Stanford University, Spring 2021, CS 448Z: Physically Based Animation and Sound

# Homework #1: Real-time Rigid-Body Dynamics with Acceleration Noise

**Due: Thursday Apr 29, 2021**

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## Abstract

The goal of this assignment is for you to implement an interactive audiovisual simulation of a simple rigid-body system which includes realistic physics-based acceleration noise ("clicking" sounds). The sounds should include realistic dependence on the mass, material and geometry of the objects, as determined by a Hertz contact model. For simplicity, we will assume small (acoustically compact) spherical objects.

## 1 Introduction

Acceleration noise is an important component in rigid-body contact sound, and is the dominant source of sound for small objects [Chadwick et al. 2012b; Chadwick et al. 2012a]. In this assignment, you will leverage analytical models of contact forces and acceleration noise to build a simple but compelling audiovisual simulation of spherical objects in contact. While larger objects, such as a bowl or a mug, can have more complex acceleration noise fields which require special methods to precompute and evaluate at runtime [Chadwick et al. 2012b], the case is much simpler for the small spherical objects that we will consider here.

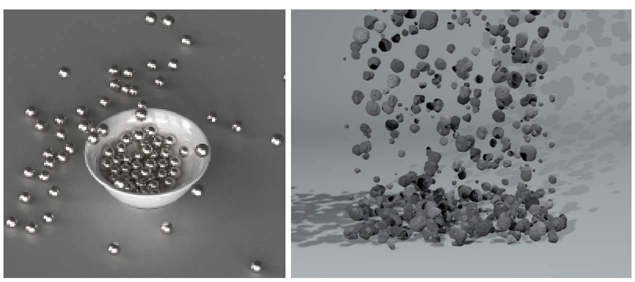

**Figure 1:** *Colliding small objects can produce significant acceleration noise: (Left) metal ball bearings (from [Chadwick et al. 2012b]); (Right) small rocks (from [Chadwick et al. 2012a]).*

## 2 Part I: Rigid-body Simulation

### 2.1 Preliminaries

You should have finished the first programming assignment, and built a real-time audiovisual particle system. In this assignment, you will extend your implementation to include more realistic collision and sound behaviors. We will assume that you have a 3D particle system, although you can adopt many of the ideas to 2D. The basic simulation will have $N$ spheres in a box, with or without gravity.

### 2.2 Spherical object model

Each object in your simulation will be a sphere with several object-specific simulation parameters for its rigid-body position and velocity 3-tuples, as well as its mass $m$, and elasticity parameters (Young's modulus $E$, and Poisson ratio $\nu$). In this assignment, we will ignore angular motion and friction. You may choose spherical objects with varying size, with materials chosen from those in Table 1, and drawn with distinctive colors.

### 2.3 Collision Detection

You are free to implement sphere-sphere and sphere-plane collision detection using a method of your choice. There are two main strategies based on either discrete-time collision checks that resolve collisions at the graphics frame level, or more involved continuous-time collision checking that resolves sub-frame collision times (described in the following subsection). You may also use a framework implementation, such as Unity, to detect and process collisions for you, although you will need to be notified of these events and the collision impulses for sound generation.

#### 2.3.1 Continuous Collision Detection

Accurate display of contact sound events requires us to resolve the exact time of collision, as opposed to simply knowing that it occurs during a timestep. To do so, you will implement continuous collision detection for spheres to determine if and when a collision occurs during a time-step interval $[t, t+\Delta t]$. You must handle two cases:

**Case I: Sphere-plane collisions:** Given the trajectory of a sphere, you can determine the first time of contact (TOC) between the sphere and plane, and use that to determine if a collision occurs during the time step. For simplicity, you can assume that the sphere has a straight-line trajectory during the timestep interval.

**Case II: Sphere-sphere collisions:** For each pair of spheres, you can estimate the first TOC by robustly solving a quadratic root-finding problem (based on the squared distance between spheres equal to zero), and checking to see if it occurs on the timestep interval. To handle all-pairs collision checks, you do not need to do anything fancy and can simply check all pairs (once) since we will not be pushing the limits of the numbers of particles for this assignment. Your design goal for the collision processor should be to produce correct behavior first and foremost.

### 2.4 Time-stepping many colliding spheres:

Each time a sphere collides with a rigid planar surface or another sphere, you will apply an appropriate rigid-body collision impulses $\pm\mathbf{j}$ (see David Baraff's course notes [Baraff 2001]) at that specific time instant to update the velocity of the object(s). For example, given a normal velocity pre-collision of $v_n^- = (\mathbf{v}_1 - \mathbf{v}_2) \cdot \mathbf{n} < 0$ where the unit contact normal is $\mathbf{n} = \text{normalize}(\mathbf{x}_1 - \mathbf{x}_2)$, the post-collision normal velocity is $v_n^+ = -\varepsilon v_n^-$ where $\varepsilon \in (0,1)$ is the restitution coefficient controlling "bounciness." The impulse applied should be $\mathbf{j} = (1+\varepsilon)\, m\, |v_n^-|\, \mathbf{n}$ where $\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2}$ is the harmonic mass of the two bodies. The updated trajectory is then used to update collision checks for those objects, and advance forward in time.

