Physically Based Sound for Computer Animation and Virtual Environments

Thin Shells

Doug James
Stanford University

“Thin Shells” References

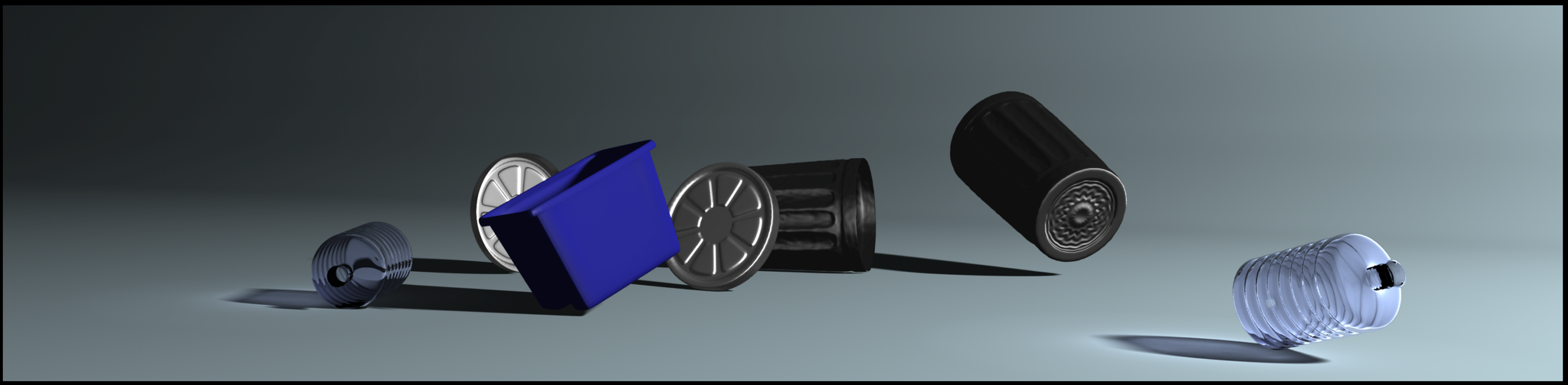
Jeff Chadwick

Steven An

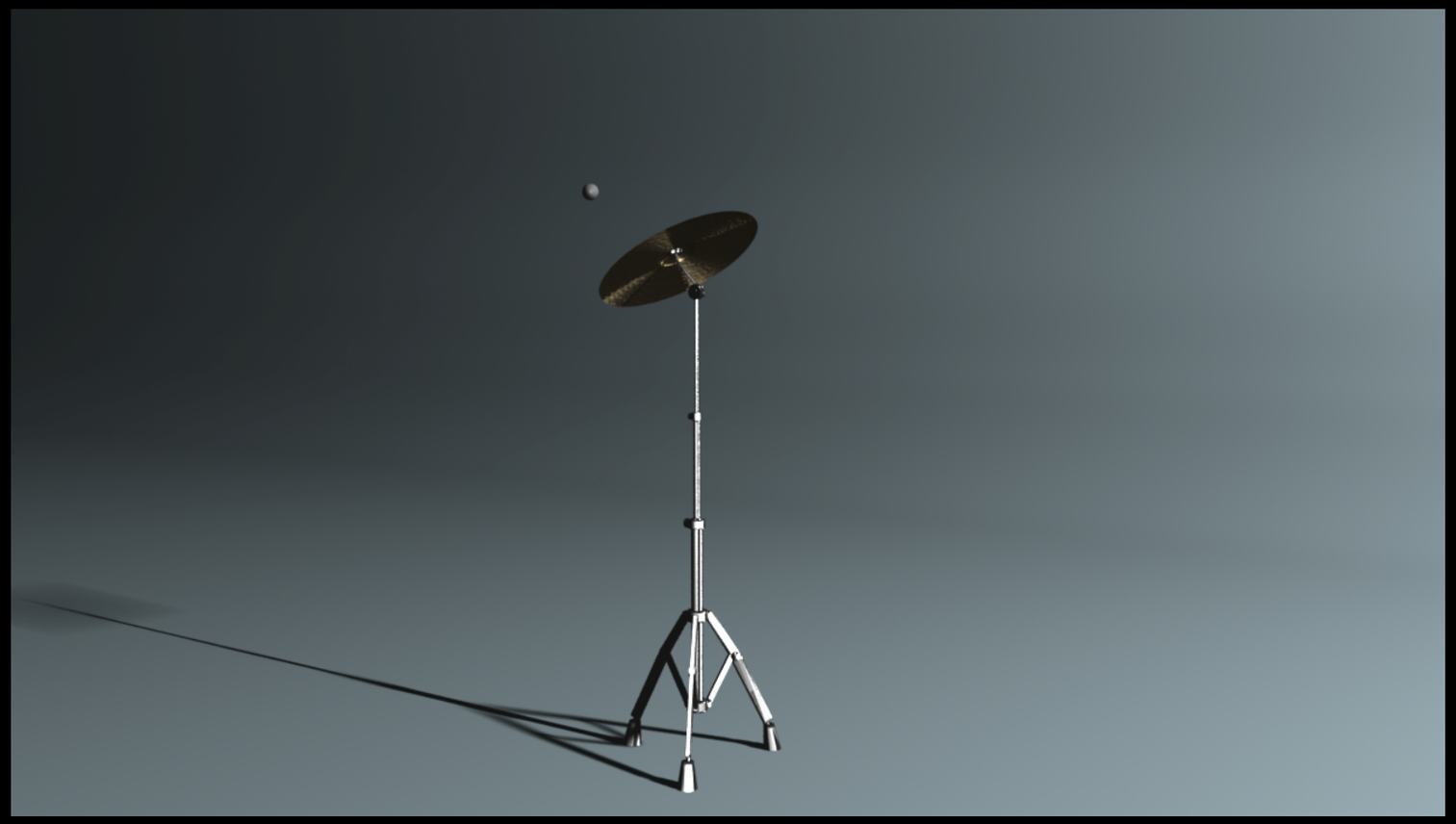
Jeffrey N. Chadwick, Steven S. An, and Doug L. James, **Harmonic Shells: A Practical Nonlinear Sound Model for Near-Rigid