CS 468

Data-driven Shape Analysis

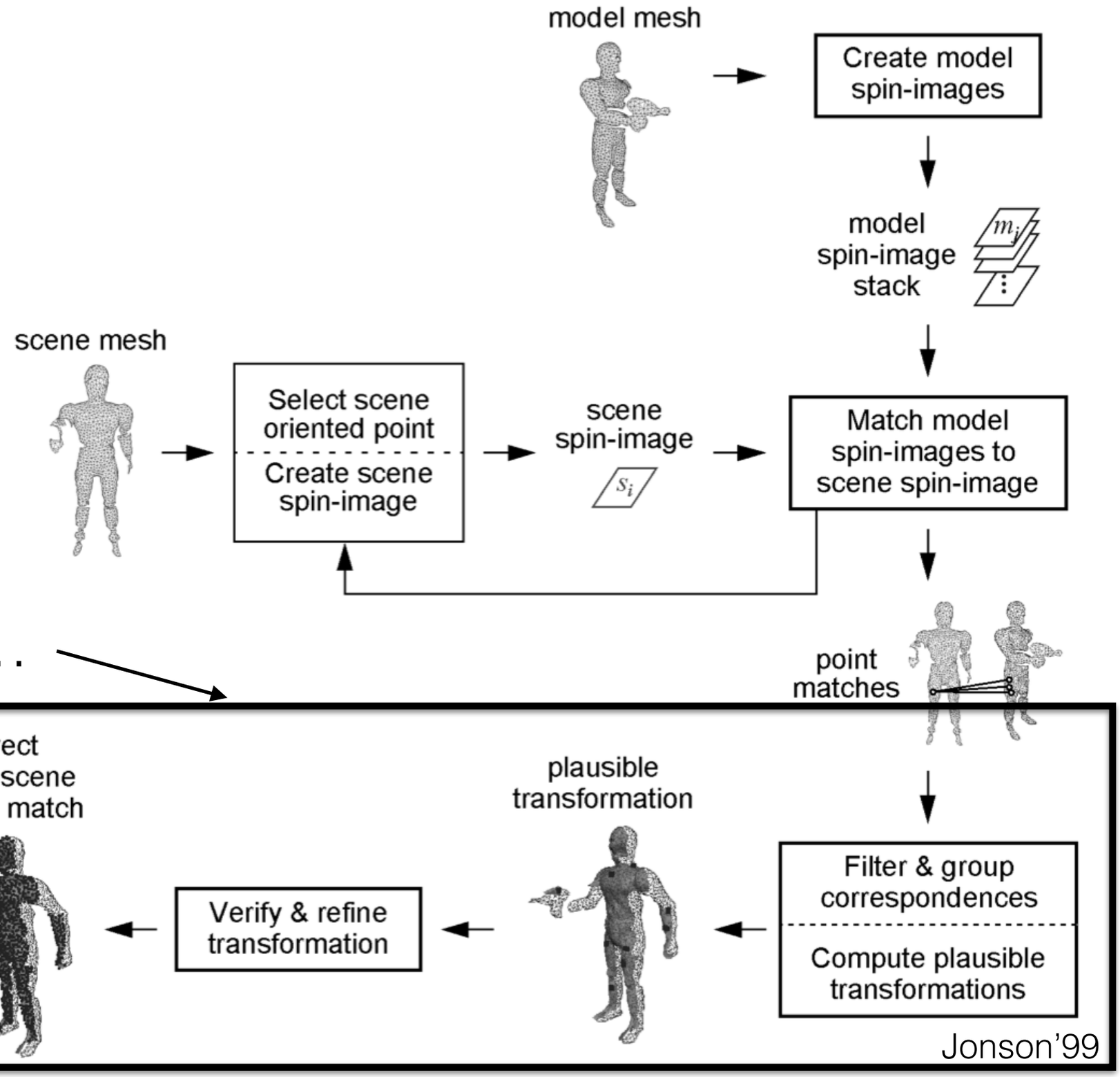
Shape Retrieval & Intrinsic Descriptors

April 3, 2014

Rigid Shape Matching

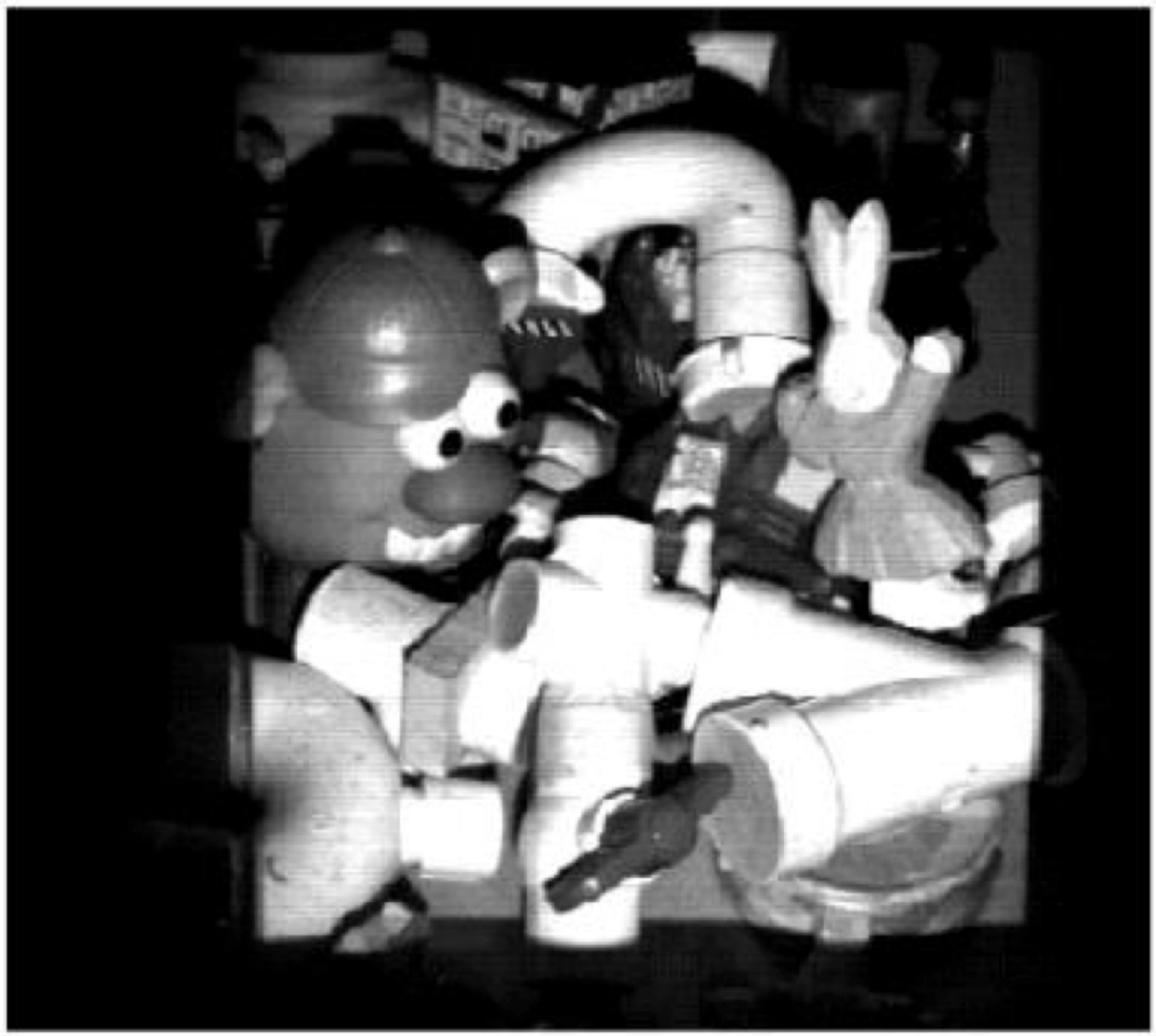
Tuesday: Spin Images

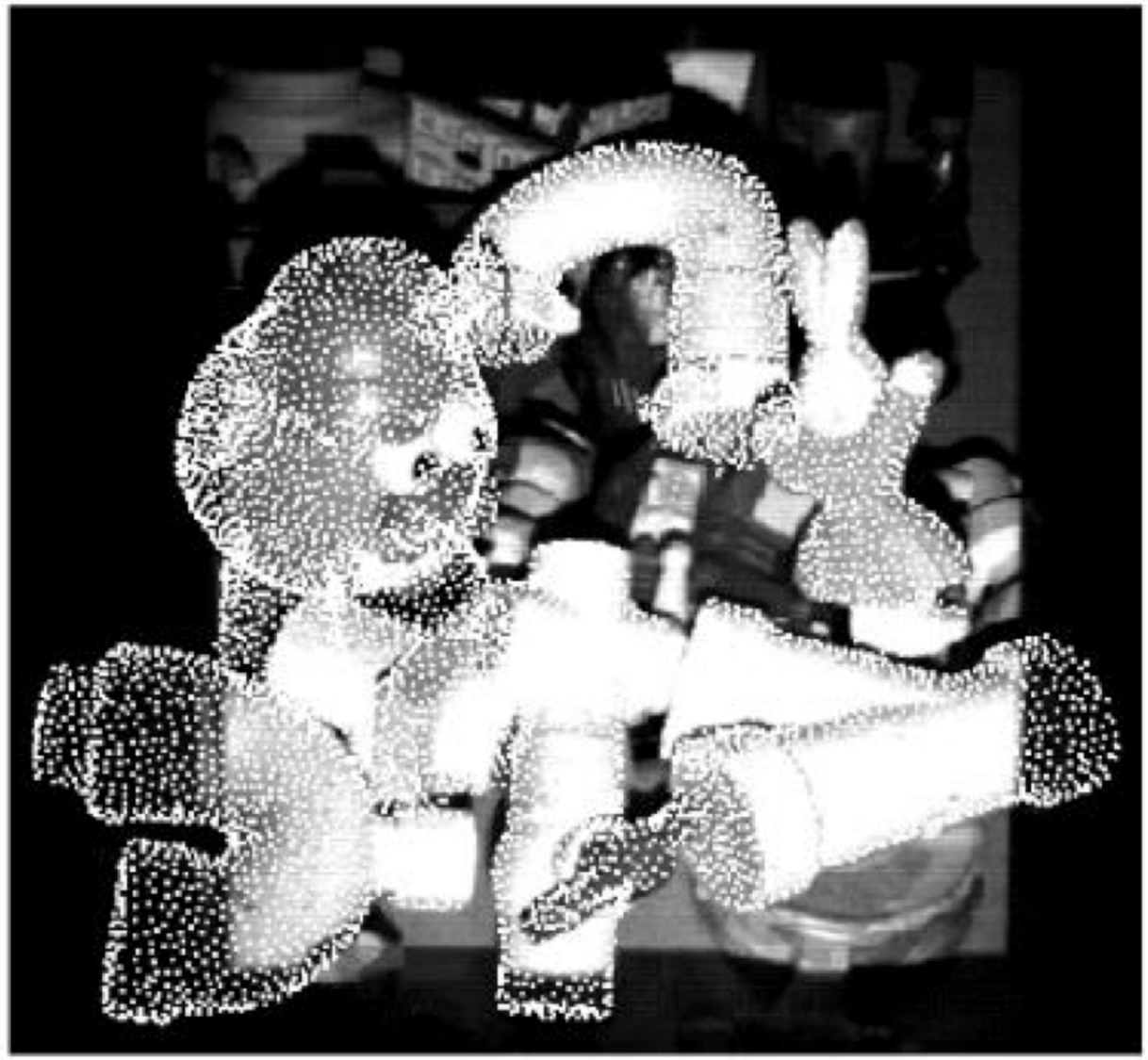
We will discuss the details of shape matching later…