*For continuous collision detection with sub-frame collision resolution* conservative advancement is used to advance the positions and velocities of the $N$ spheres from time $t$ to $t + \Delta t$, so as to not miss acoustically important collisions. In practice, this means advancing the simulation time from $t$ to each of the intermediate collision times $t_1, t_2, \ldots$, until you reach the end of the timestep, $t + \Delta t$. Note that for small $N$, you can perform all-pairs checks at intermediate times, $\{t_i\}$, however you can be more efficient by using a

| Material | Density, $\rho$ (kg/m$^3$) | Young's modulus, $E$ (GPa) | Poisson ratio, $\nu$ |
|---|---|---|---|
| Steel | 8940 | 123.4 | 0.34 |
| Ceramic | 2700 | 72 | 0.19 |
| Glass | 2700 | 62 | 0.20 |
| Plastic | 1200 | 2.4 | 0.37 |

**Table 1:** *Four materials (from [Chadwick et al. 2012b])*

spatial bound on the sphere's linear trajectory during the remainder of the timestep. In practice, you can use a priority queue to keep track of the candidate TOCs. Keep in mind that each impulse you apply will modify future interactions with affected objects.

# 3 Part II: Synthesizing Acceleration Noise

Once you have a functioning rigid-body simulation, you can now convert the contact events into sound rendered at your listening position. Each collision event involving a sphere will generate a sound waveform at a specific future time which you will composite into a large (circular) sound buffer. The details are now described.

## 3.1 Acceleration noise model for a spherical object

As discussed in class, the far-field acoustic pressure emitted by a small acoustically compact spherical object undergoing small oscillations with speed $V(t)$ along an axis $\hat{V}$, can be approximated by a point dipole of strength $2\pi a^3 V(t)$ along the axis $\hat{V}$, where $a$ is the radius of the sphere. The rendered pressure for the motion is [Howe 2002] (3.8.5),

$$p(\boldsymbol{x},t) \approx \frac{\rho_0 a^3 \cos\theta}{2c\|\boldsymbol{x}-\boldsymbol{x}_0\|}\,\frac{\partial^2 V}{\partial t^2}\left(t - \frac{\|\boldsymbol{x}-\boldsymbol{x}_0\|}{c}\right), \quad (1)$$

where $\boldsymbol{x}$ is the listening position, $\boldsymbol{x}_0$ is the center of mass of the sphere, $c$ is the speed of sound in air ($\approx 330m/s$), $\rho_0$ is the density of air at STP (at 20 $^oC$ and 101.325 kPa, dry air has a density of $1.20\ kg/m^3$), and $\cos\theta = \cos(\hat{V}, \boldsymbol{x}-\boldsymbol{x}_0)$ is the cosine of the angle between the direction of oscillation and the direction to the listener.

Note that the pressure depends on the second time derivative of the object's velocity, or the "jerk." Thus only non-constant acceleration, such as during contact, will result in acoustic pressure contributions.

## 3.2 Hertz contact force modeling

We will model the rate of change of the object's acceleration, i.e., "jerk," using a Hertz contact model for spherical surfaces. To be specific, we quote [Chadwick et al. 2012b]:

> Hertz contact theory states that the normal contact force between two colliding elastic bodies is given by
>
> $$f = Kd^{1.5} \quad (2)$$
>
> where $d$ is the penetration depth at the contact point and $K$ is a constant depending on the material properties and local contact geometry of the colliding bodies. For a collision between two frictionless spheres, $K = (4/3)\sqrt{r}E^*$ and the time dependence of the contact force for a collision beginning at time $t_0$ can be approximated by a half-sine pulse [Johnson 1985]
>
> $$S(t;t_0,\tau) = \begin{cases} \sin\left(\frac{\pi(t-t_0)}{\tau}\right) & \text{if } t_0 \le t \le t_0+\tau \\ 0 & \text{otherwise} \end{cases} \quad (3)$$
>
> where $\tau$ is a time scale defined by
>
> $$\tau = 2.87\left(\frac{m^2}{rE^{*2}V}\right)^{1/5}. \quad (4)$$
>
> $V$ is the normal impact speed and the other constants used in (4) are defined as follows:
>
> $$\frac{1}{r} = \frac{1}{r_1}+\frac{1}{r_2}, \quad \frac{1}{m} = \frac{1}{m_1}+\frac{1}{m_2}, \quad \frac{1}{E^*} = \frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}. \quad (5)$$
>
> $R_i$, $m_i$, $\nu_i$ and $E_i$ are the radius, mass, Poisson ratio and Young's modulus for spheres $i = 1, 2$.

Since we are dealing with spheres, we can skip the material on general rigid bodies in §3.2 of [Chadwick et al. 2012b].

