_j) \right\| = w_{ij}$
- The Embeddability problem (r-embeddability problem) is the problem of determining whether $G = (V, E, \omega)$ is embeddable (r-embeddable).

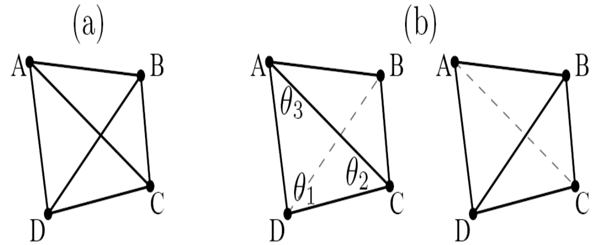
Features of the algorithm

1. Support noisy distance measurements
2. Distributed and requiring no beacons or anchors
3. Localize each node correctly with high probability or not at all
4. Cluster-based and support dynamic node insertion and mobility (a cluster is a node and its set of neighbors)

Proposed Algorithm (high level)

- Phase 1. Cluster Localization
- Phase 2. Cluster Optimization (Optional)
- Phase 3. Cluster Transformation

- Robust quadrilateral
  - The four sub-triangles satisfy ($b \sin^2 \theta > d_{\min}$) where b is the shortest side and θ is the smallest angle



Proposed Algorithm – Phase 1

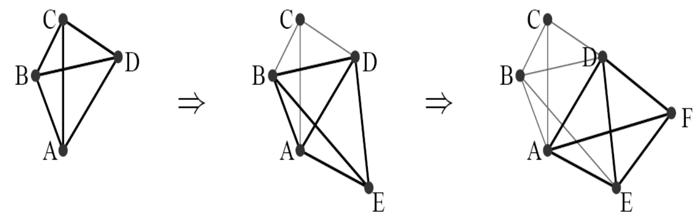
- Distance measurements from each one-hop neighbors are broadcast to the origin node
- The complete set of robust quadrilaterals in the cluster is computed (Alg. 1) and the overlap graph (robust quads are the vertices and insert an edge if two vertices share three nodes) is generated
- Position estimates are computed for as many nodes as possible via a breadth-first search in the overlap graph (Alg. 2).

Proposed Algorithm – Phase 3

- After Phase 1 is complete for the two clusters, the positions of each node in each local coordinate system are shared.
- As long as there are at lease three non-collinear nodes in common between the two localizations, the transformation can be computed
- By testing if these three nodes form a robust triangle, we simultaneously guarantee non-collinearity and the same resistance to flip ambiguities.

Related Background Knowledge

- The r-embeddability problem is NP-hard for r>=2



- Trilateration
- Non-rigid or rigid graphs
- Flip ambiguities
- Discontinuous flex ambiguities

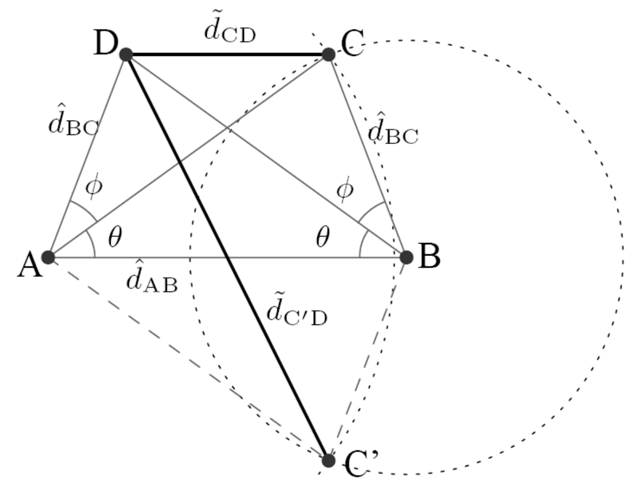
Proof of Correctness – Flex ambi.

- Flex ambiguity occurs only when a rigid graph becomes non-rigid by the removal of a single edge.
- Laman's theorem: Let a graph G have exactly 2n-3 edges where n is the number of vertices. G is generically rigid in R^2 if and only if every subgraph G' with n' vertices has 2n'-3 or fewer edges.

Proof of Correctness – Flip ambi.