Rigid Shape Matching

Goal: retrieve and align a shape

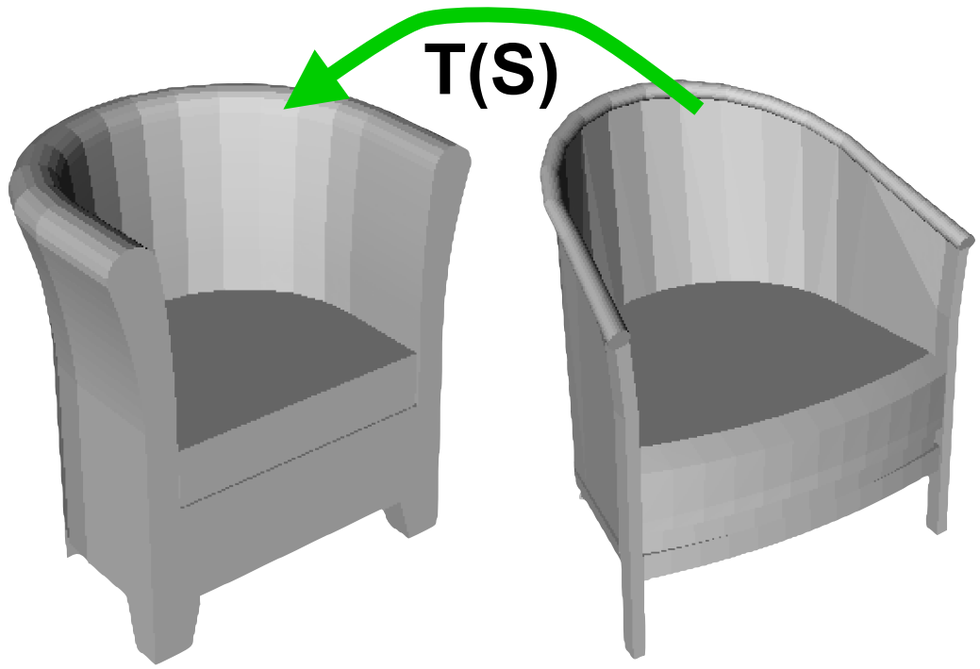


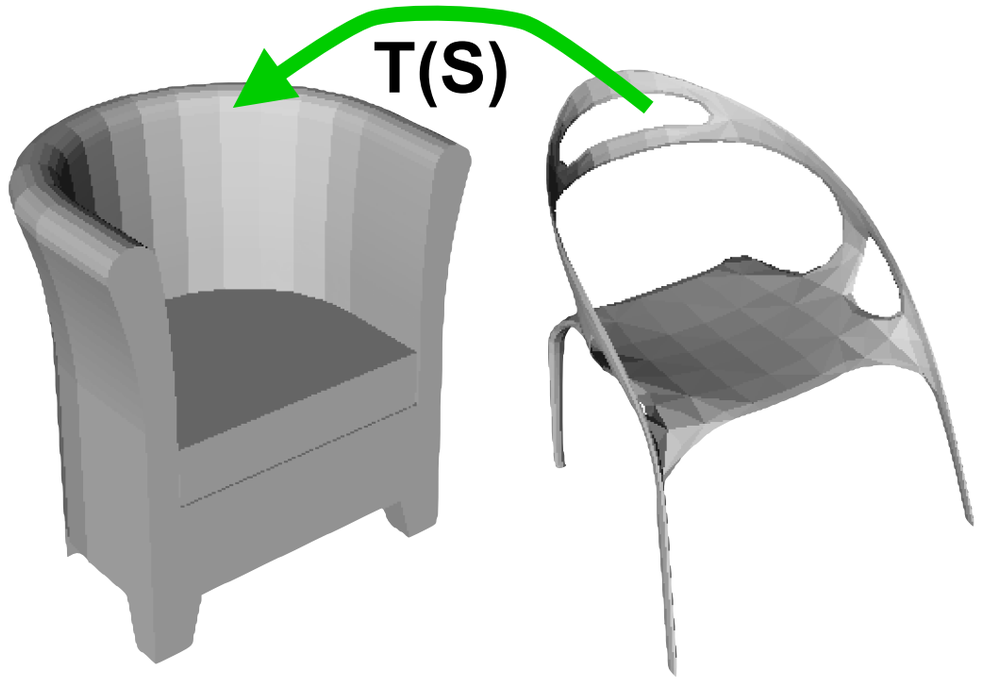


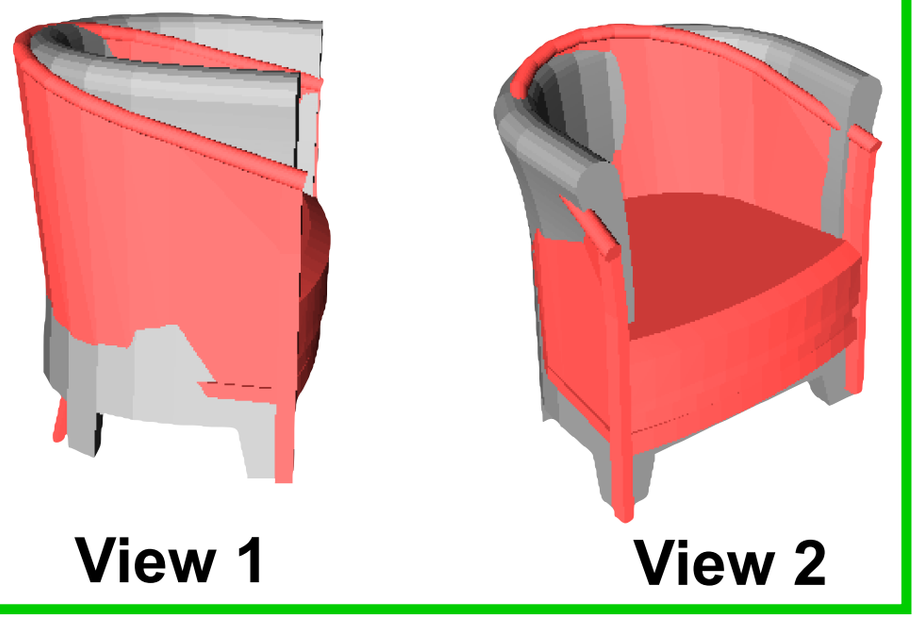
Jonson'99

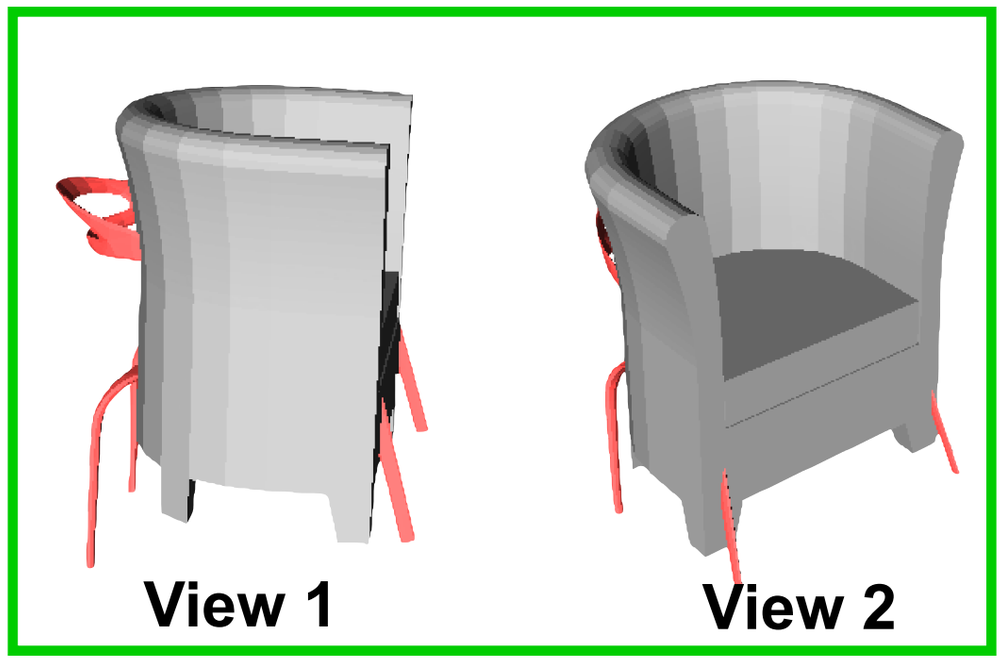
Rigid Shape Matching

Search for a transformation





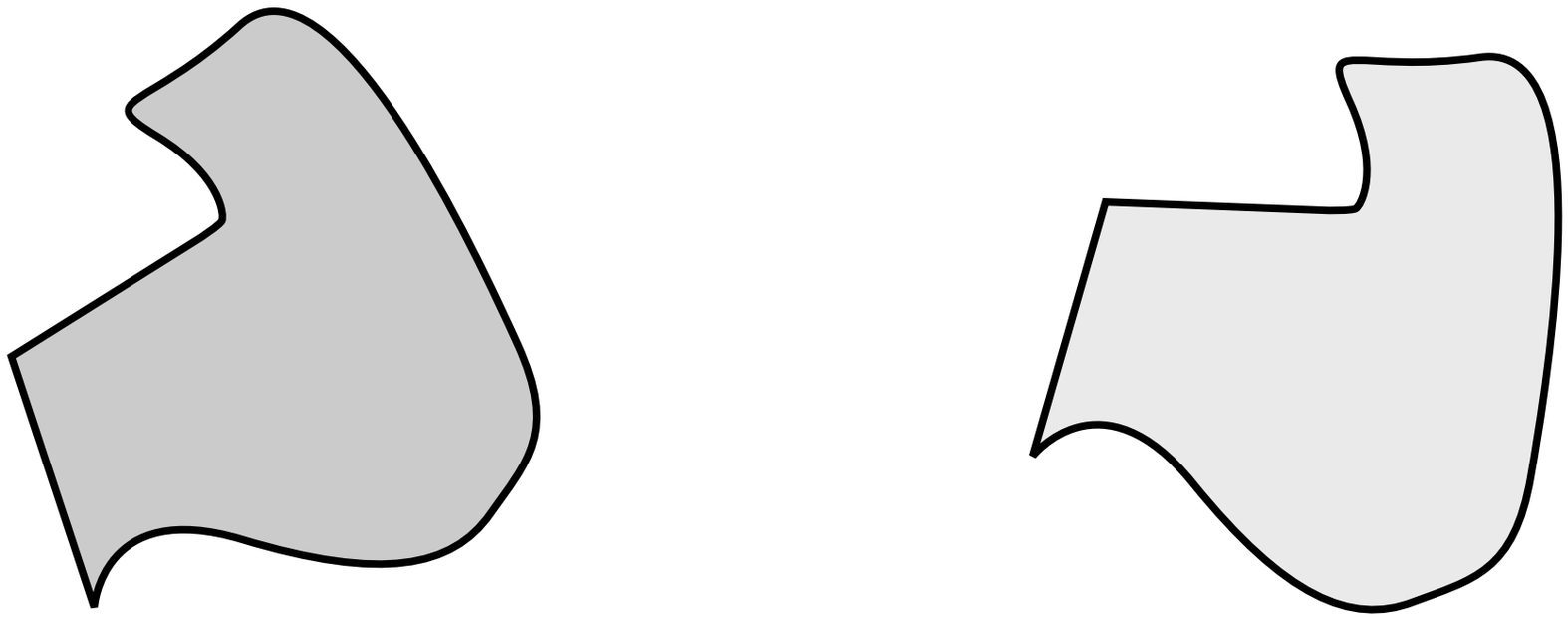




Rigid Shape Matching

M_1

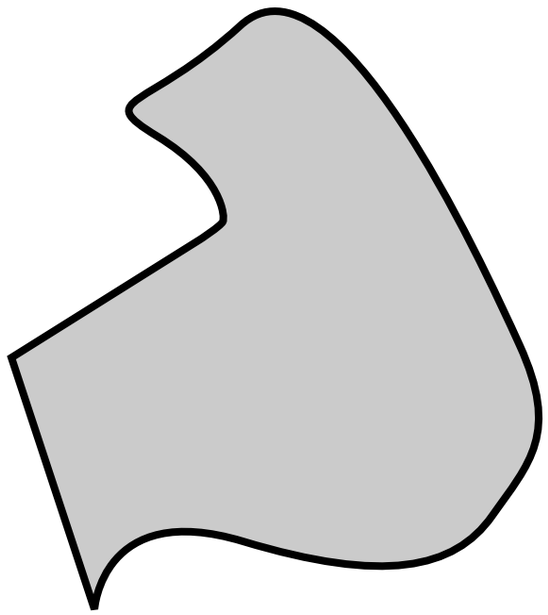
M_2



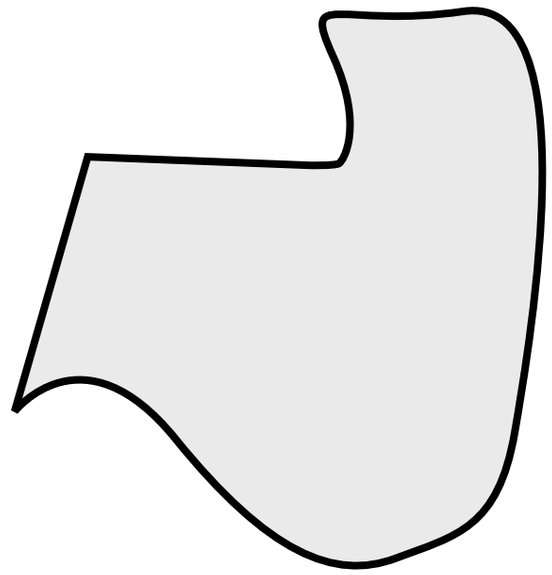
$$M_1 \approx T(M_2)$$

Rigid Shape Matching

M_1



M_2



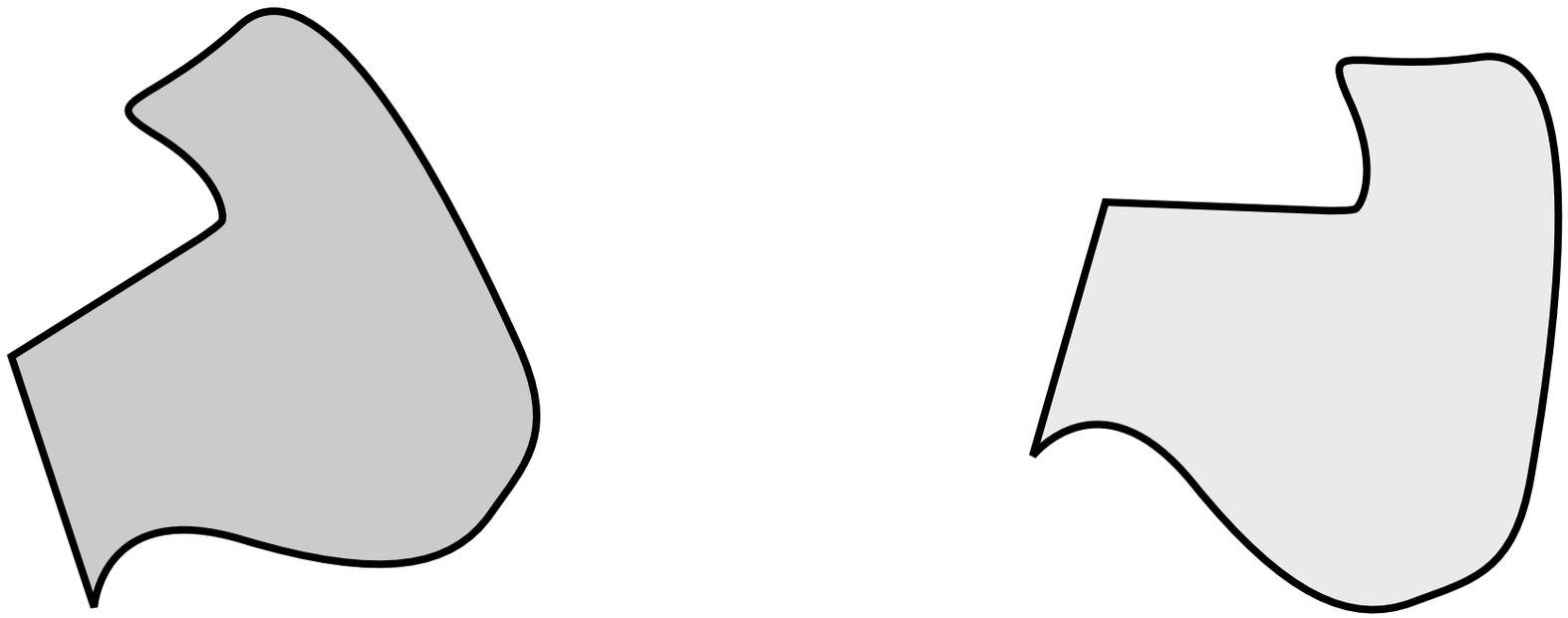
$$M_1 \approx T(M_2)$$

T : **???**

Rigid Shape Matching

M_1

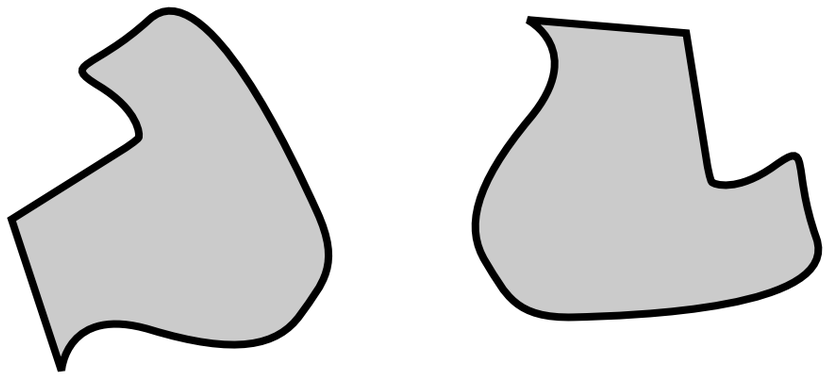
M_2



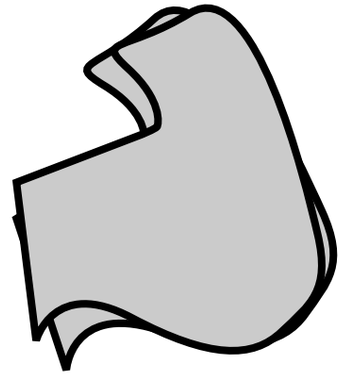
$$M_1 \approx T(M_2)$$

$$T : \text{translation} + \text{rotation}$$

Local vs Global Matching



global registration
any rigid transform



local registration
nearly aligned

Given $M_1, \ldots, M_n$, find $T_2, \ldots, T_n$ such that

$$M_1 \approx T_2(M_2) \cdots \approx T_n(M_n)$$

Mitra

Find Rigid Transform

How many points do we need?

Find Rigid Transform

How many points do we need?

$$\mathbf{p}_1 \rightarrow \mathbf{q}_1$$ Translation

Find Rigid Transform

How many points do we need?

$$\mathbf{p}_1 \rightarrow \mathbf{q}_1$$

$$\mathbf{p}_2 \rightarrow \mathbf{q}_2$$

Still ambiguity in rotation

Find Rigid Transform

How many points do we need?

$$\mathbf{p}_1 \rightarrow \mathbf{q}_1$$

$$\mathbf{p}_2 \rightarrow \mathbf{q}_2$$

$$\mathbf{p}_3 \rightarrow \mathbf{q}_3$$

$$R\mathbf{p}_i + t \approx \mathbf{q}_i$$

Find Rigid Transform

How many points do we need?

$$\mathbf{p}_1 \rightarrow \mathbf{q}_1$$

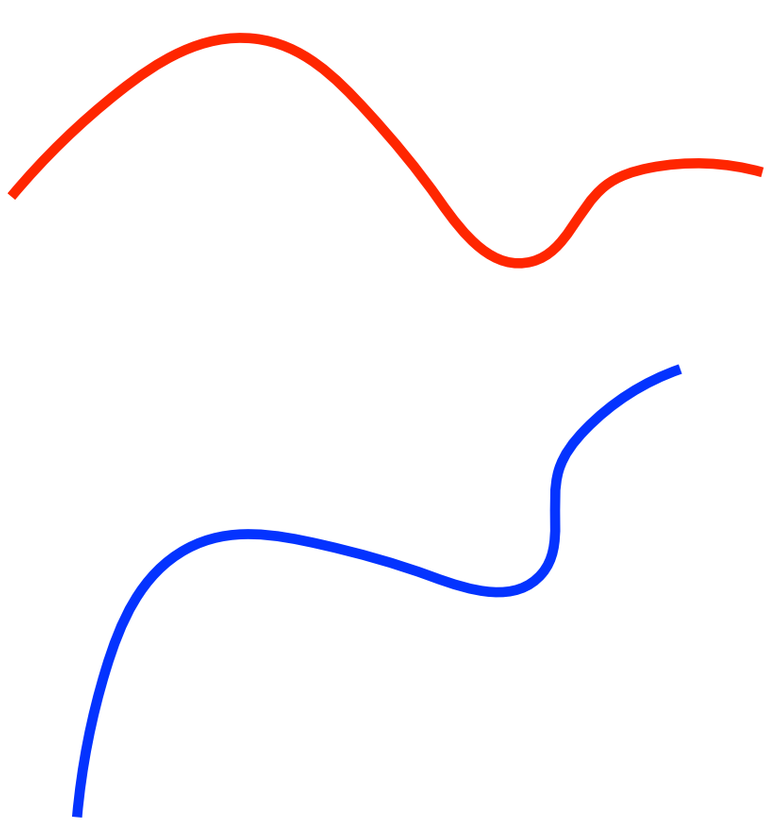
$$\mathbf{p}_2 \rightarrow \mathbf{q}_2$$