Thin Shells**, ACM Transactions on Graphics (SIGGRAPH ASIA Conference Proceedings), December 2009.

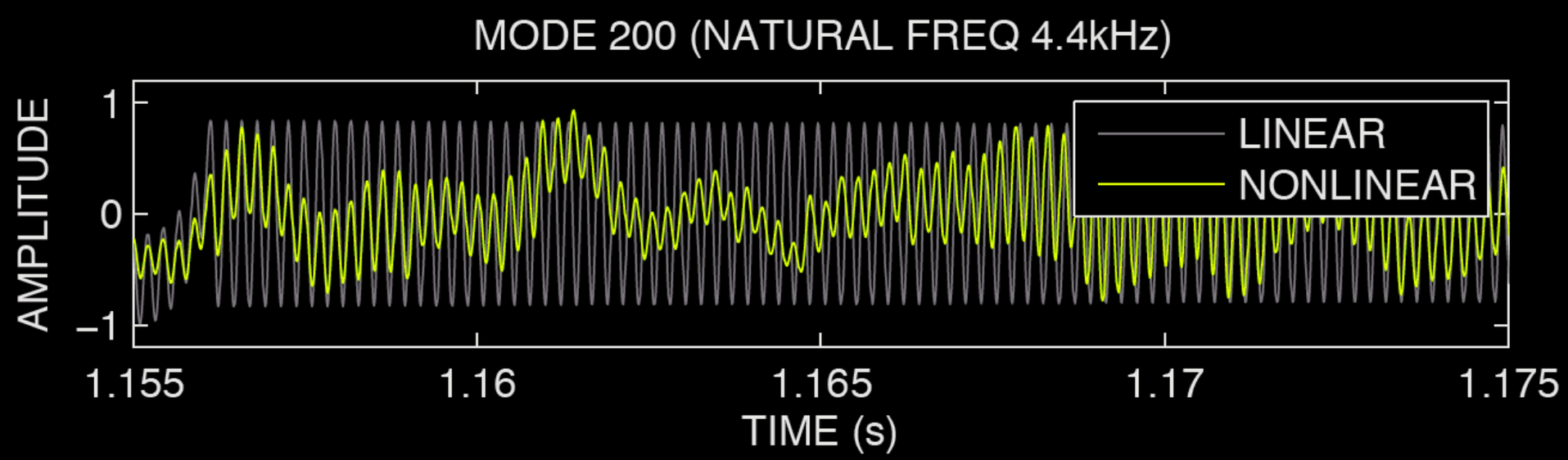
Steven S. An, Theodore Kim and Doug L. James, **Optimizing Cubature for Efficient Integration of Subspace Deformations**, ACM Transactions on Graphics (SIGGRAPH ASIA Conference Proceedings), 27(5), December 2008, pp. 164:1-164:11.