- Assuming the noise is zero-mean Gaussian distributed

$$P(X > d + d_{err}) = \Phi(\frac{d_{err}}{\sigma})$$



- For simplicity, the graph is

left-right symmetric although the probability of error will only decrease by breaking this symmetry

$$d_{err} = \frac{d_{C'D} - d_{CD}}{2} = d_{AB} \frac{\sqrt{\sin^2\phi + 4\sin^2(\theta+\phi)\sin^2\theta} - \sin\phi}{2\sin(2\theta+\phi)}$$

$$\frac{\partial d_{err}}{\partial \phi} = 0 \Rightarrow d_{err} = d_{AB}\sin^2\theta$$

Computational Complexity

- The algorithm avoids nodes that may have position ambiguities at the cost of failing to find all possible realizations.
- T(Alg. 1)=$O(m^3)$: m is the maximum node degree.
- T(Alg. 2)=O(q): q is the number of robust quadrilaterals (<=$O(m^3)$)
- T(Finding the inter-cluster transformations for one cluster)=$O(m^2)$.
- Communication overhead: $O(m^2)$

Experiment – Evaluation criteria

$$\sigma_p^2 = \sum_{i=1}^{N} \frac{(\hat{x}_i - x_i)^2 + (\hat{y}_i - y_i)^2}{N}$$

position error

$$\sigma_d^2 = \sum_{i=1}^{M} \frac{(\hat{d}_i - d_i)^2}{M}$$

distance measurement error

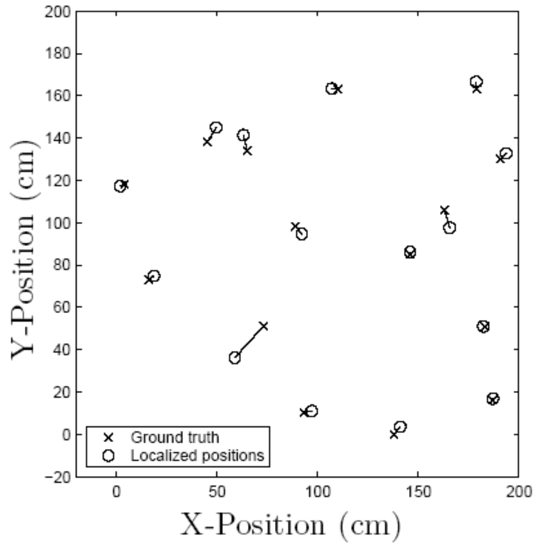
$$\bar{R} = \frac{1}{N} \sum_{i}^{N} \frac{L_i}{K_i}$$

cluster success rate

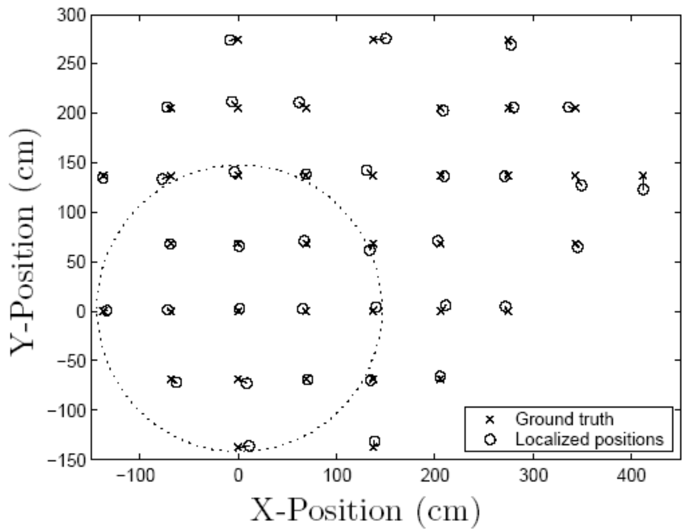
$$\tilde{R} = \frac{\max | forest_i |}{| nodes |}$$