$$\mathbf{p}_3 \rightarrow \mathbf{q}_3$$

$$R\mathbf{p}_i + t \approx \mathbf{q}_i$$

Correspondence problem: $\mathbf{p}_i \xrightarrow{?} \mathbf{q}_j$

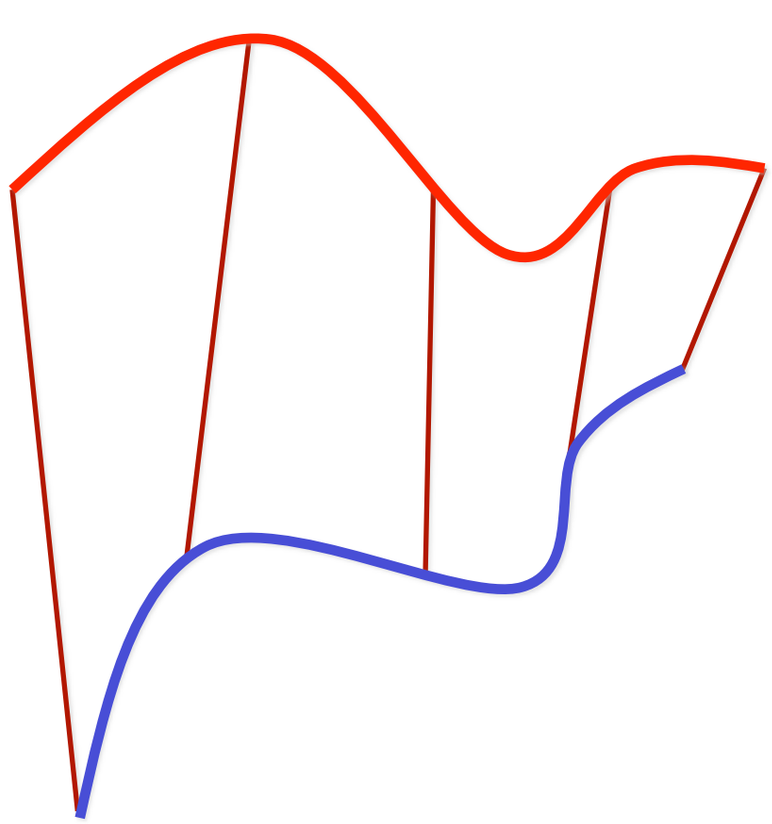
Toy Example



Mitra

Toy Example

Given correspondences, can find the best rigid transform



Mitra

Toy Example

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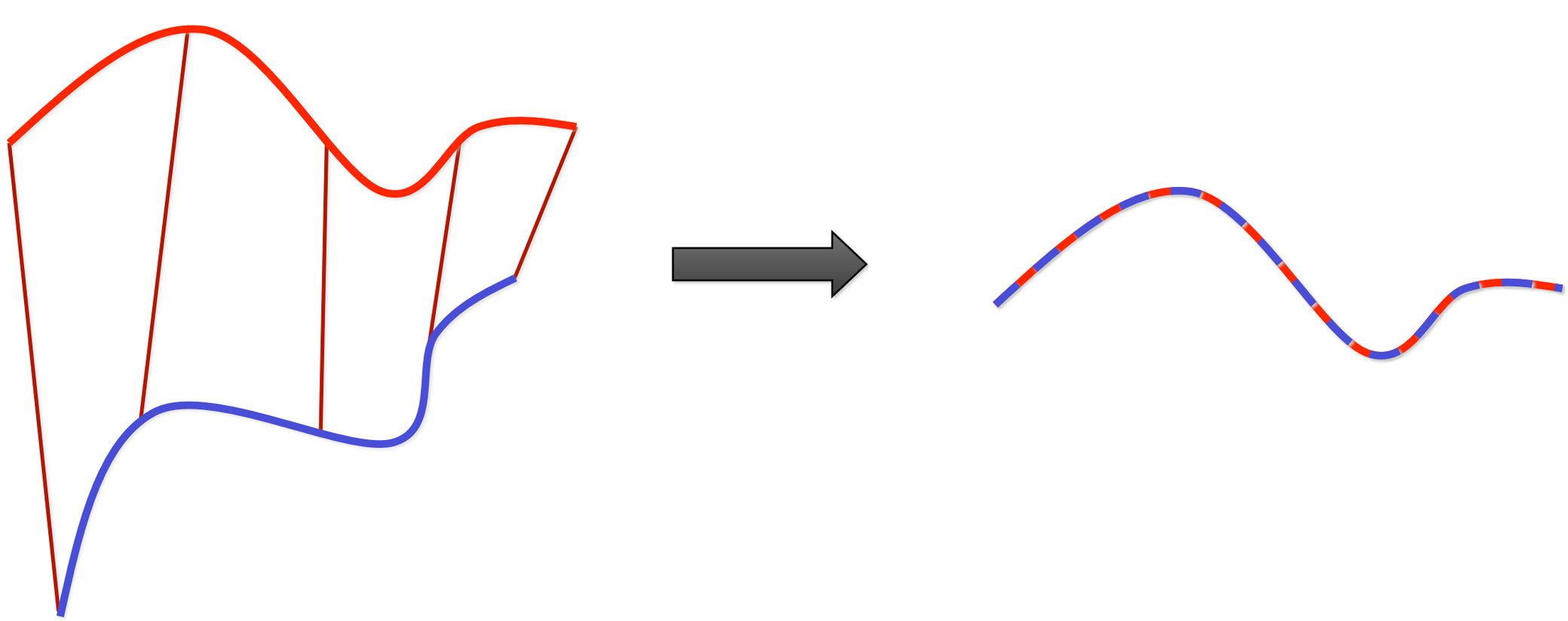
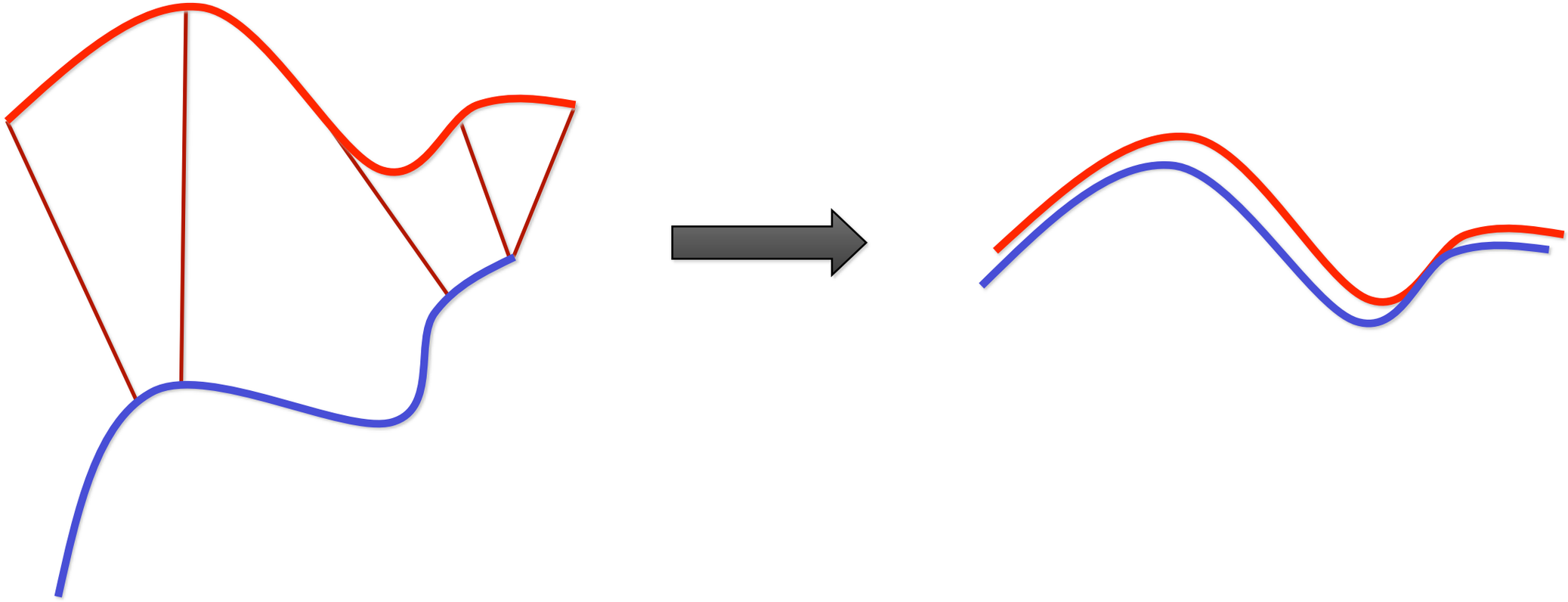


Toy Example

Given correspondences, can find the best rigid transform

Why do we use more than 3 points?

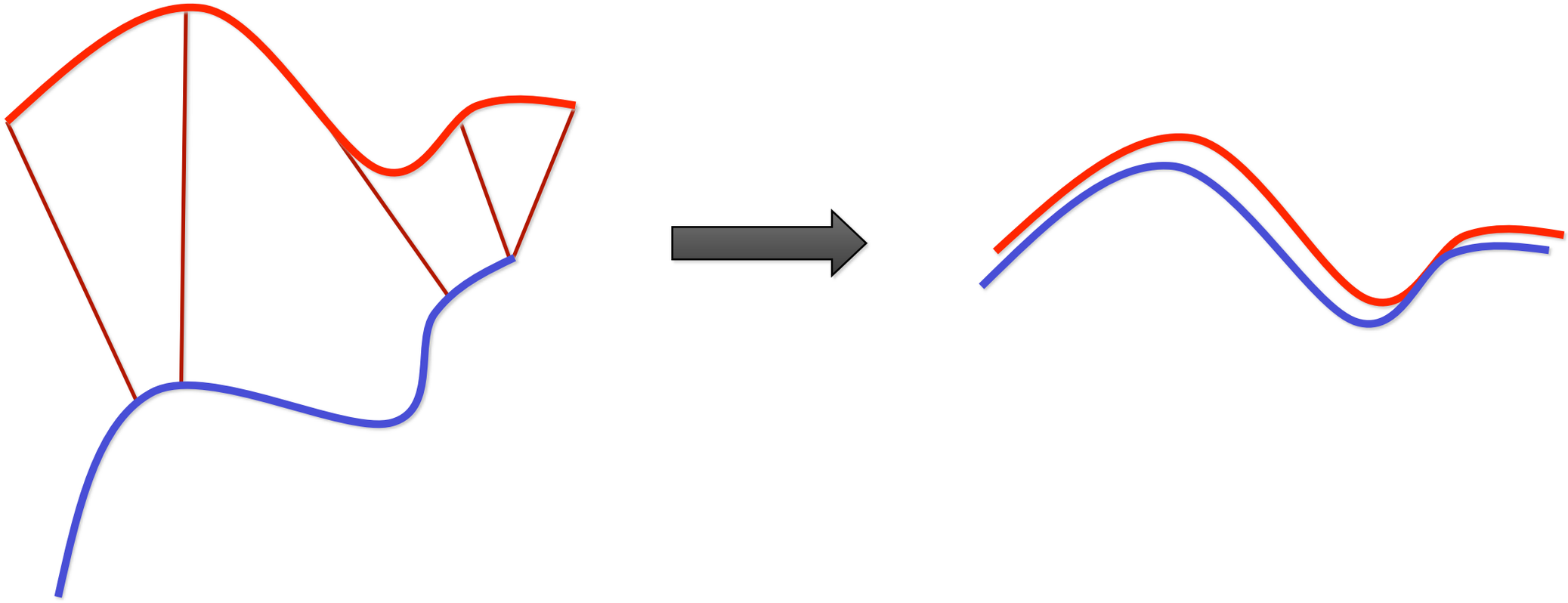


Mitra

Toy Example

Given correspondences, can find the best rigid transform

Why do we use more than 3 points?
How to get correspondences?



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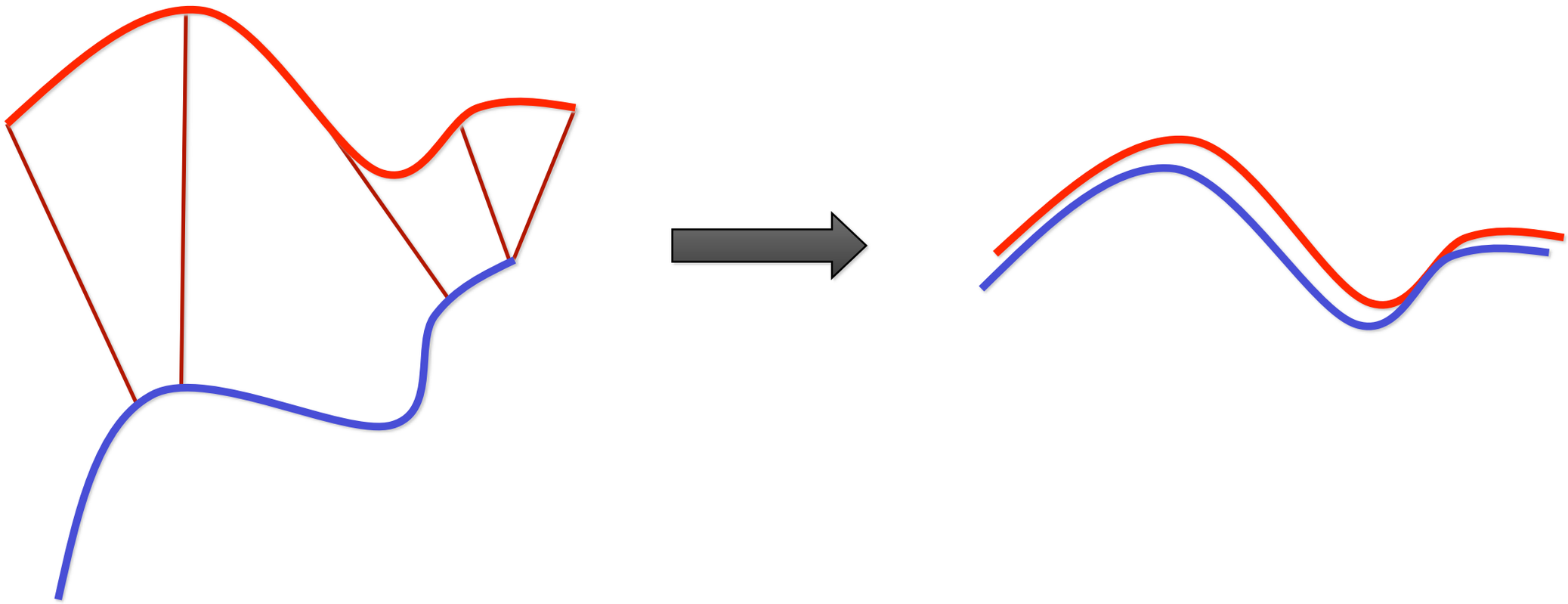
Toy Example

After initial correspondences, refine with closest points

$$\mathbf{p}_i \rightarrow \mathcal{C}(\mathbf{p}_i)$$



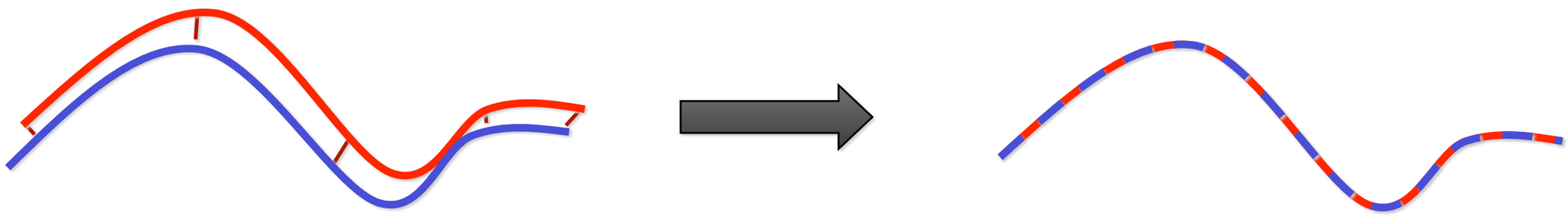
Mitra

Toy Example

... and iterate to find alignment

Iterative Closest Points (ICP) [Besl and McKay 92]

Converges if starting poses are *close enough*



Rigid Shape Matching

Basic ICP algorithm

Select (e.g., 1000) random points

Match each to closest point on other scan, using data structure such as *k*-d tree

Reject pairs with distance > k times median

Construct error function:

$$E := \sum_i (R\mathbf{p}_i + t - \mathbf{q}_i)^2$$

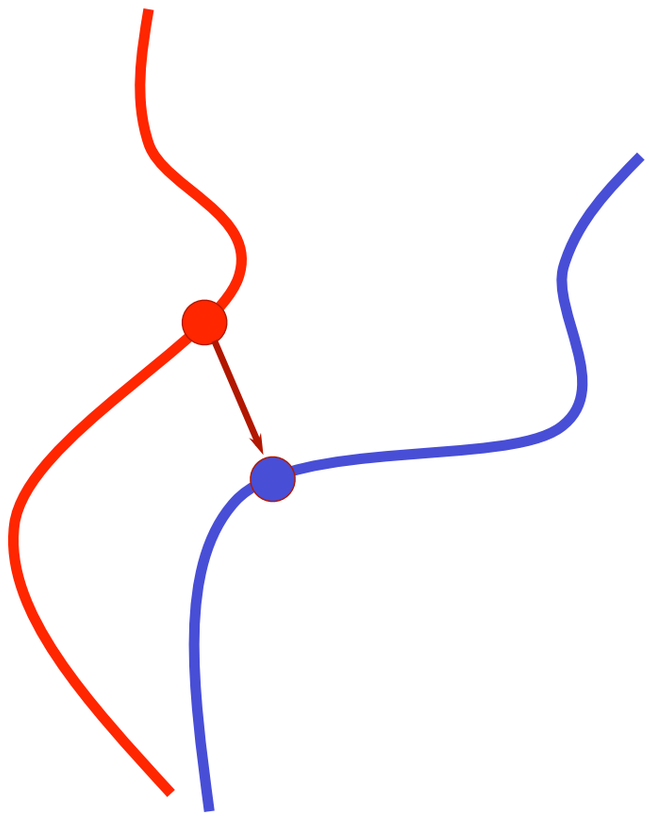
Minimize (closed form solution in [Horn 87])

Also this writeup

Rigid Shape Matching

Finding closest point is most expensive stage of the ICP algorithm

- Brute force search – O(n)
- Spatial data structure (e.g., k-d tree) – O(log n)

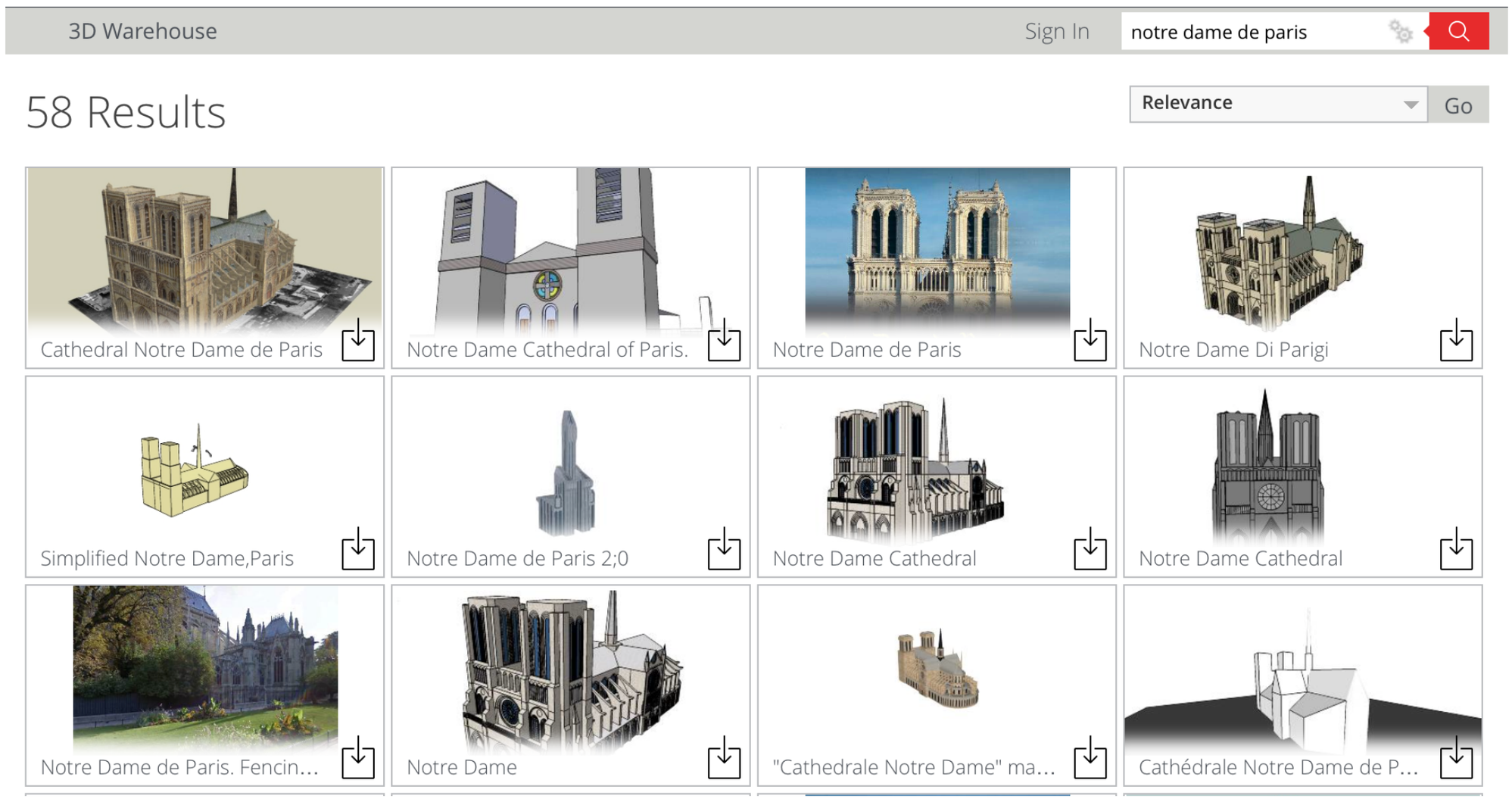


Applications

Shape Retrieval

Retrieval Options

- ➡ **Text Query (e.g. Trimble 3D Warehouse)**
- Shape Query
- Sketch Query (e.g. Princeton Search Engine, SBSR)