**Contact time-scale $\tau$ evaluation:** In your simulation, you will have sphere-sphere and also sphere-plane collisions. Estimating the Hertz contact parameters for sphere-sphere collisions is straightforward. For sphere-plane collisions you can assume that the planar halfspace is a sphere of infinite radius and mass, and therefore $r$ and $m$ simplify to the radius and mass of the finite sphere, but $E^*$ depends on both materials.

**Acceleration profile:** For sound synthesis we need the acceleration profile for (1), and we need its derivative to be continuous for sound rendering. It follows that the linear acceleration of body $i$ is given by (11) of [Chadwick et al. 2012b]:

$$\mathbf{a}_i(t) = \frac{\pi}{2\,m_i\,\tau}\,\mathbf{j}_i\,S(t;t_0,\tau), \quad (6)$$

where $\mathbf{j}_i \in \mathbb{R}^3$ is the total impulse applied to body $i$. (Note that the world and body frame have the same orientation since we have zero angular velocity, $\boldsymbol{\omega}_i = 0$, for frictionless spheres.) Therefore assuming oscillations along the $\hat{V} = \hat{\mathbf{j}}_i$ axis, we have that the jerk profile for body $i$ is

$$\frac{\partial^2 V}{\partial t^2}\left(t - \frac{\|\boldsymbol{x}-\boldsymbol{x}_i\|}{c}\right) = \hat{V} \bullet\ \dot{\mathbf{a}}_i\left(t - \frac{\|\boldsymbol{x}-\boldsymbol{x}_i\|}{c}\right) \quad (7)$$

$$= \frac{\pi}{2\,m_i\,\tau}\,\|\mathbf{j}_i\|\,\dot{S}\left(t - \frac{\|\boldsymbol{x}-\boldsymbol{x}_i\|}{c};t_0,\tau\right). \quad (8)$$

Substituting (8) into (1) allows us to estimate the acceleration noise contribution, once for each body.

**Smooth jerk profile approximation:** One complication in evaluating the jerk profile in (8) is that the $S(t)$ acceleration profile has discontinuous derivatives, i.e., where zero meets the half-cosine hump. In [Chadwick et al. 2012b], S(t) was approximated using smooth bump-like basis functions from a Mitchell-Netravali cubic filter, and similar things could be done here. However, for simplicity we encourage you to use the following approximation:

$$S(t;t_0,\tau) \approx exp\left(-\frac{6}{\tau^2}\left(t-t_0-\frac{\tau}{2}\right)^2\right), \quad (9)$$

which has the derivative

$$\dot{S}(t;t_0,\tau) = -\frac{12}{\tau^2}\left(t-t_0-\frac{\tau}{2}\right)\,S(t;t_0,\tau), \quad (10)$$

which has $O(\frac{1}{\tau})$ magnitude (and applies slightly too much jerk due to the smoothing). A plot of $S$ (3) and its Gaussian approximation (9) is shown in Figure 2. Finally, using (10) in (8), one can evaluate (1) to get the sound produced by *one* sphere during a collision event.

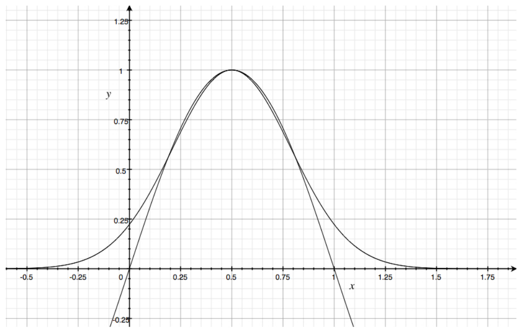


**Figure 2:** *The half-sine pulse $S$ ($\tau = 1$) compared to a Gaussian approximation.*

## 4 Extensions

After you have implemented the core system, and if you are so inclined, here are some other extensions you might try:

- *Adjustable viewing and/or listening location(s):* Move around to better investigate the collision sounds. Or support multiple listening locations.
- *"Instant replay":* Evaluating sounds in real time can be tricky without clear comparisons. If you cache your simulation you can rewind and replay events, and listen to them from different locations.
- *Sound reflections* can really improve the realism of acceleration noise, and are usually very audible given the sparseness of the "click" sound. You might try to include image sources to approximate the scattering of the sphere accleration noise pulse from the planar surfaces, e.g., ground plane, in the simulation. The image sources are at reflected positions, and the dipole vectors also need to be reflected.
- *Reverberation:* Add reverberation to your simulation using a plug-in, or otherwise experiment with the sound capabilities of your framework to enchance the simulation.
- *Build something interesting* by combining the assemblage of spheres, with springs, constraints, and other force models. For example, you could simulate a "Newton's Cradle," or a pool ball "trick shot," or a virtual kinetic sculpture.

## 5 What to Submit

You should submit (1) your code, (2) a brief 1-page write-up on your findings, (3) video captures of your results. Upload your final submission as a zip file to Canvas (`http://canvas.stanford.edu`).

## References


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