largest forest

Experiment – Data

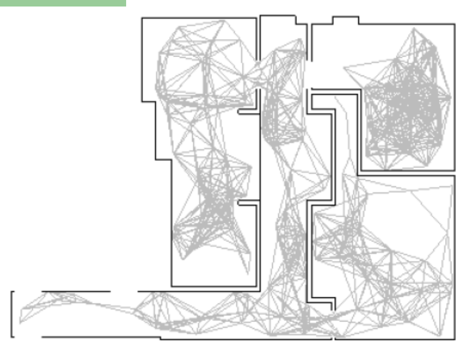


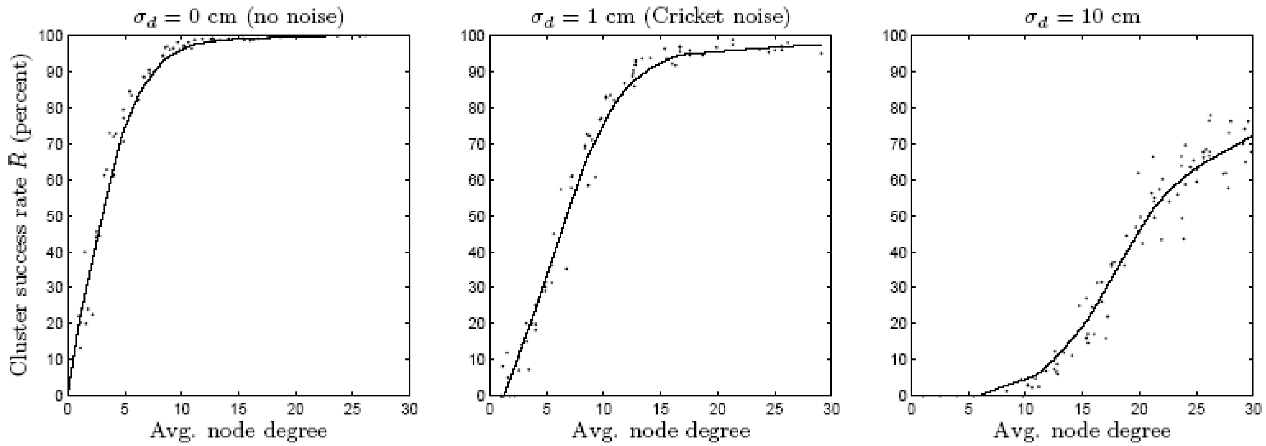
metric	value
σ_d	4.38 cm
σ_p	6.82 cm
$\bar{R}$	0.97
$\tilde{R}$	$38/40 = 0.95$



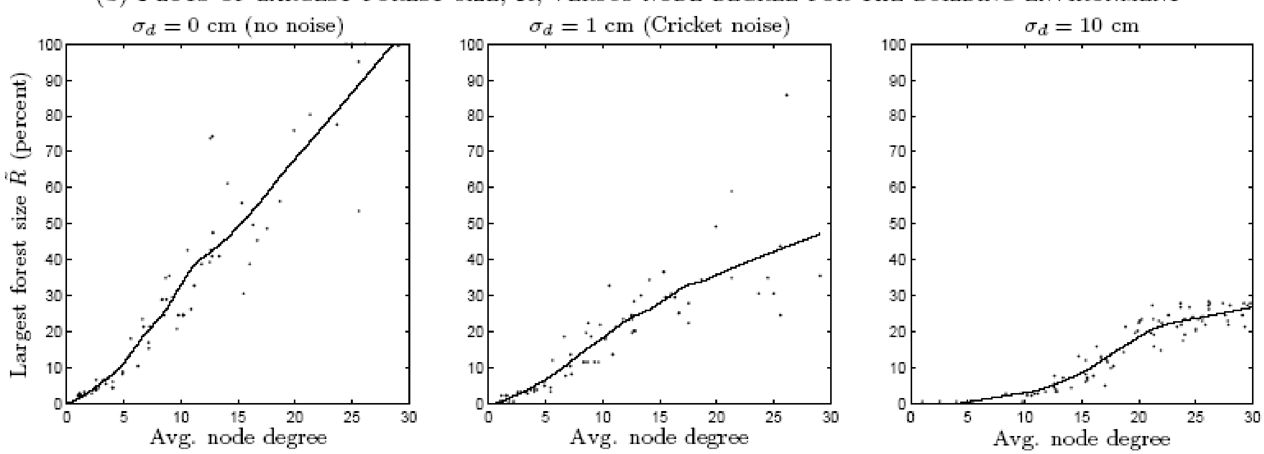
metric	value
σ_d	5.18 cm
σ_p	7.02 cm
$\bar{R}$	$15/16 = 0.94$
$\tilde{R}$	$15/16 = 0.94$

Experiment – data (cont.)