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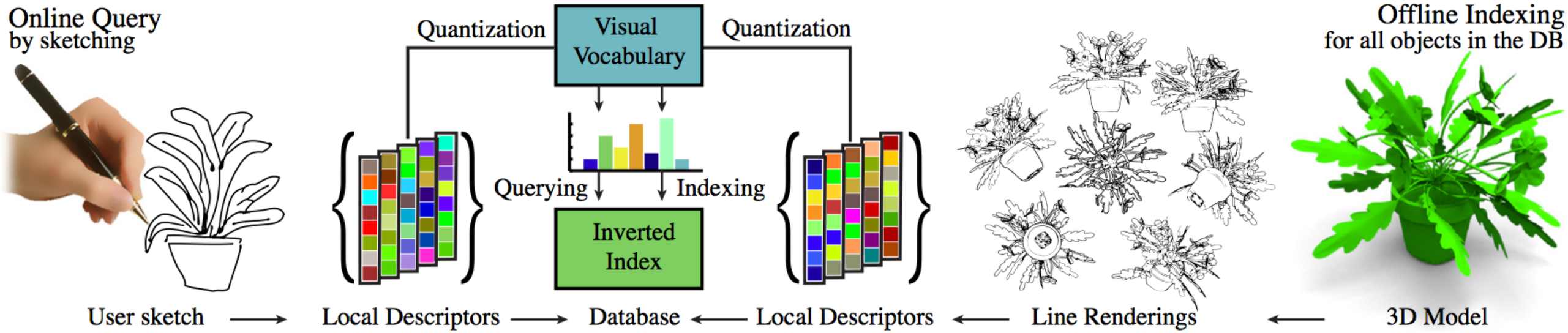
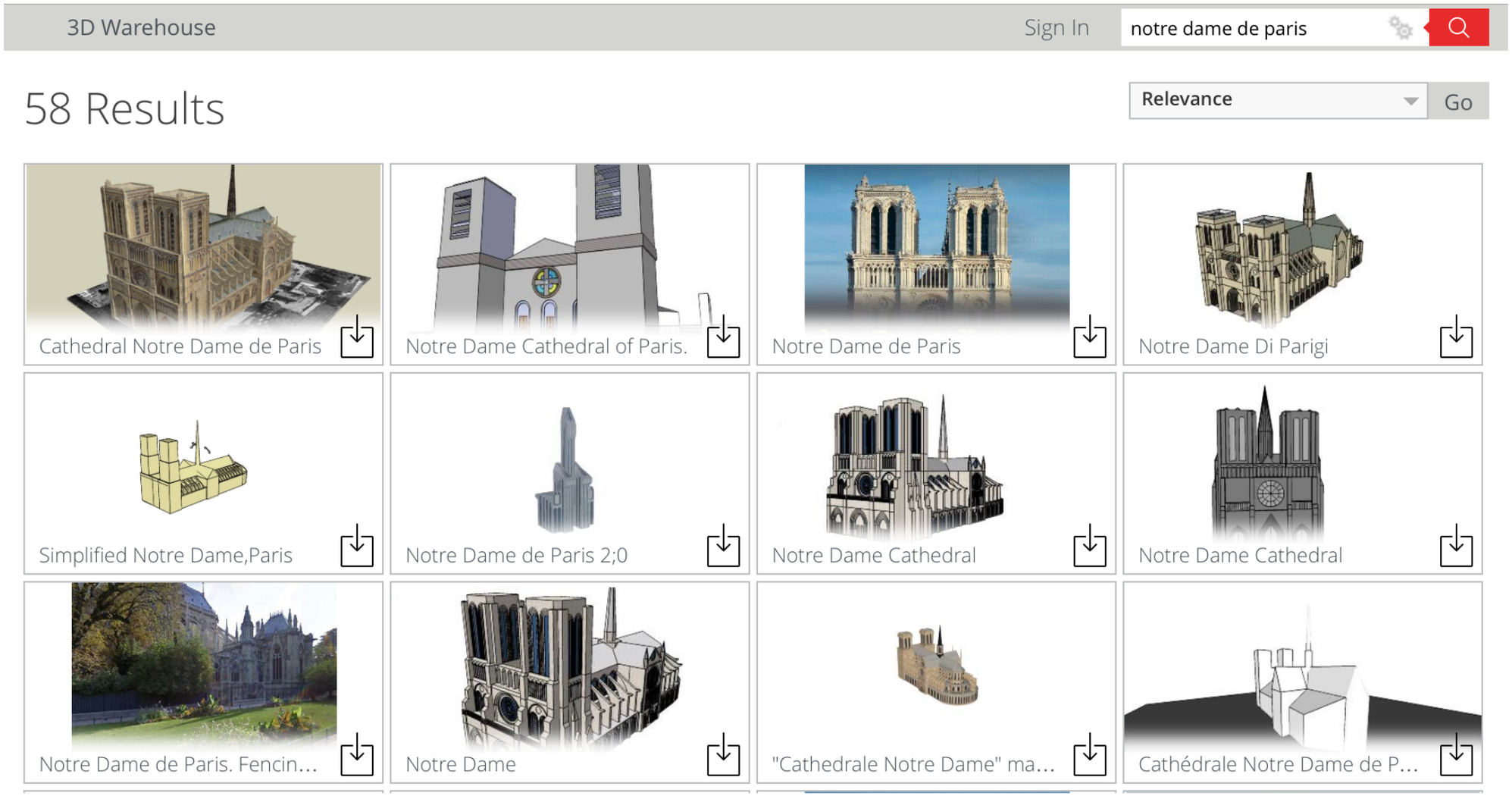


Eitz et al. 2012

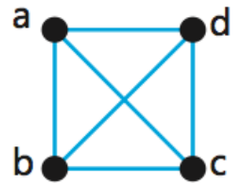
Exploration

Is this the best way to understand a collection?



Quartets

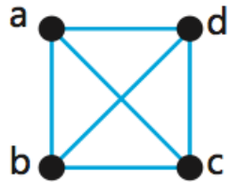
Organizing a diverse collection



1. Pick four shapes and compute shape descriptors

Quartets

Organizing a diverse collection



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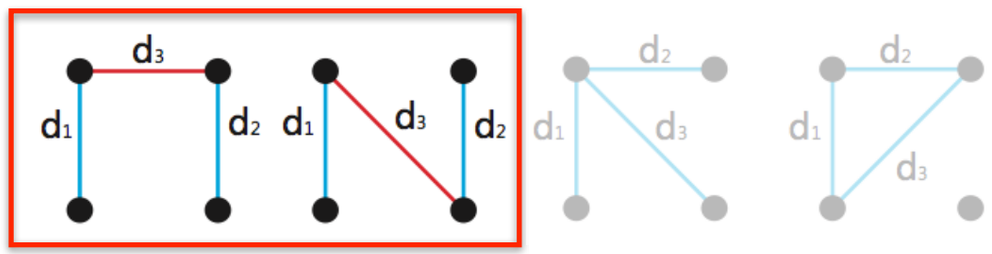
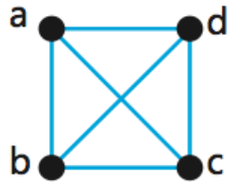


2. Remove three worst edges

Quartets

Organizing a diverse collection



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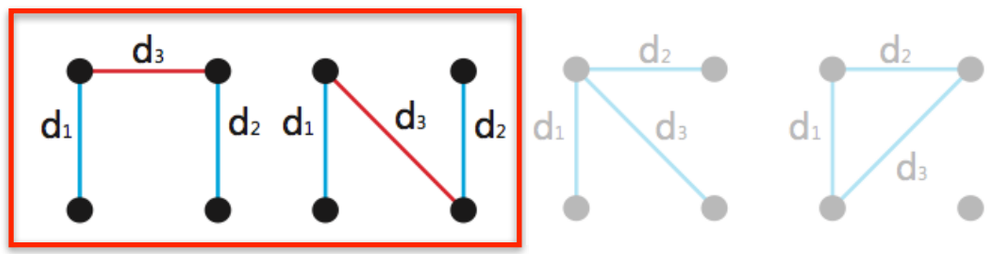
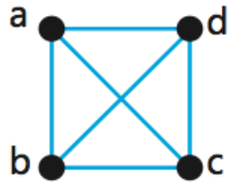


2. Remove three worst edges

3. Keep configs with a “bridge”

Quartets

Organizing a diverse collection



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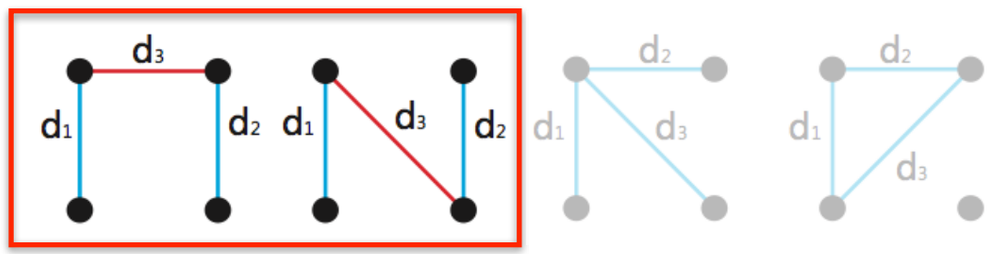


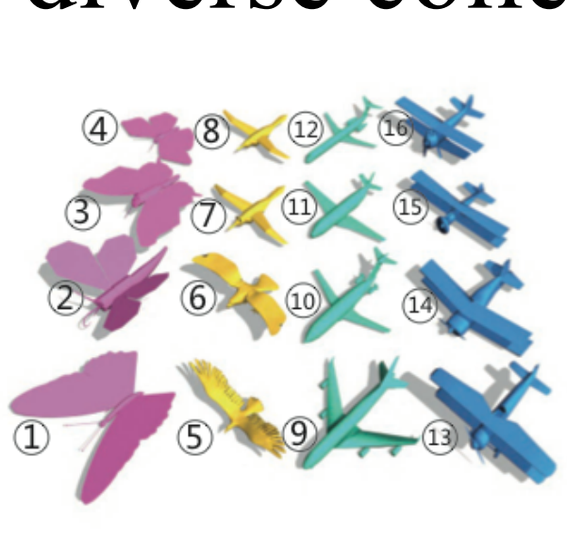
2. Remove three worst edges

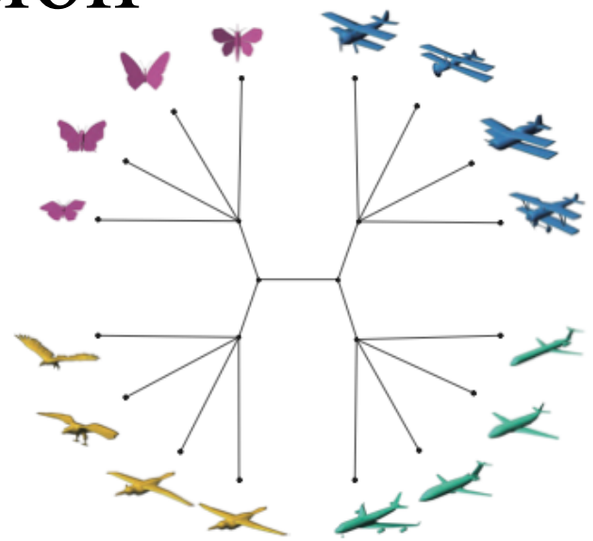
3. Keep configs with a “bridge”

4. Repeat 1-3 for many quartets, construct a graph that respects quartets (partition so that ‘good’ edges are not cut)

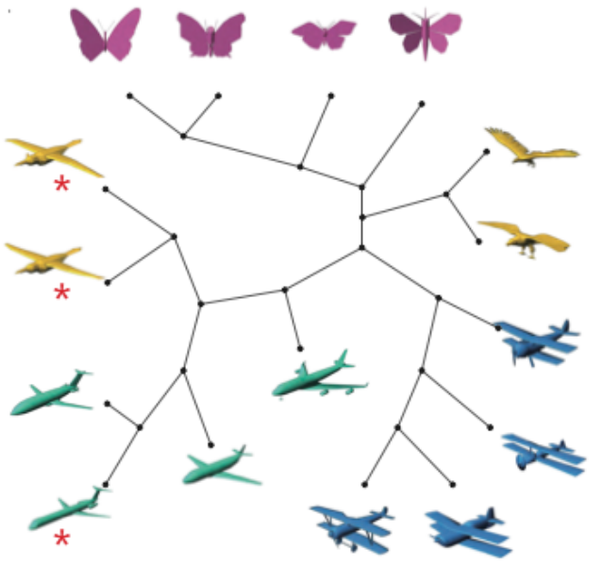
Quartets

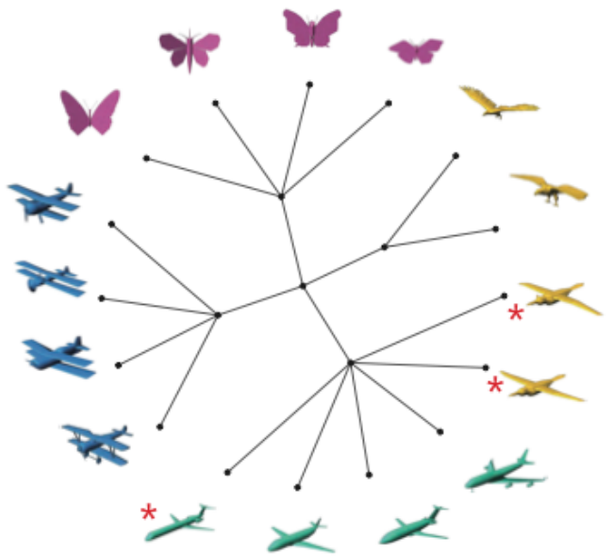
Organizing a diverse collection





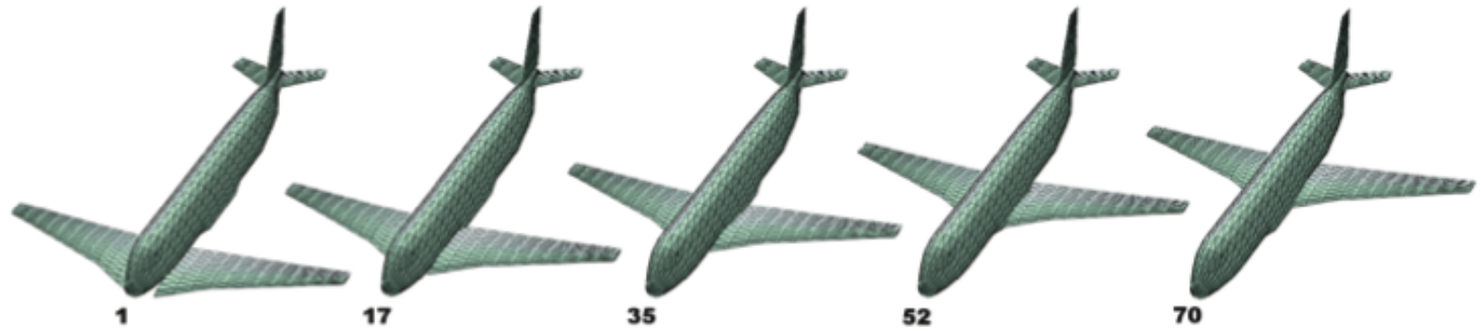
Input Shapes Quartet-based

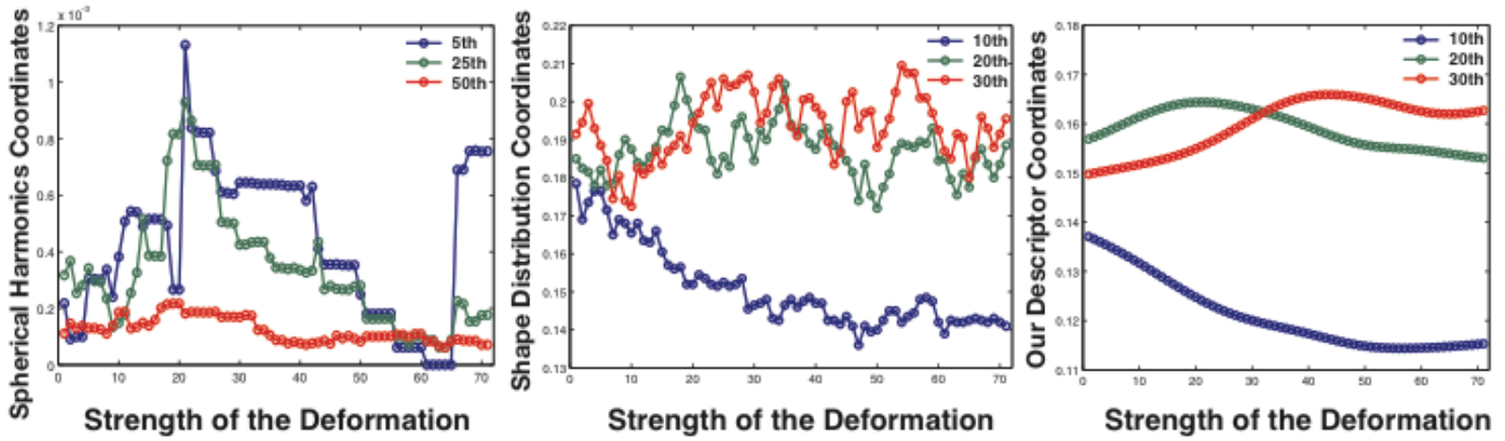




Cluster using Lightfield Descr

Templated Exploration





Spherical Harmonic

Shape Distr. (D2)

Smoothed D2

Idea: see how descriptor changes due to continuous deformation of a template

Exploration of Continuous Variability in Collections of 3D Shapes

Maks Ovsjanikov
Stanford University

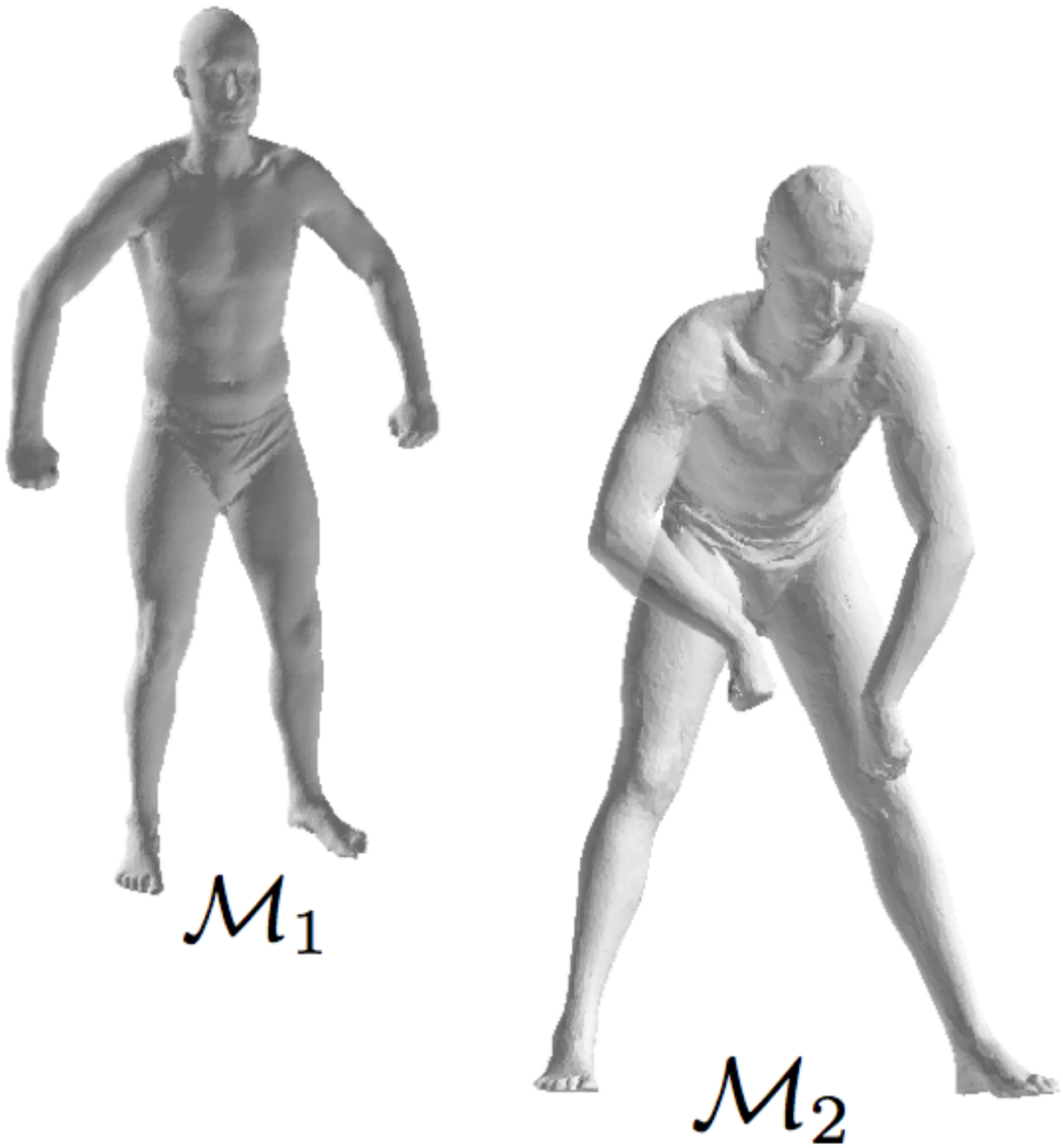
Wilmot Li
Adobe Systems

Leonidas Guibas
Stanford University

Niloy J. Mitra
KAUST

Intrinsic Descriptors

Why Intrinsic?