Crash!

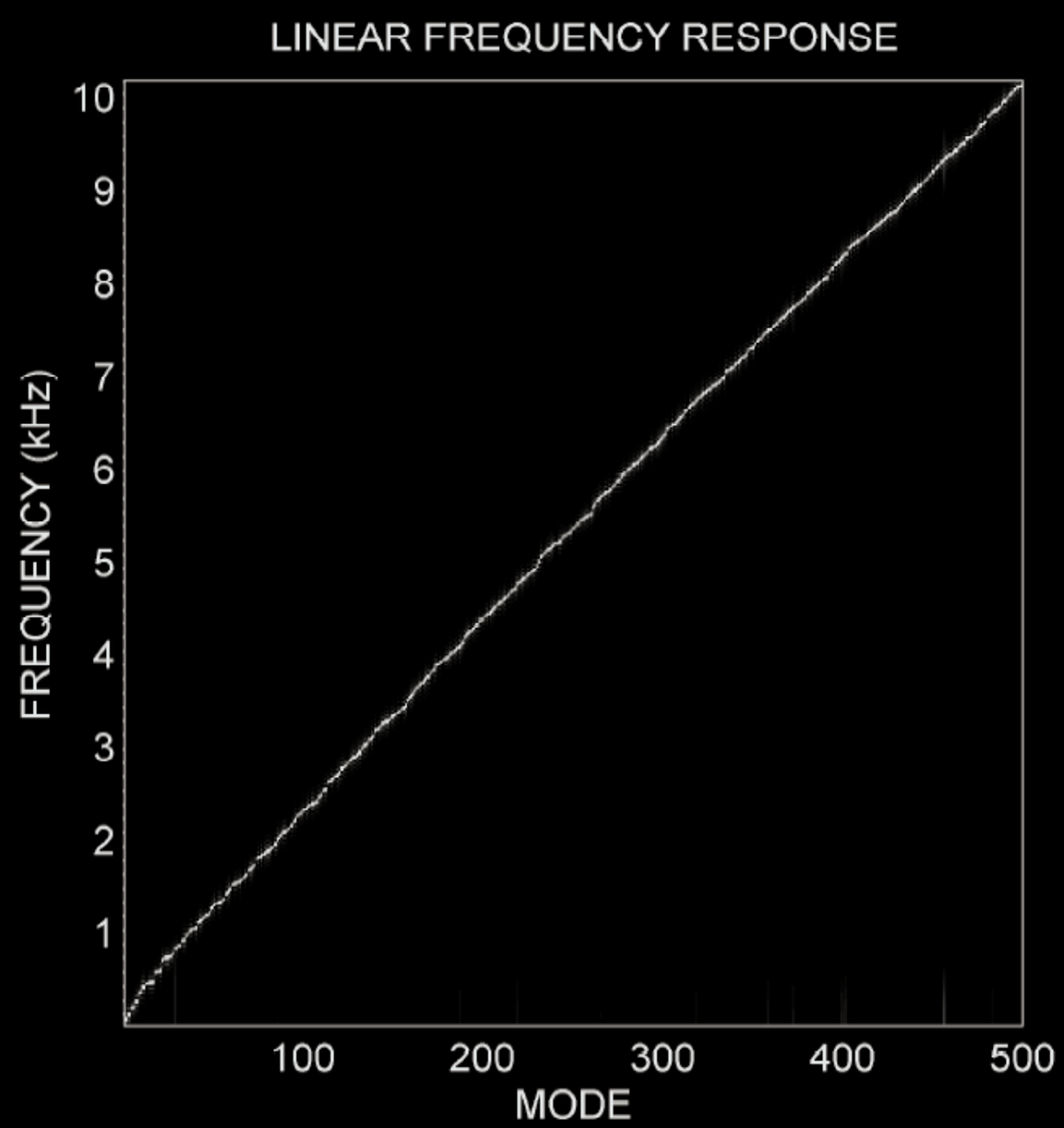


Nonlinear mode coupling

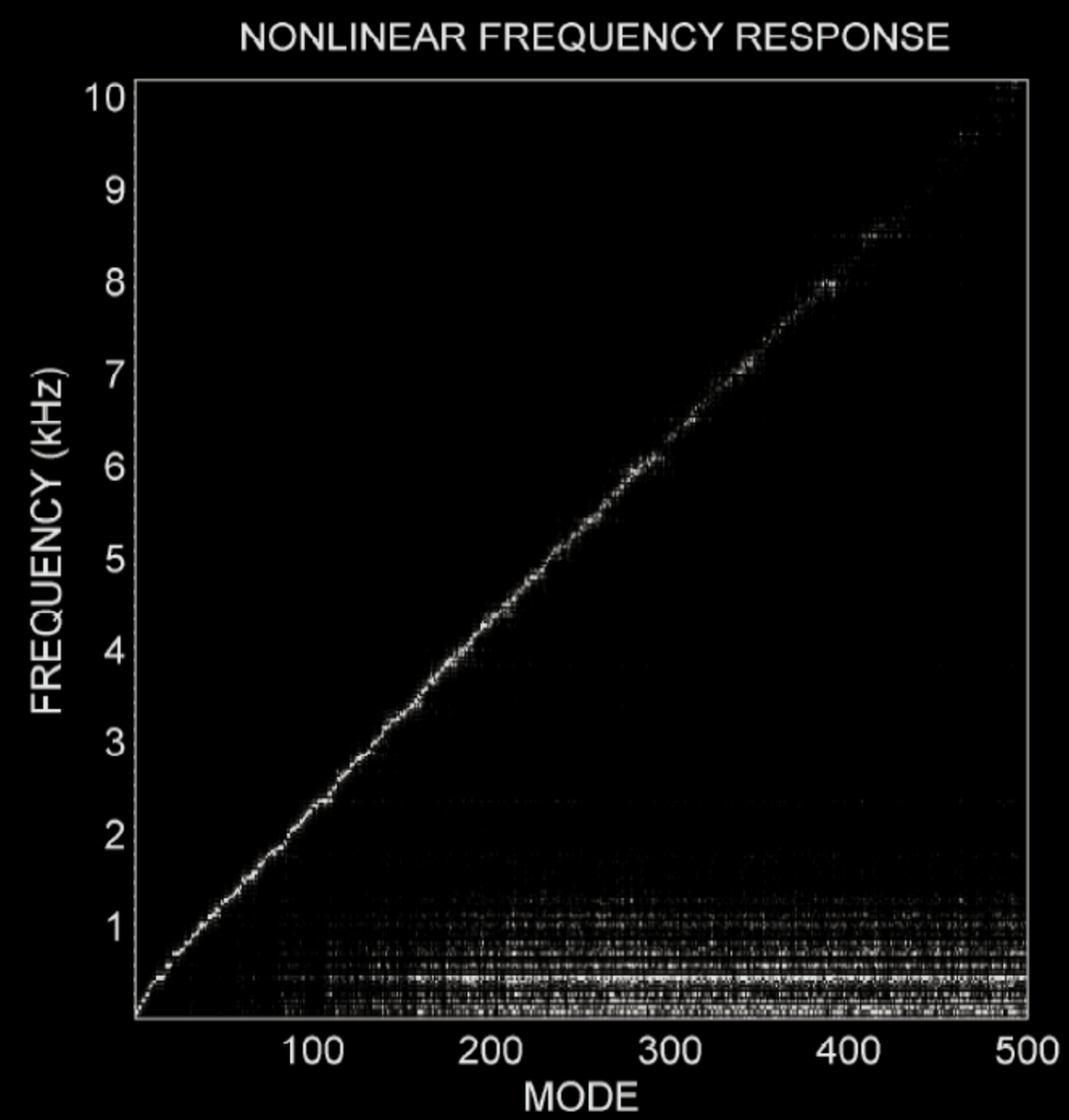




Nonlinear mode coupling



Linear Response



Nonlinear Response

Thin-Shell Dynamics

- Thin-shell membrane + bending energy

$$W_m = \frac{Y\,h}{2(1-\nu^2)}\,\left[(1-\nu)\,\mathrm{tr}(\boldsymbol{\epsilon}_m^2) + \nu\,\mathrm{tr}(\boldsymbol{\epsilon}_m)^2)\right]$$

$$W_b = \frac{Y\,h^3}{24(1-\nu^2)}\,\left[(1-\nu)\,\mathrm{tr}(\boldsymbol{\epsilon}_b^2) + \nu\,\mathrm{tr}(\boldsymbol{\epsilon}_b)^2)\right]$$

- Internal energy + force