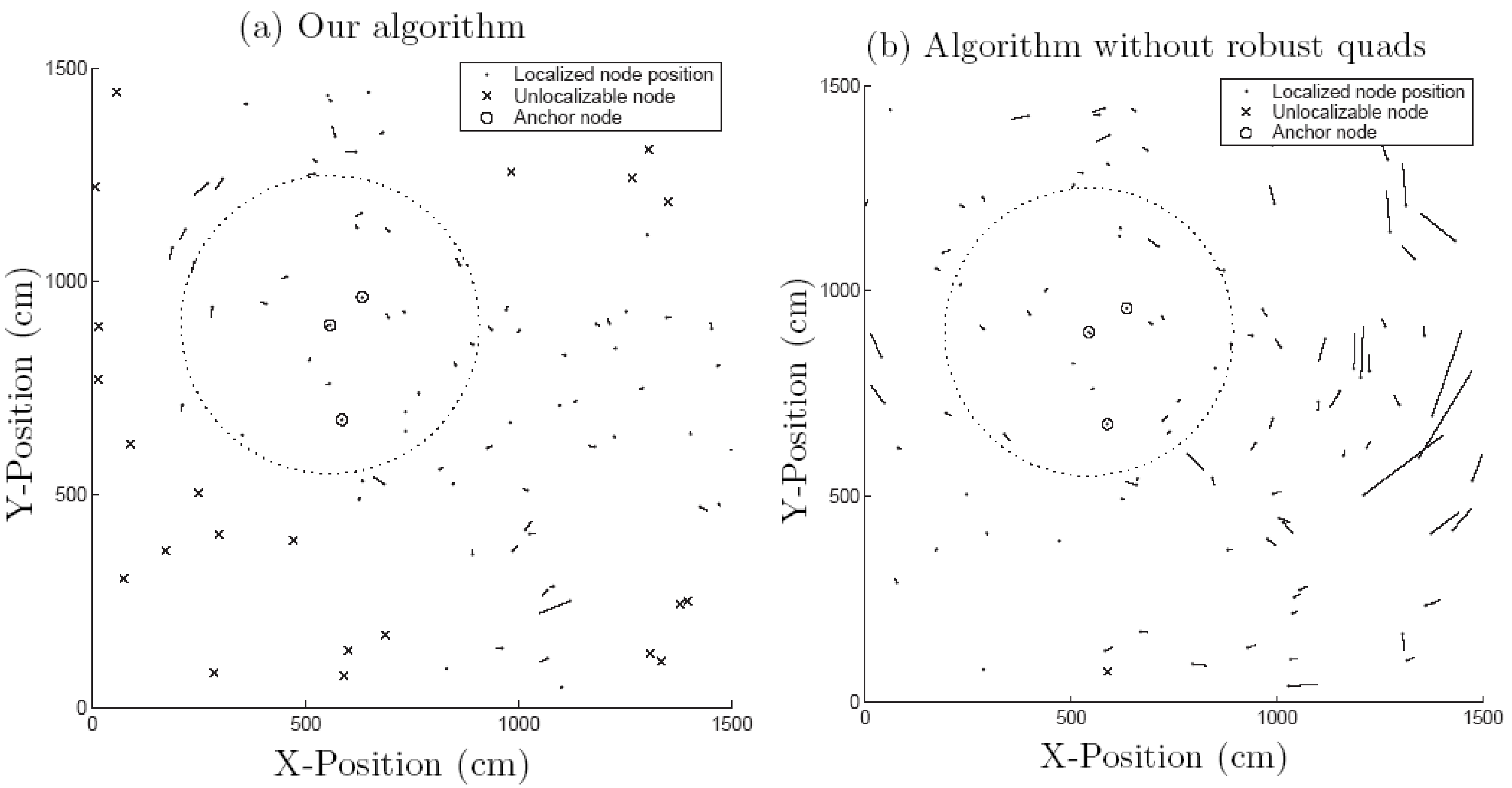




(b) Plots of largest forest size, $\tilde{R}$, versus node degree for the building environment