Why Intrinsic?

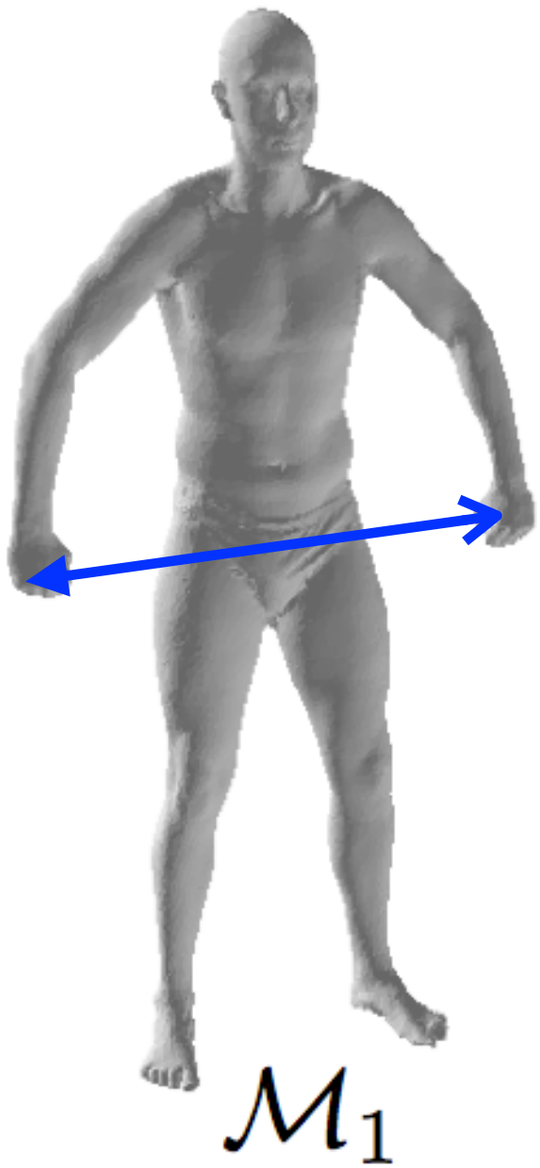
Would D2 work to describe a human vs an animal?

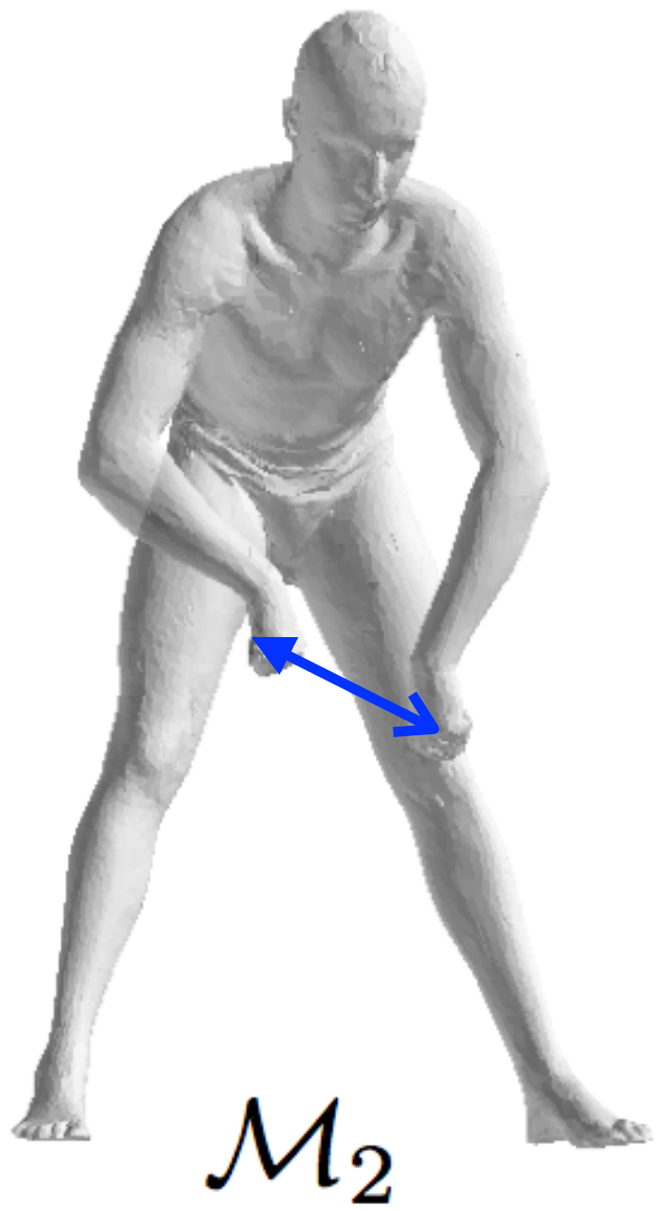
$\mathcal{M}_1$

$\mathcal{M}_2$

Why Intrinsic?

Would D2 work to describe a human vs an animal?

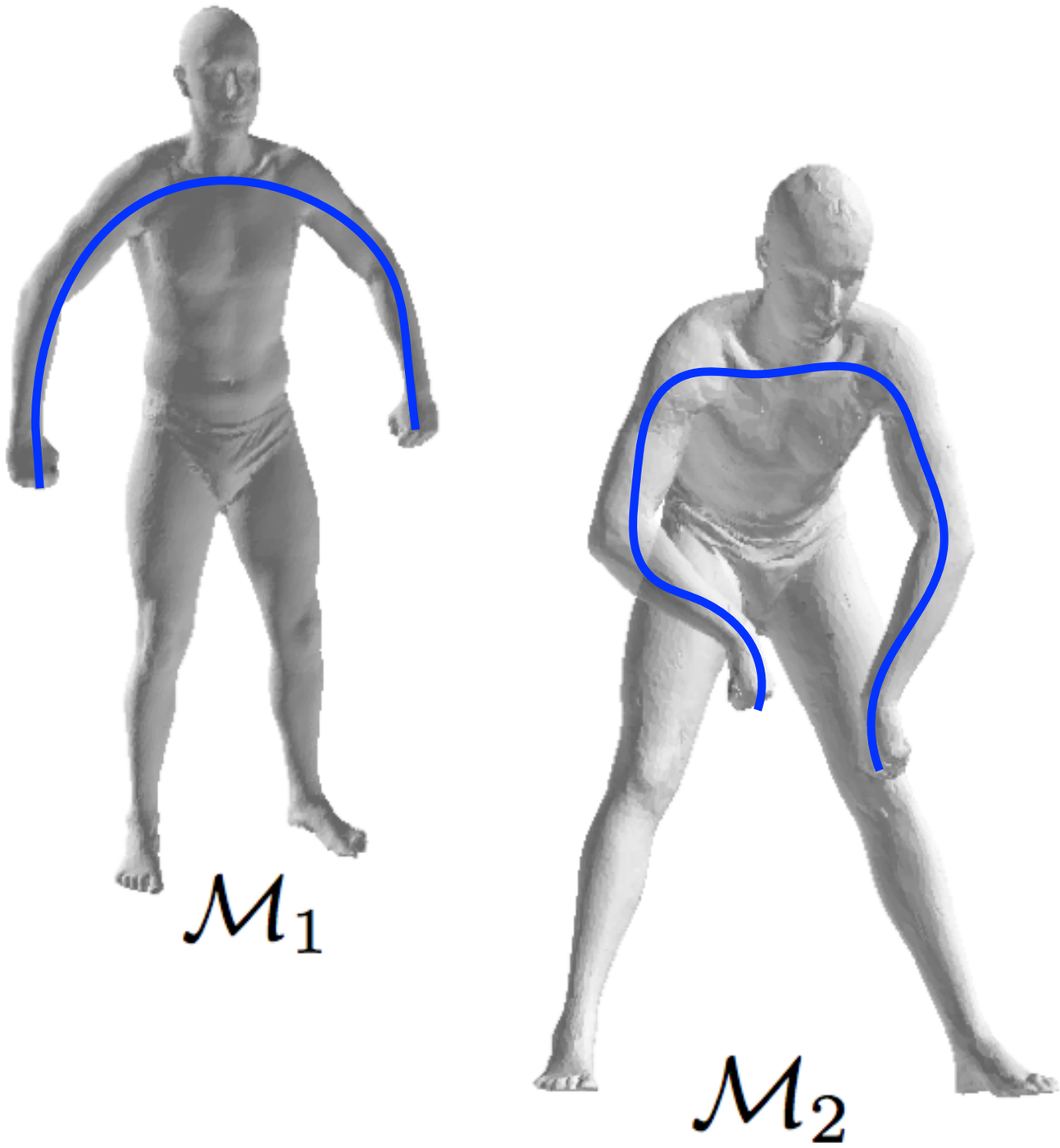




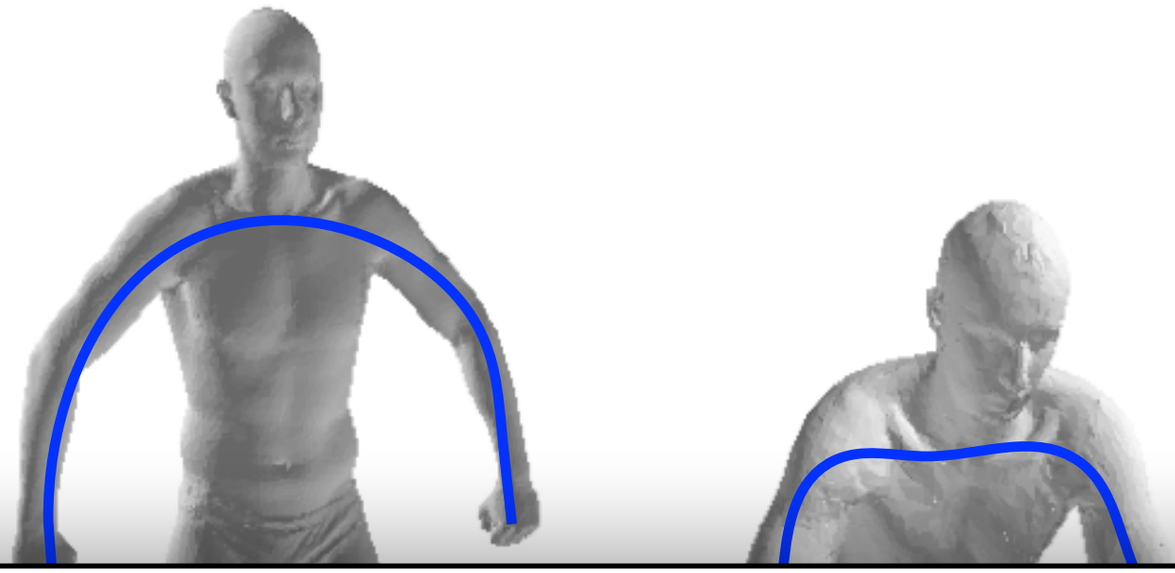
IDEAS?

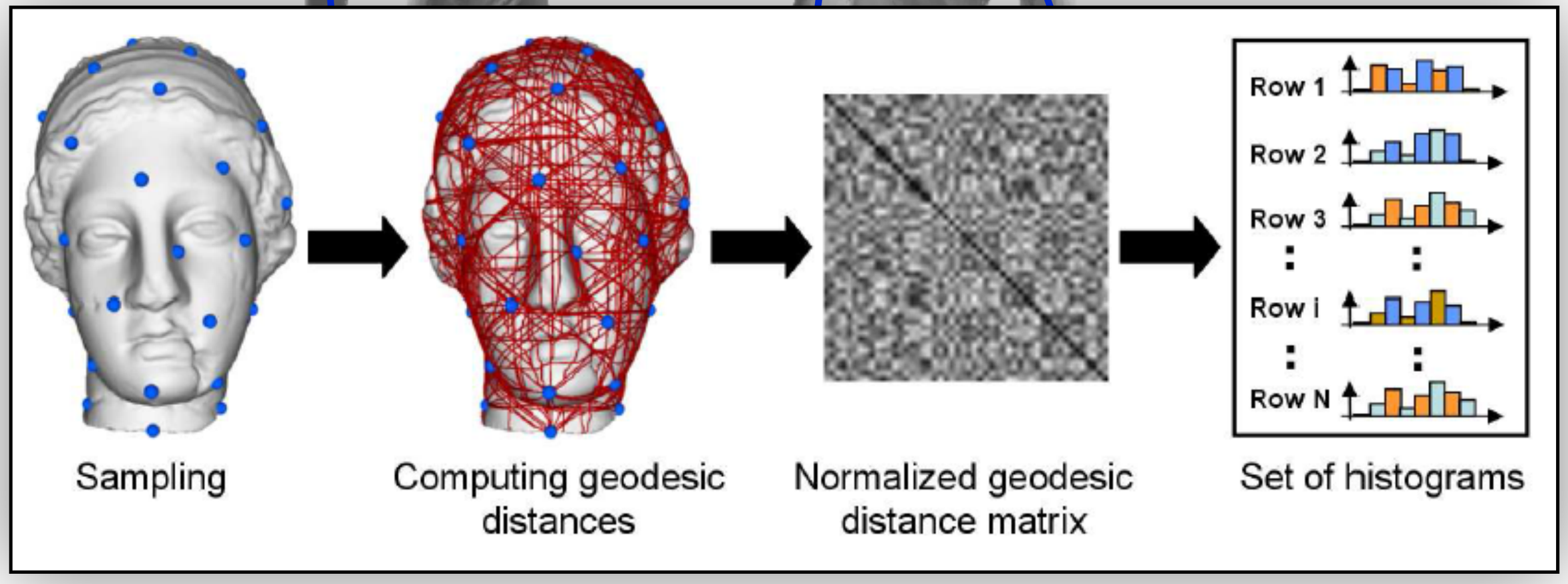
Same person, but distances change drastically depending on the pose