$$E(\mathbf{u}) = \int_S W(\mathbf{u}; X)\,dS_X = \sum_{i=1}^{N_\Delta} A_i W_i(\mathbf{u})$$

$$\mathbf{f}_{int} = \nabla_{\mathbf{u}} E(\mathbf{u}) = \sum_{i=1}^{N_\Delta} A_i\,\nabla_{\mathbf{u}} W_i(\mathbf{u})$$

Dynamics interlude

Dimension Model Reduction
(a.k.a. “subspace integration”)

Dimensional Model Reduction

[Bathe 1996; Krysl, Lall, Marsden 2001]

$$\mathbf{M}\ddot{\boldsymbol{u}} + \boldsymbol{f}^{int}(\boldsymbol{u}) = \boldsymbol{f}^{ext}$$

Substitute $\boldsymbol{u} = \mathbf{U}\mathbf{q}$

Project $\mathbf{U}^T$

$$\overbrace{\mathbf{U}^T\mathbf{M}\mathbf{U}}^{\text{identity}}\ddot{\mathbf{q}} + \mathbf{U}^T\boldsymbol{f}^{int}(\mathbf{U}\mathbf{q}) = \mathbf{U}^T\boldsymbol{f}^{ext}$$

$$\ddot{\mathbf{q}} + \mathbf{f}(\mathbf{q}) = \mathbf{f}_{ext}$$

Reduced Equations of Motion

Reduced Internal Forces: Linear Modal Analysis

$$\mathbf{f}(\mathbf{q}) = \mathbf{U}^T \boldsymbol{f}^{int}(\mathbf{U}\mathbf{q})$$

$$= \begin{pmatrix} \omega_1^2 \, q_1 \\ \omega_2^2 \, q_2 \\ \\ \vdots \\ \\ \omega_r^2 \, q_r \end{pmatrix}$$

Linear force terms.

r decoupled oscillators

*Fast O(r) time-stepping using **IIR filters***
[Hamming 1983;
van den Doel and Pai]

Reduced Internal Forces

[Bathe 1996; Krysl, Lall, Marsden 2001]