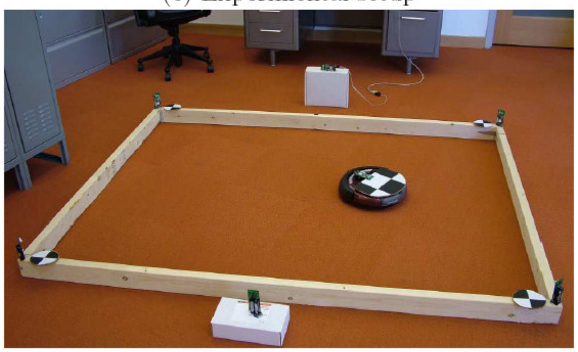


Experiment – data (cont.)

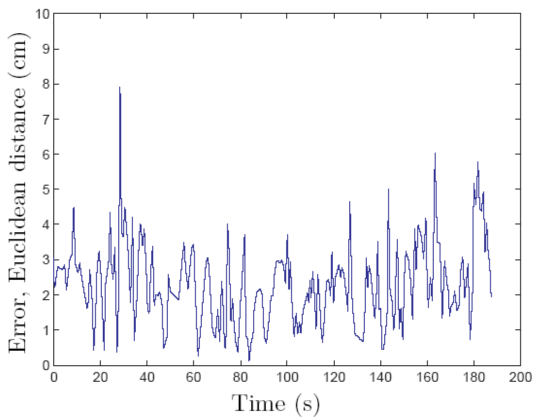


Experiment – data (cont.)

(c) Experimental setup

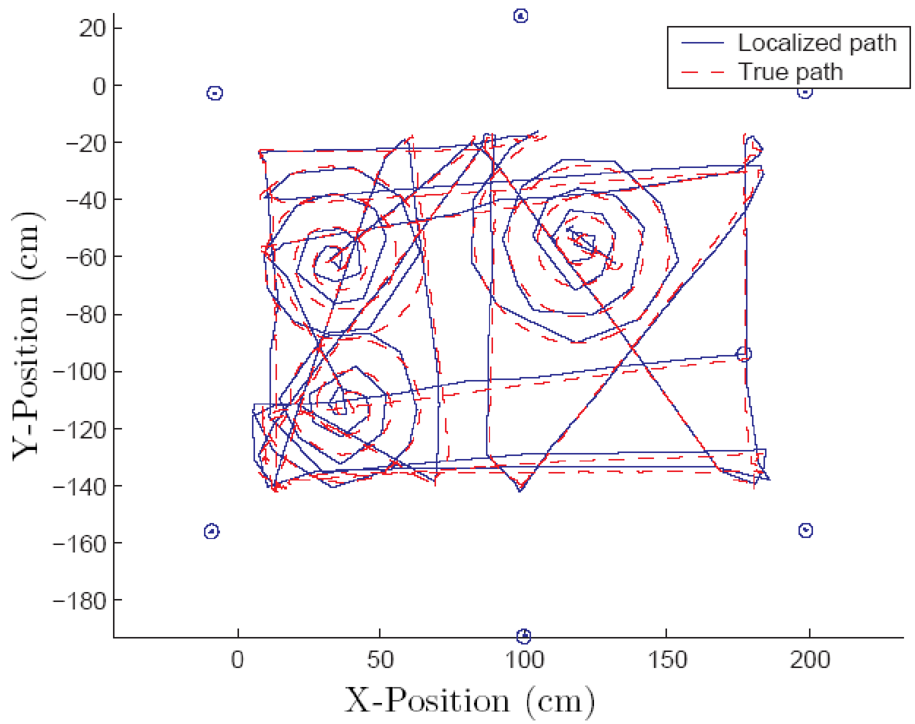


(b) Magnitude of localization error vs. time



(c) Experimental setup

(a) Localized path of mobile node



Conclusion

- The criteria for quadrilateral robustness can be adjusted to cope with arbitrary amounts of measurement noise in the system.
- Under conditions of low node connectivity or high measurement noise, the algorithm may be unable to localize a useful number of nodes.
- Even as noise goes to 0, nodes in large networks must have degree >= 10 on average to achieve 100% localization.