Geodesic Distances



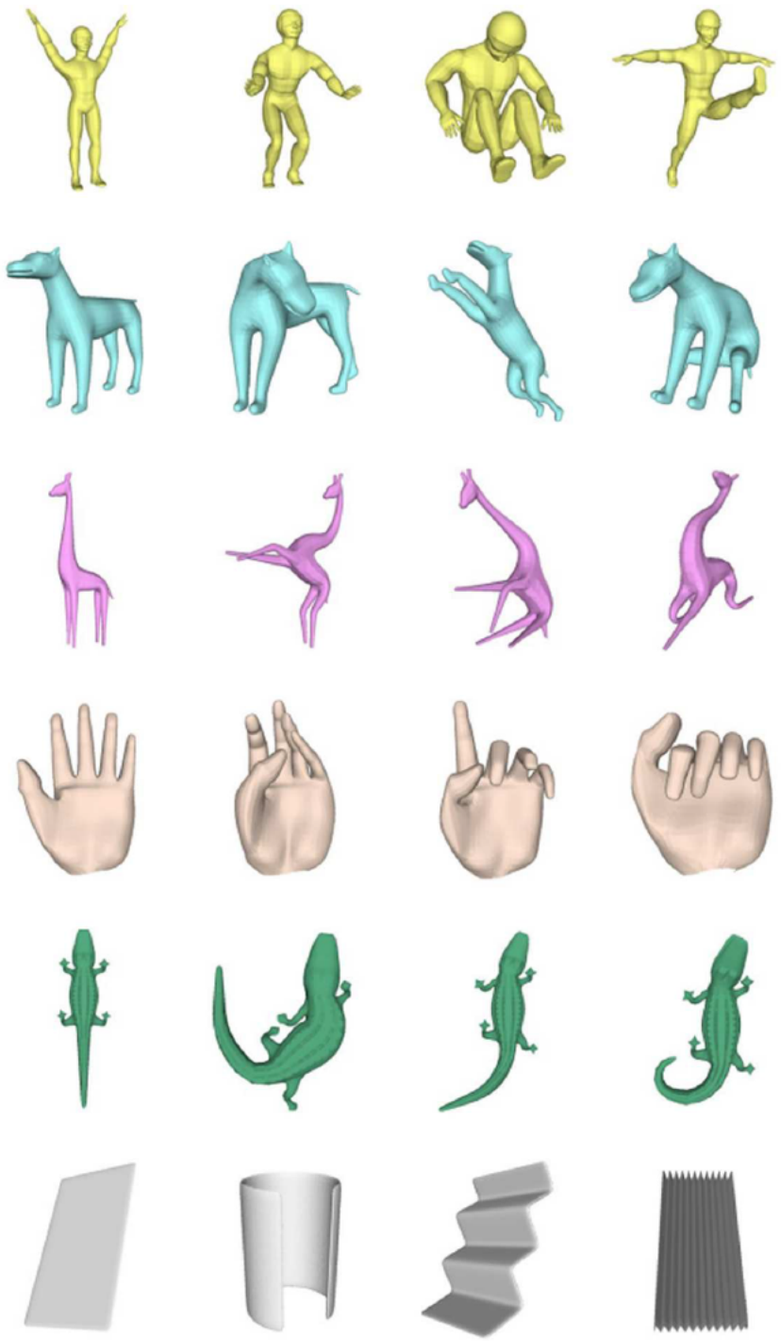
Geodesic Distances

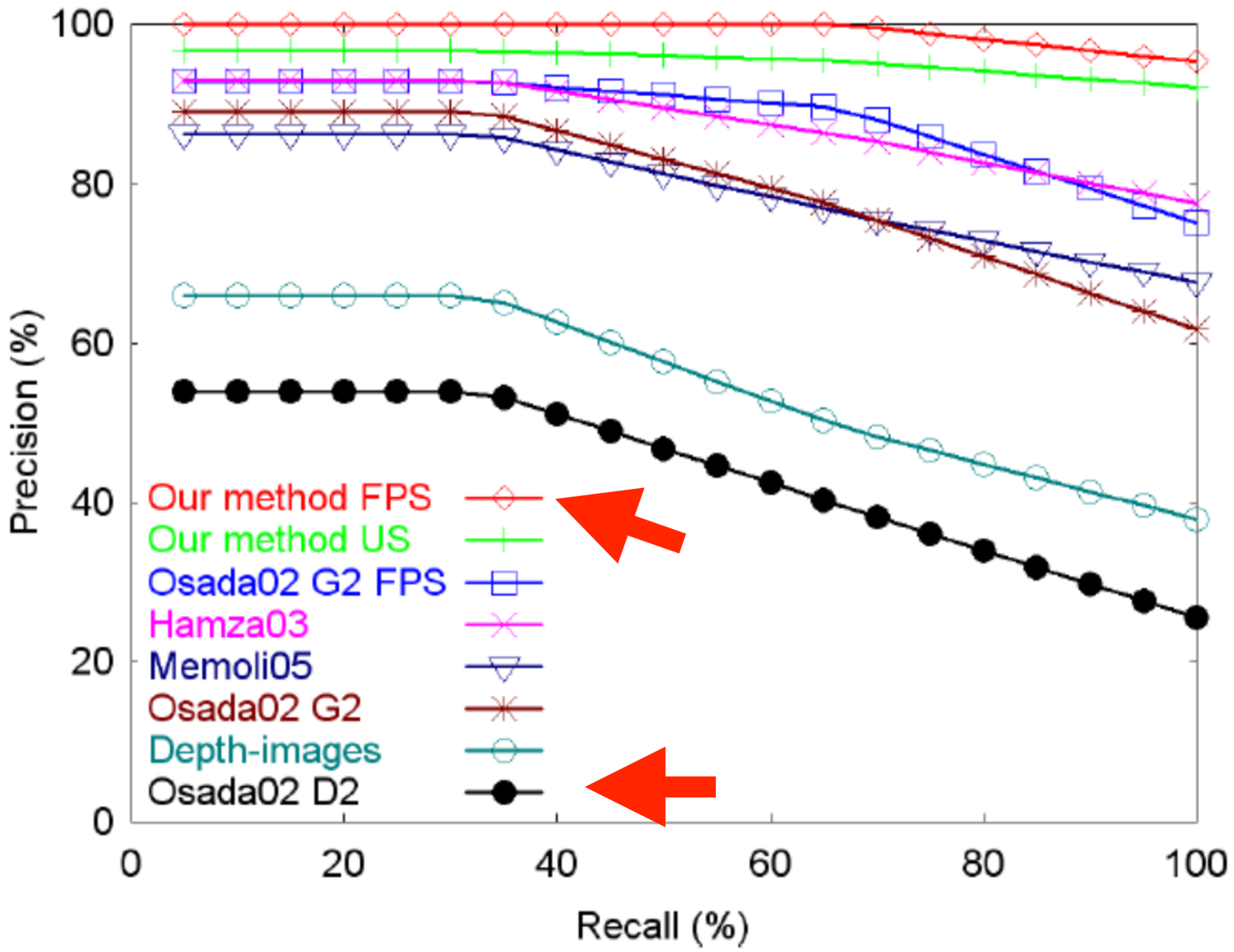




Ruggeri et al. 2008

Geodesic Distances





Ruggeri et al. 2008

Isometry

Geodesic distances are preserved
OR
Shape is bent, but not stretched

Isometry

Hope

http://www.flickr.com/photos/melvinvoskuijl/galleries/72157624236168459

Isometry

Hope

Reality

http://www.flickr.com/photos/melvinvoskuijl/galleries/72157624236168459

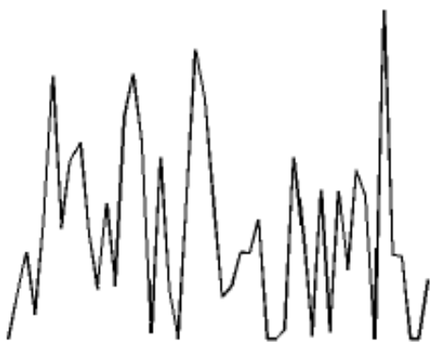
Need to deal with approximate isometry

Isometry-Invariant Retrieval

(Feature detection)

Feature description

Vector quantization

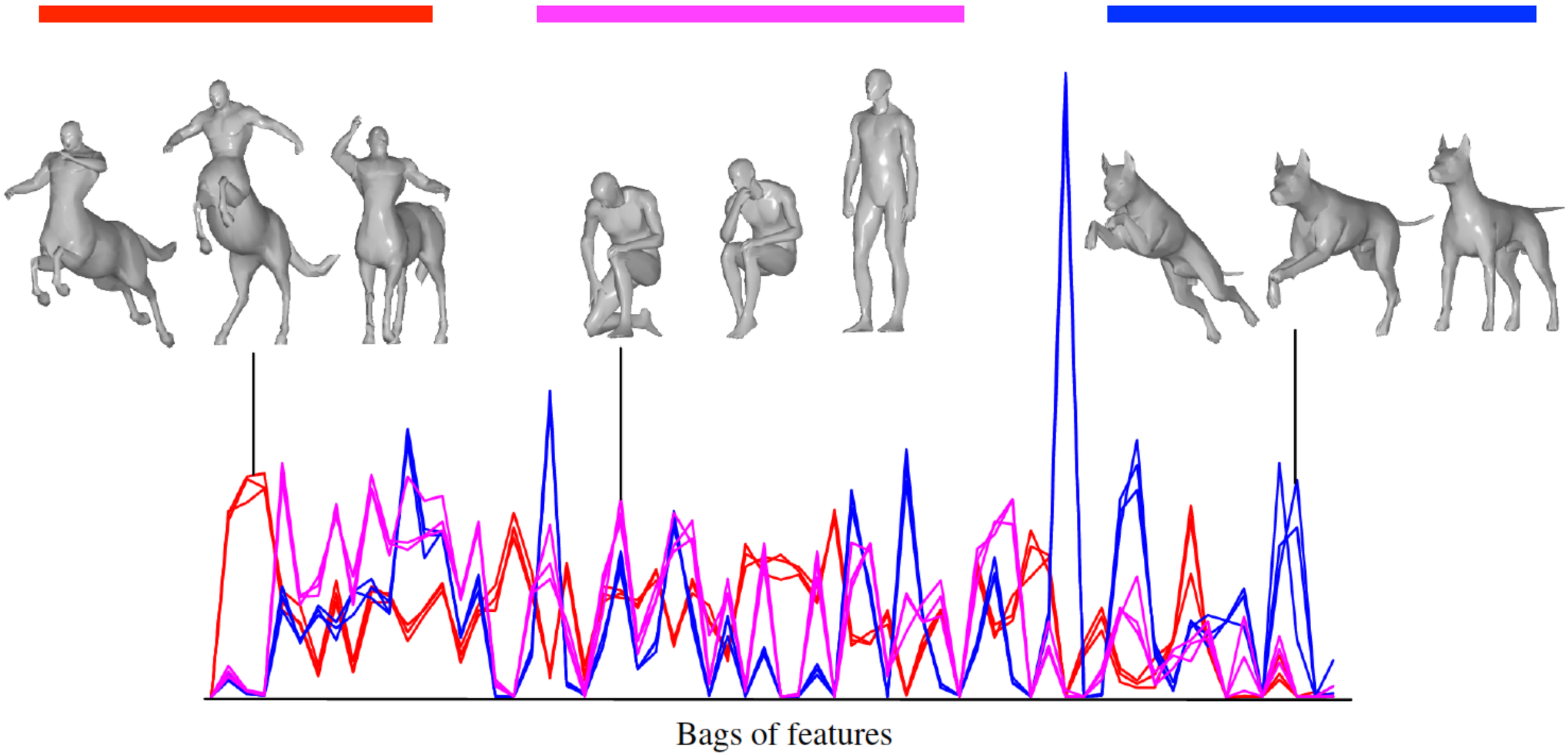


Bag of words

Some Stats

Bronstein et al. 2011

Isometry-Invariant Retrieval



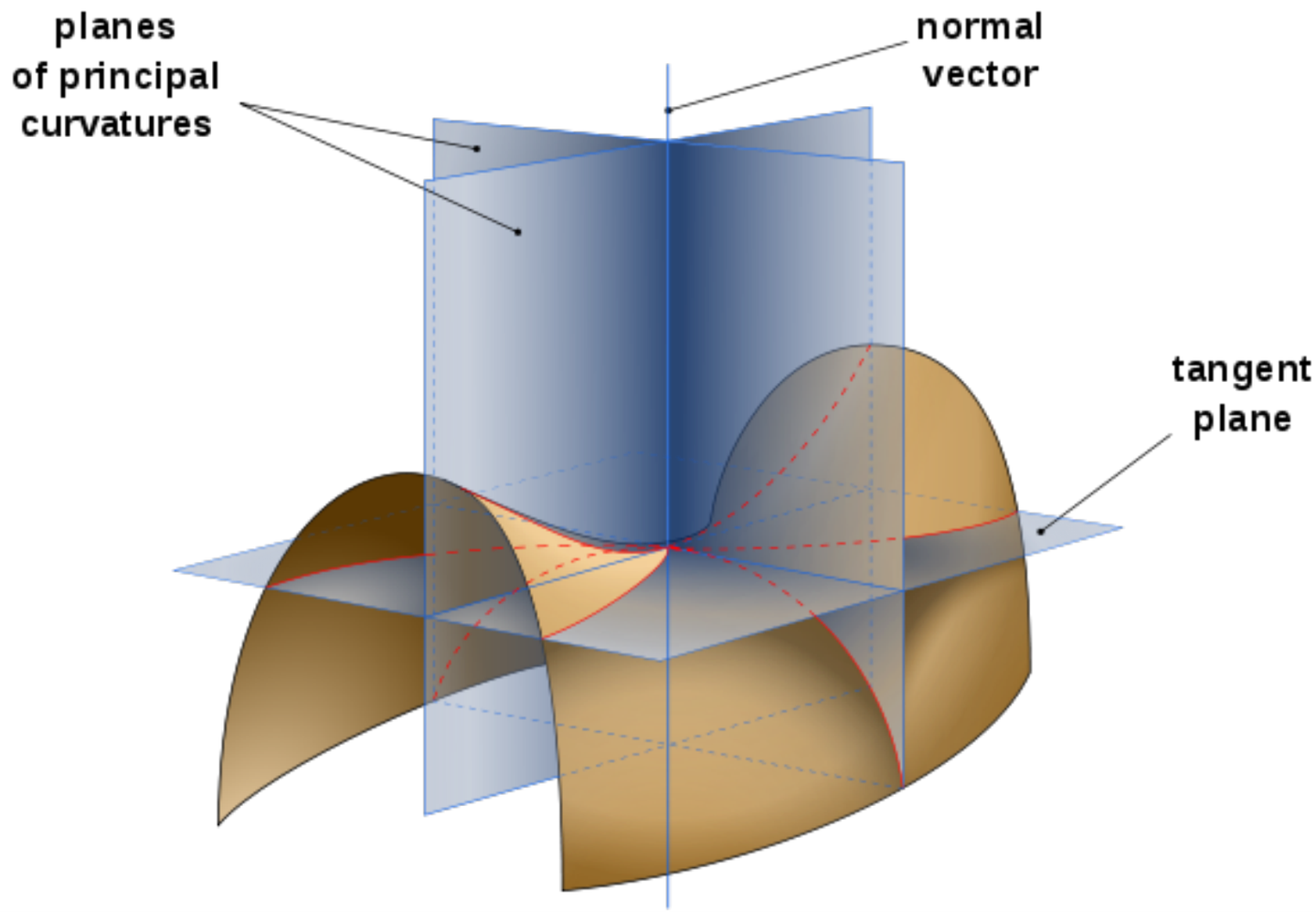
Bronstein et al. 2011

Per-Point Features

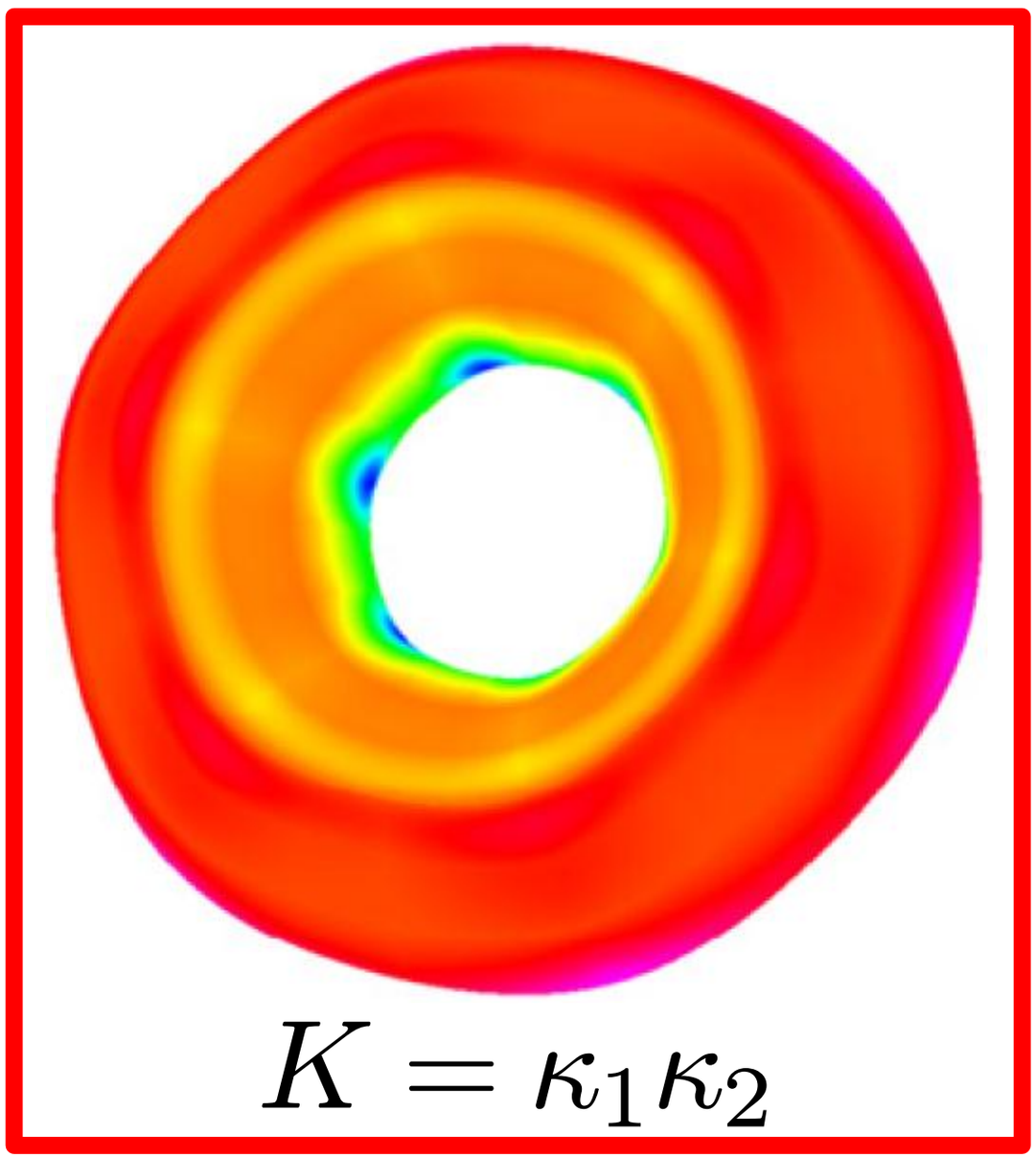
Isometrically Invariant Features

➡ **Curvature**

- Average Geodesic Distance
- Histogram of Geodesic Distances (similar to D2)
- Global Point Signature
- Heat Kernel Signature
- Wave Kernel Signature



Curvature



$$K = \kappa_1 \kappa_2$$

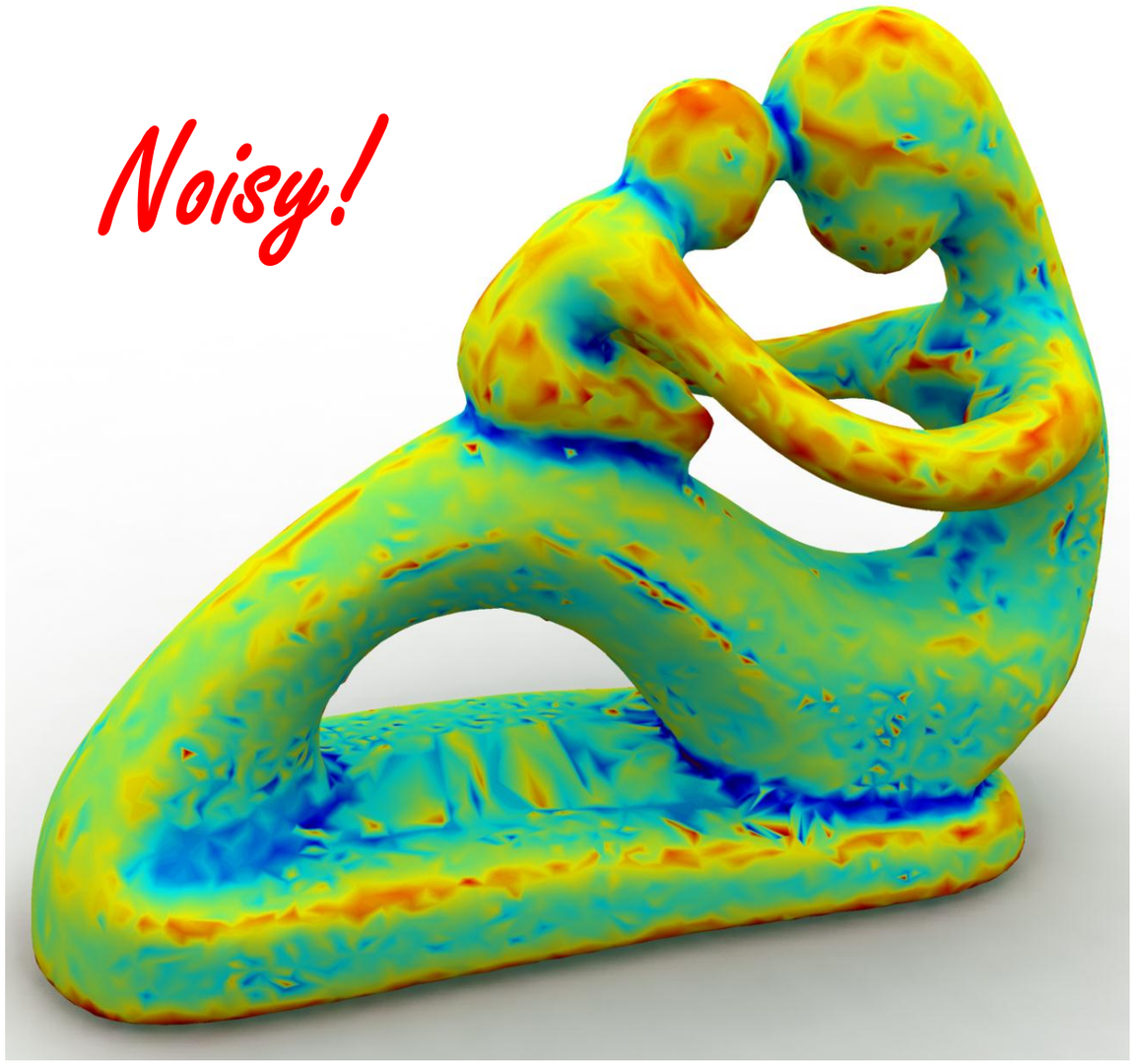
Theorema Egregium ("Remarkable Theorem"): Gaussian curvature is intrinsic.

http://www.sciencedirect.com/science/article/pii/S0010448510001983

Curvature

Problems

Noisy!