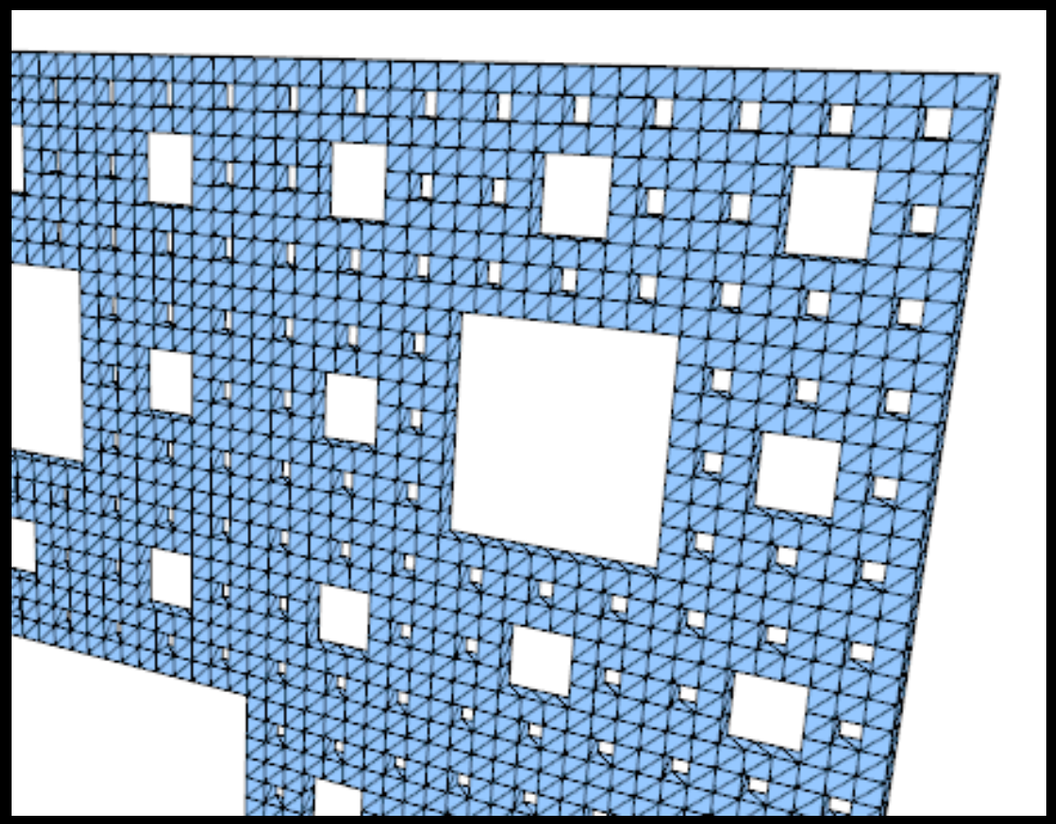
$$\mathbf{f}(\mathbf{q}) = \mathbf{U}^T \boldsymbol{f}^{int}(\mathbf{U}\mathbf{q})$$

Slow $O(rN)$ computation

(but less stiff, so larger explicit time-steps)

Reduced Internal Forces

[Bathe 1996; Krysl, Lall, Marsden 2001]



$O(rN)$

$\mathbf{U}$

$\mathbf{U}^T$

$\mathbf{q} \longrightarrow \mathbf{f}$

Optimizing Cubature

for Efficient Integration of Subspace Deformations

(with Steven An and Ted Kim)