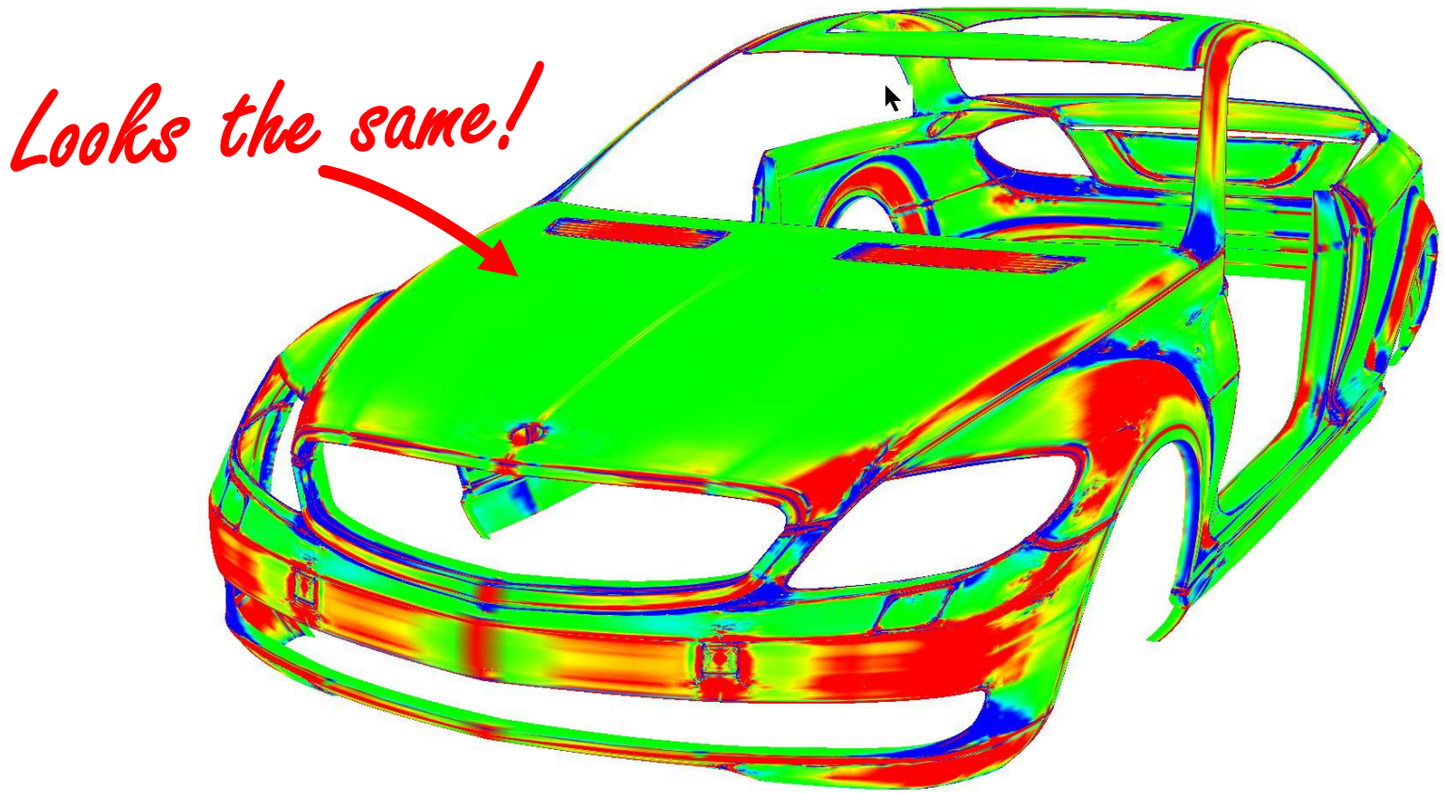


$$K = \kappa_1 \kappa_2$$

Solomon

Curvature

Problems



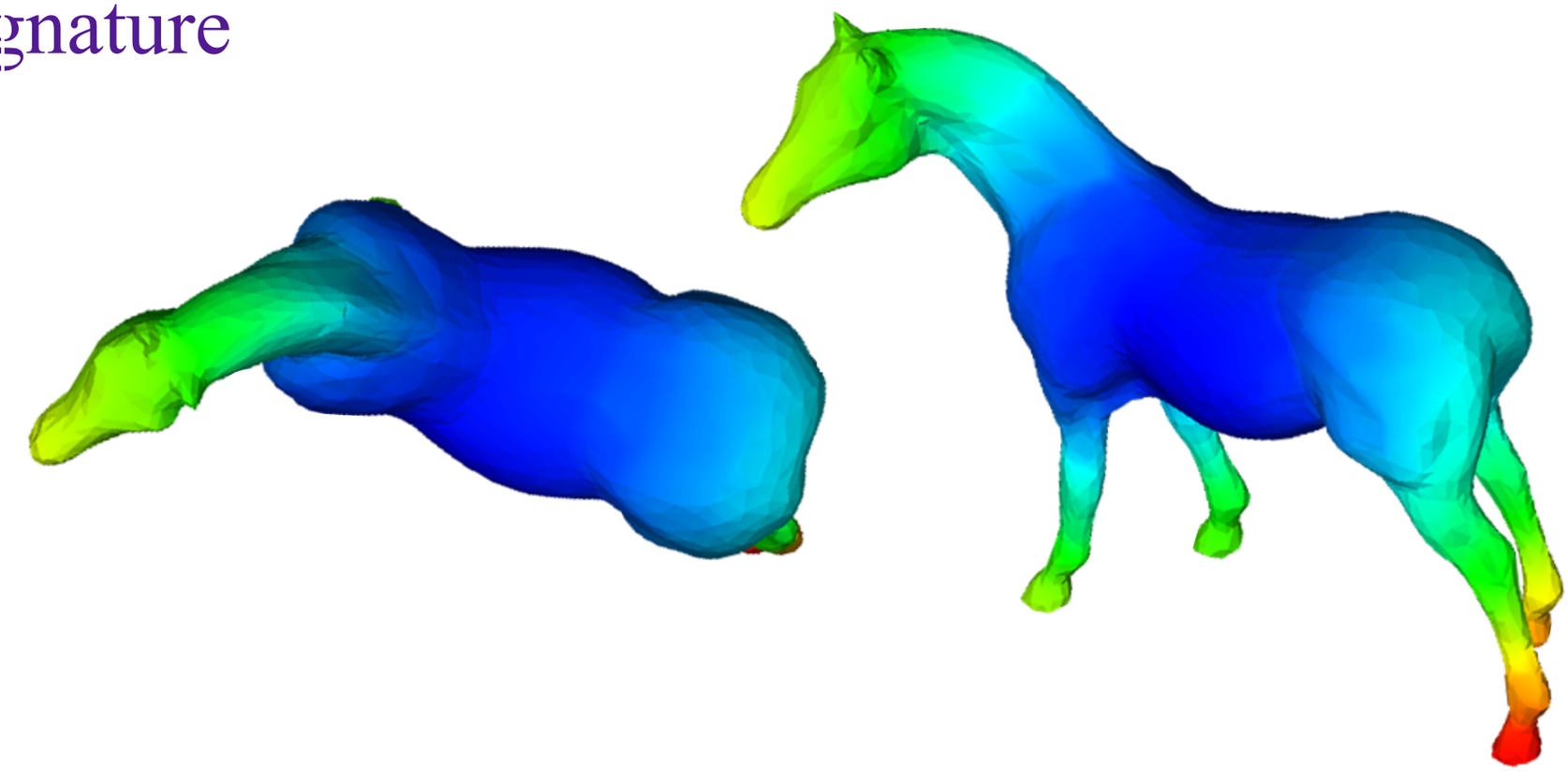
http://www.integrityware.com/images/MerceedesGaussianCurvature.jpg

$$K = \kappa_1 \kappa_2$$

Invariant Features

Isometrically Invariant Features

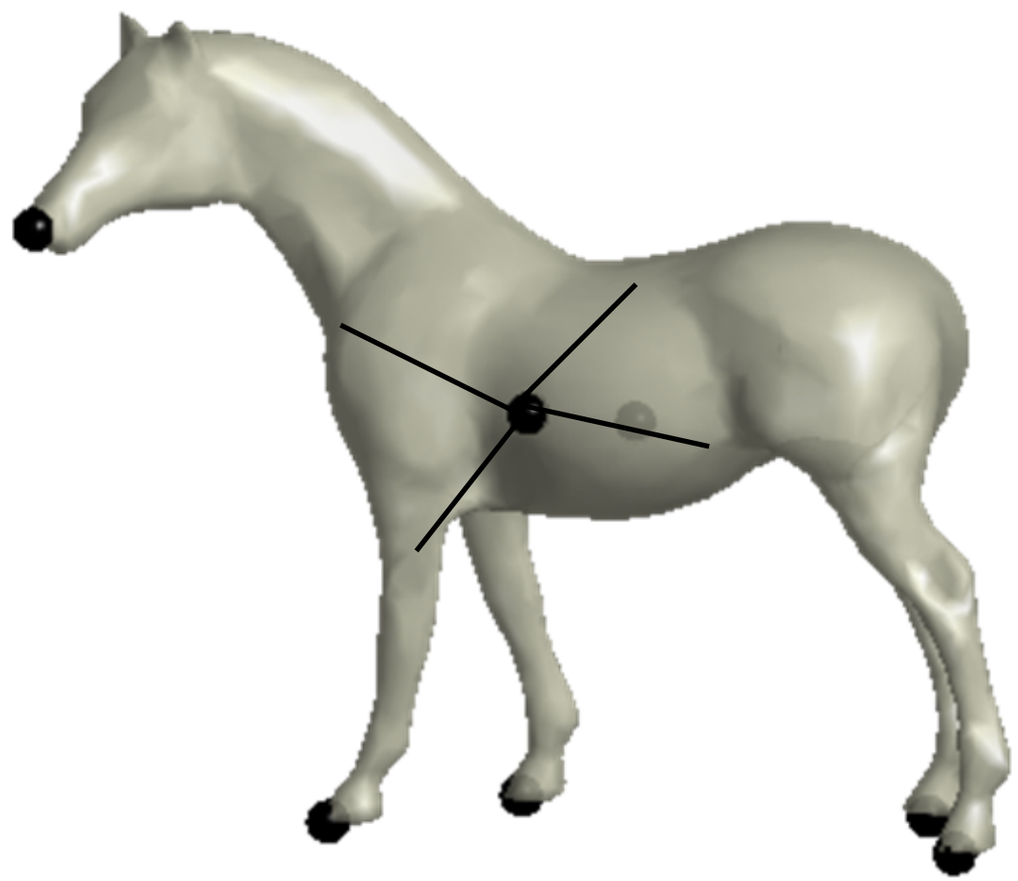
- Curvature
- ➡ **Average Geodesic Distance**
- Histogram of Geodesic Distances (similar to D2)
- Global Point Signature
- Heat Kernel Signature
- Wave Kernel Signature



Invariant Features

Isometrically Invariant Features

- Curvature
- Average Geodesic Distance
- ➡ **Histogram of Geodesic Distances (per point)**
- Global Point Signature
- Heat Kernel Signature
- Wave Kernel Signature



Invariant Features

Isometrically Invariant Features

- Curvature
- Average Geodesic Distance
- Histogram of Geodesic Distances (per point)

➡ **Global Point Signature**

➡ **Heat Kernel Signature**

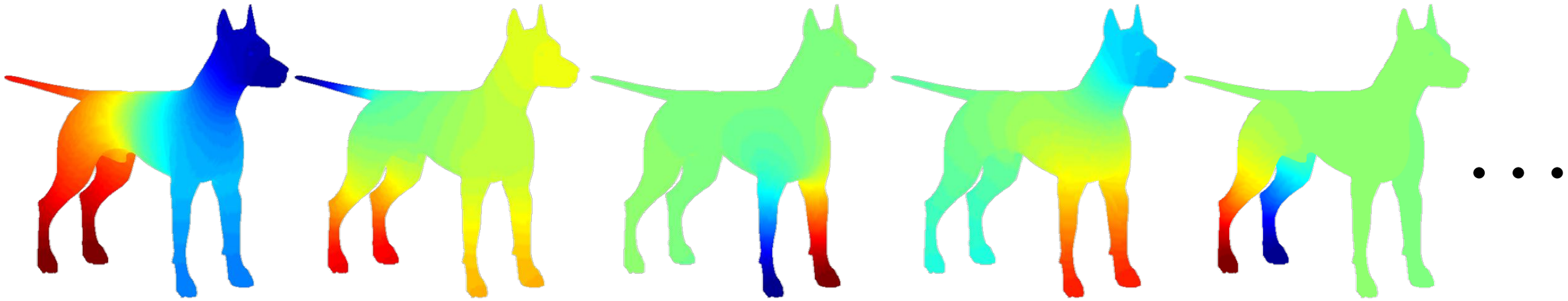
➡ **Wave Kernel Signature**

Laplace-Beltrami Operator

Laplace-Beltrami Operator

Functional basis on a surface

- Invariant to isometric deformations
- Low-frequency to high-frequency
- Has physics interpretation



$\Delta\phi_1 = \lambda_1\phi_1$ $\Delta\phi_2 = \lambda_2\phi_2$ $\Delta\phi_3 = \lambda_3\phi_3$ $\Delta\phi_4 = \lambda_4\phi_4$ $\Delta\phi_5 = \lambda_5\phi_5$

$(\Delta\phi_0 = 0)$

eigenvalues **eigenfunctions**

Solomon

Global Point Signature

$$GPS(p) = \left(\frac{1}{\sqrt{\lambda_1}}\phi_1(p), \frac{1}{\sqrt{\lambda_2}}\phi_2(p), \frac{1}{\sqrt{\lambda_3}}\phi_3(p), \cdots \right)$$

Rustamov et al. 2007

Global Point Signature

$$GPS(p) = \left(\frac{1}{\sqrt{\lambda_1}}\phi_1(p), \frac{1}{\sqrt{\lambda_2}}\phi_2(p), \frac{1}{\sqrt{\lambda_3}}\phi_3(p), \cdots \right)$$

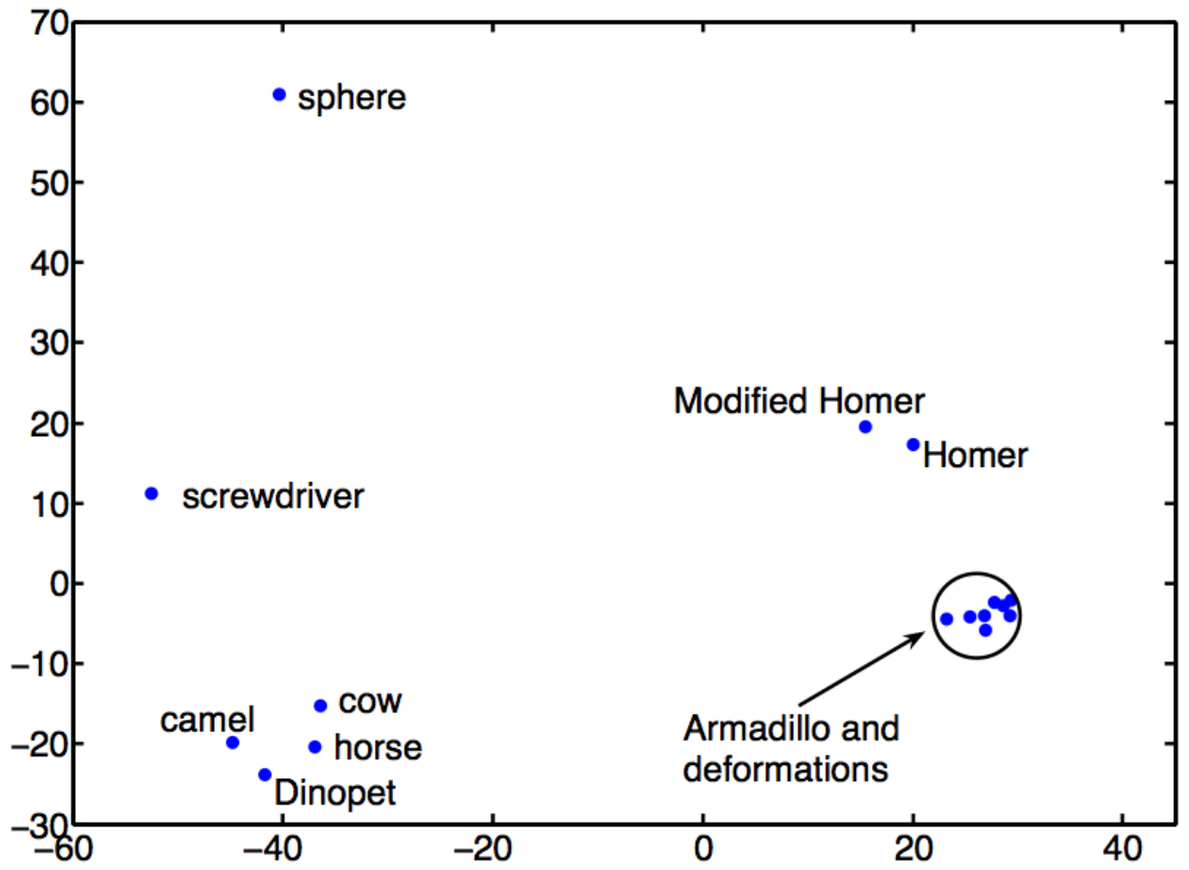


Figure 4: *Armadillo and its deformations.*

Similar to D2, but use histograms in embedded space (rather than euclidean)

Rustamov et al. 2007

Global Point Signature

$$GPS(p) = \left(\frac{1}{\sqrt{\lambda_1}}\phi_1(p), \frac{1}{\sqrt{\lambda_2}}\phi_2(p), \frac{1}{\sqrt{\lambda_3}}\phi_3(p), \cdots \right)$$

Pros

- Isometry-invariant
- Global (each point feature depends on entire shape)

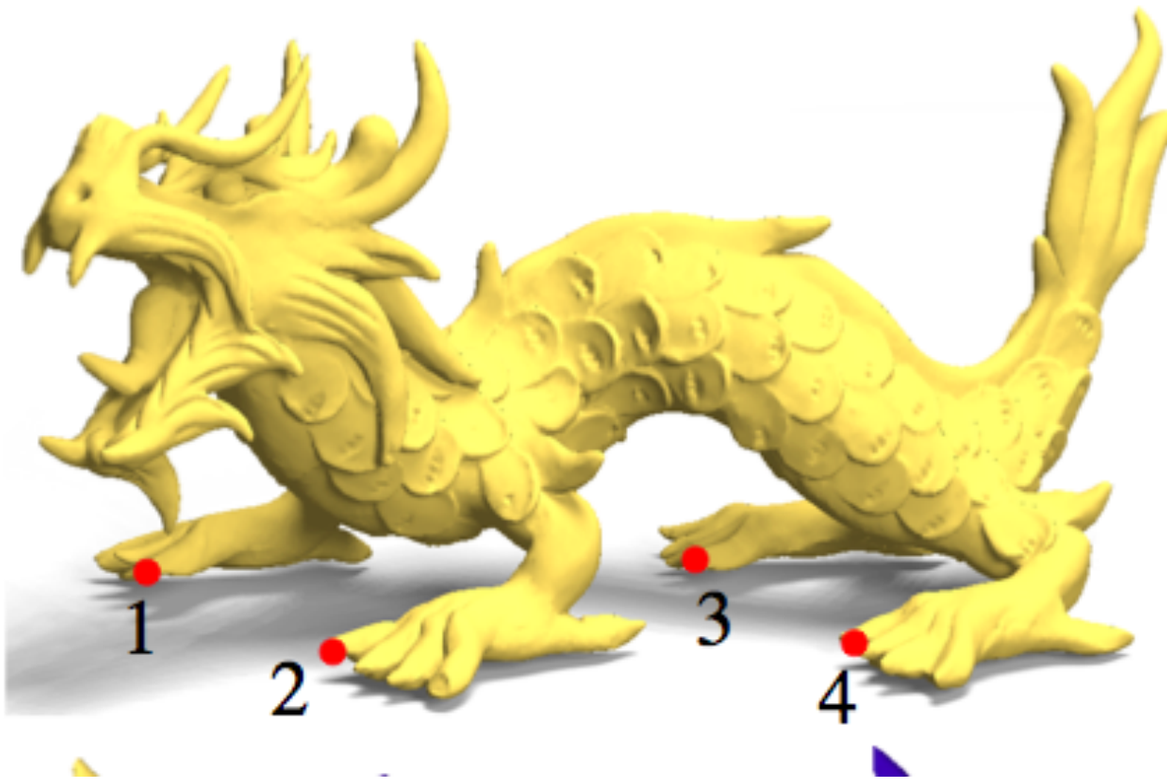
Cons

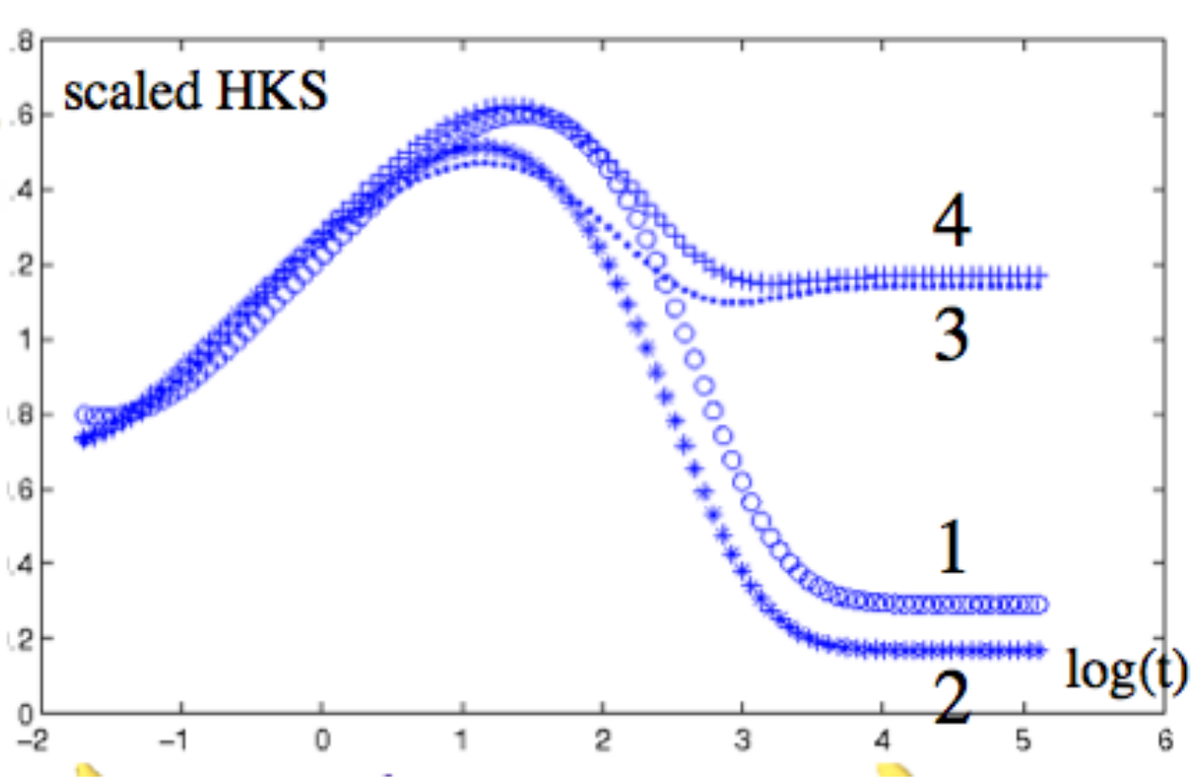
- Eigenfunctions might change positions due to deformations
- Only global

Rustamov et al. 2007

Heat Kernel Signature

$$HKS(x) : \mathbb{R}^+ \rightarrow \mathbb{R}, HKS(x,t) = k_t(x,x).$$



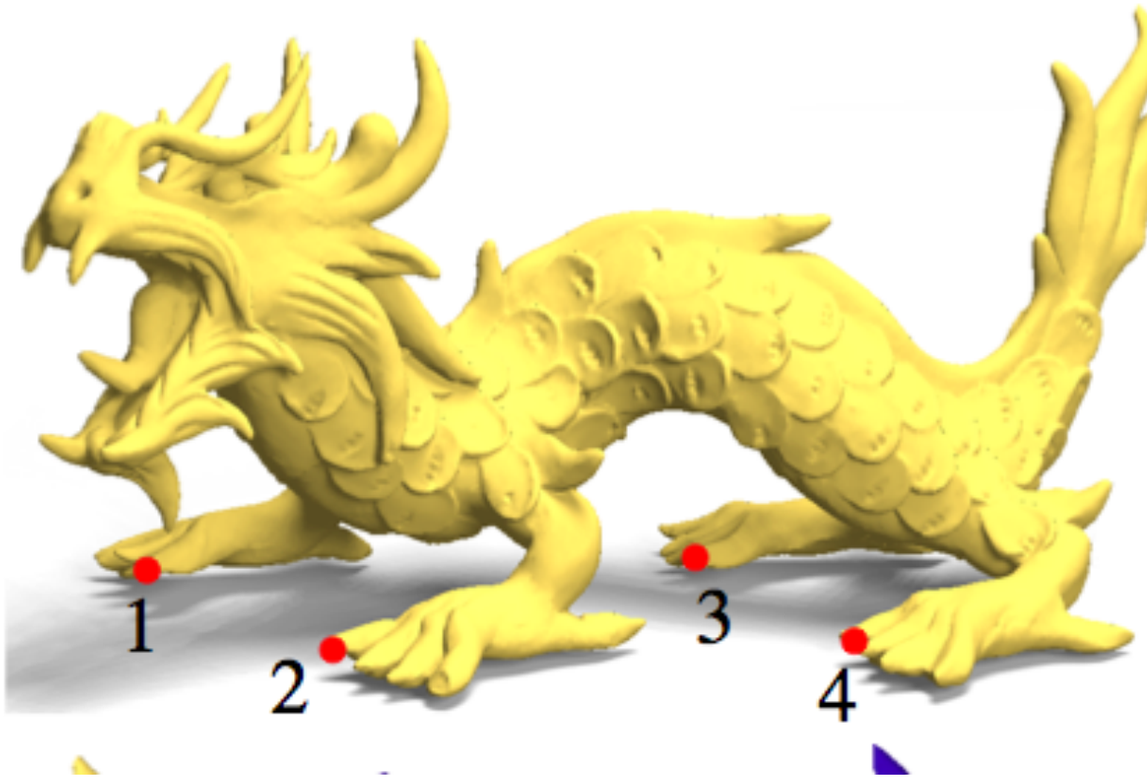


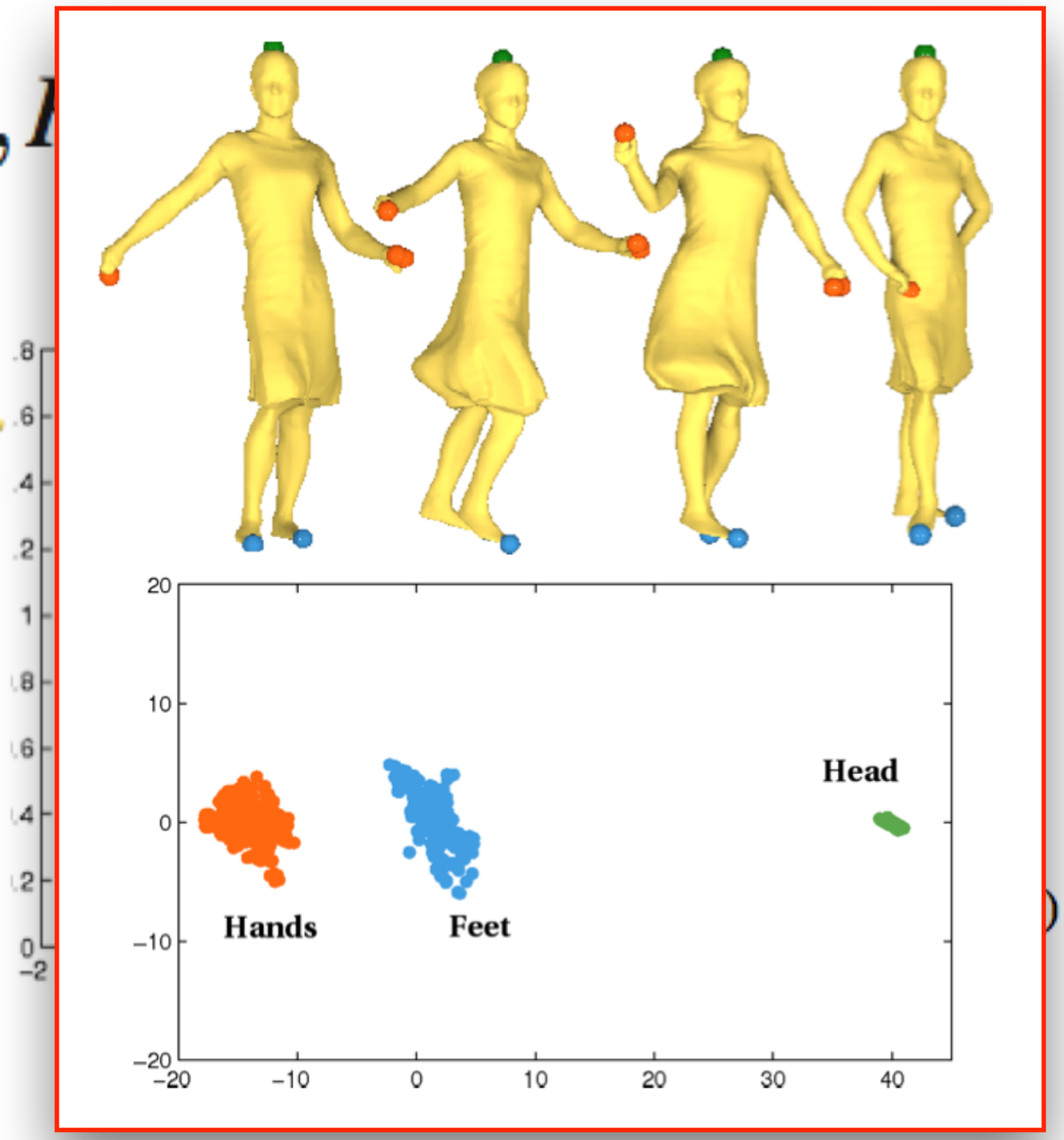
At time t how much heat comes back to x

Sun et al. 2008

Heat Kernel Signature

$$HKS(x) : \mathbb{R}^+ \rightarrow \mathbb{R}, I$$





At time t how much heat comes back to x

Sun et al. 2008

Heat Kernel Signature

$$HKS(x) : \mathbb{R}^+ \longrightarrow \mathbb{R}, HKS(x,t) = k_t(x,x).$$

Physics:

$$k_t(x,x) = \sum_{n=0}^{\infty} e^{-\lambda_i t} \phi_n(x)^2$$

Sun et al. 2008

Heat Kernel Signature

$$k_t(x,x) = \sum_{n=0}^{\infty} e^{-\lambda_i t} \phi_n(x)^2$$

Pros

- Isometry-invariant
- Multi-scale (depends on range of t)
- Change in eigenfunction positions is not a problem

Cons

- Theoretical invariance only for isometric deformations

Sun et al. 2008

Wave Kernel Signature

$$WKS(E,x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T |\psi_E(x,t)|^2 \, dt = \sum_{n=0}^{\infty} \phi_n(x)^2 f_E(\lambda_k)^2$$

Initial energy distribution

Average probability over time that particle is at *x*.

WKS: Aubury et al. Slide: Solomon

Wave Kernel Signature

$$WKS(E,x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T |\psi_E(x,t)|^2 \, dt = \sum_{n=0}^{\infty} \phi_n(x)^2 f_E(\lambda_k)^2$$

Initial energy distribution

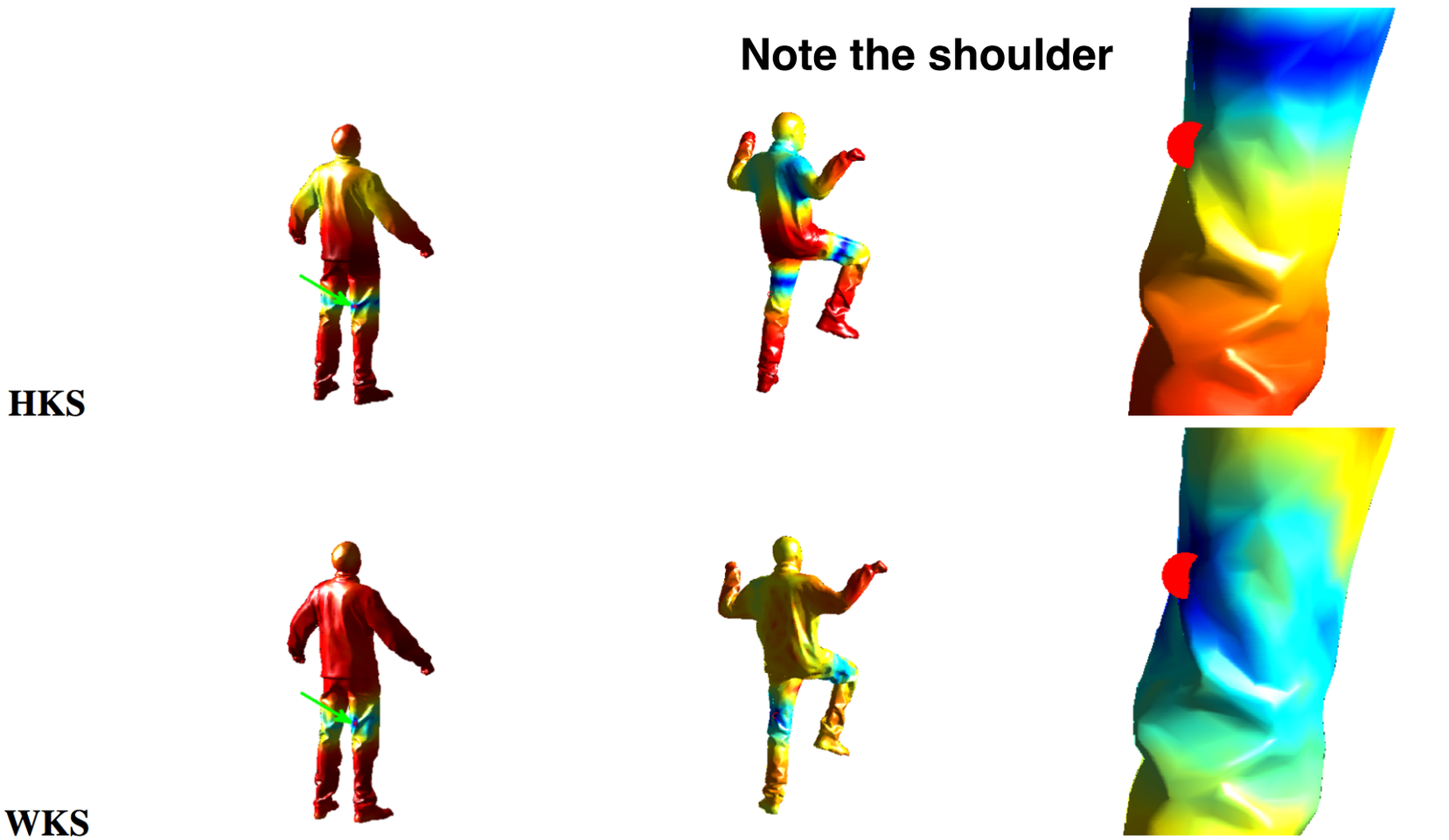
Comes from stability analysis

Average probability over time that particle is at *x*.

WKS: Aubury et al. Slide: Solomon

Wave Kernel Signature

$$WKS(E,x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T |\psi_E(x,t)|^2 \, dt = \sum_{n=0}^{\infty} \phi_n(x)^2 f_E(\lambda_k)^2$$



Aubury et al. 2011

Wave Kernel Signature

$$WKS(E,x) = \lim_{T\to\infty} \frac{1}{T} \int_0^T |\psi_E(x,t)|^2 \, dt = \sum_{n=0}^{\infty} \phi_n(x)^2 f_E(\lambda_k)^2$$

Pros

- Isometry-invariant
- More accurate in higher frequencies

Cons

- Similar to HKS

Aubury et al. 2011

Homework #1

Shape Descriptors — due Apr 20

- Point Sampling
- Descriptor I: Shape Distributions (D2)
- Descriptor II: Shape Distributions with Geodesic Distances
- “Descriptor” III: Shape Matching (ICP)
- Evaluation (retrieval, confusion matrix, precision-recall)
- Exploration (MDS Embedding)
- Write a (short, but structured) report

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