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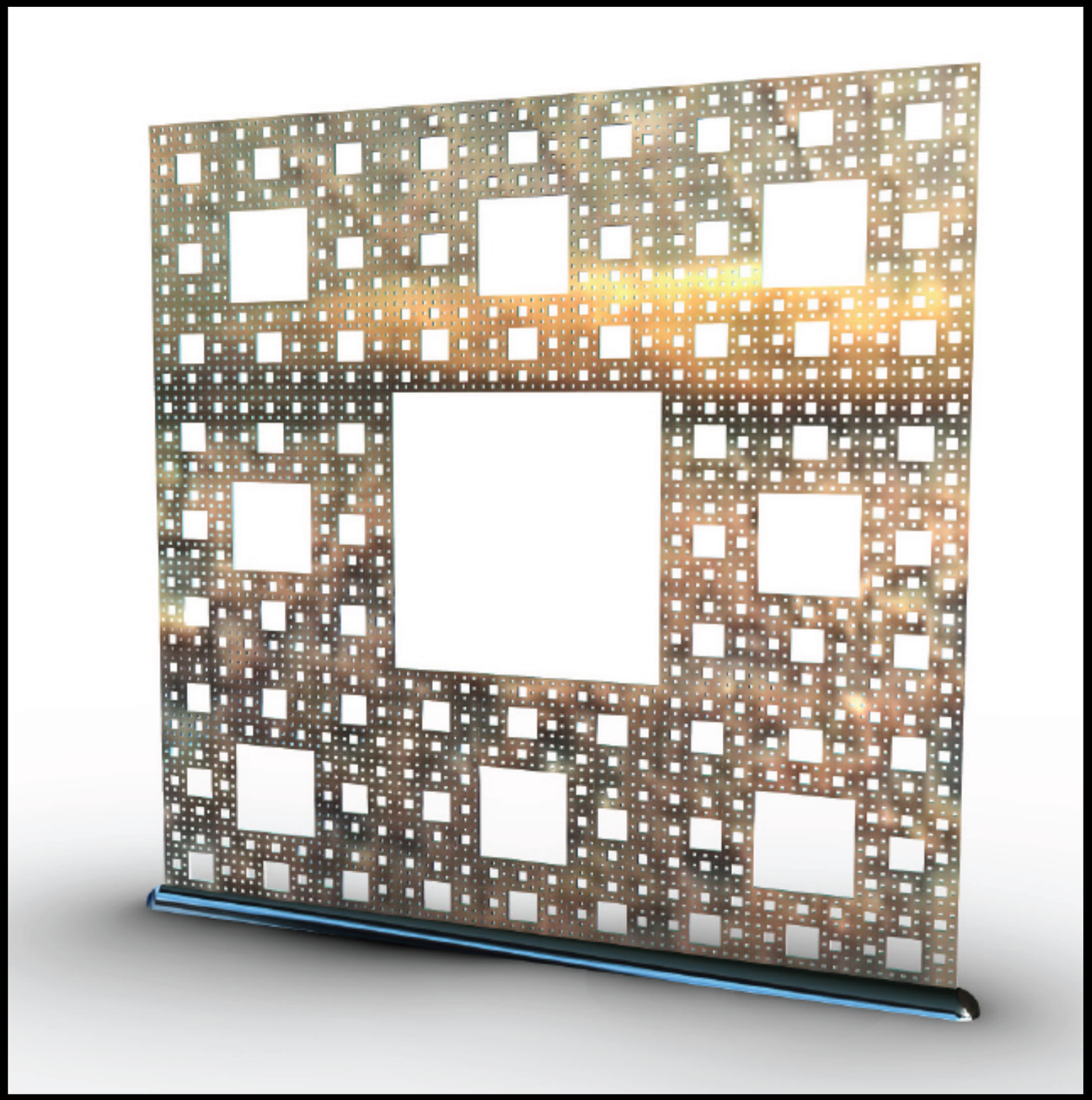
NEW HORIZONS

$$\mathbf{f}(\mathbf{q}) = -\nabla_{\mathbf{q}} E(\mathbf{q})$$

$$= \int_{\Omega} \mathbf{g}(X;\mathbf{q})\ d\Omega_X$$

$$\approx \sum_{i=1}^{n} w_i\ \mathbf{g}(X_i;\mathbf{q})$$



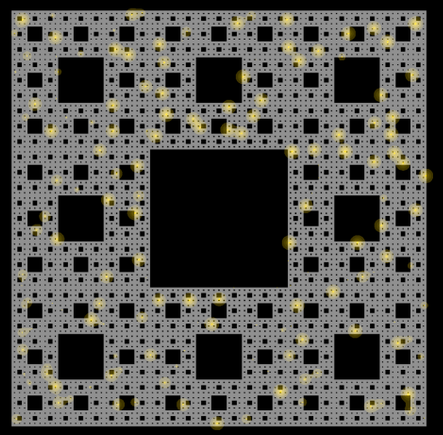
Optimizing Cubature

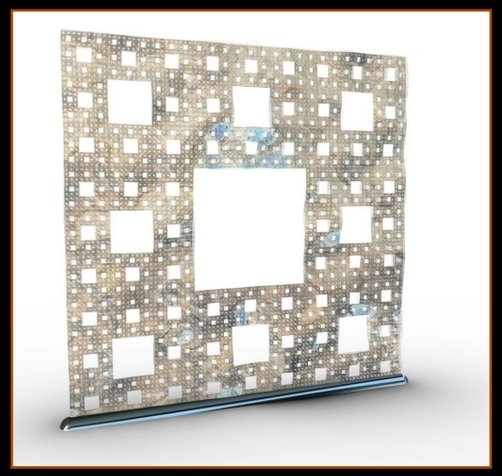
SIGGRAPHASIA2008
NEW HORIZONS

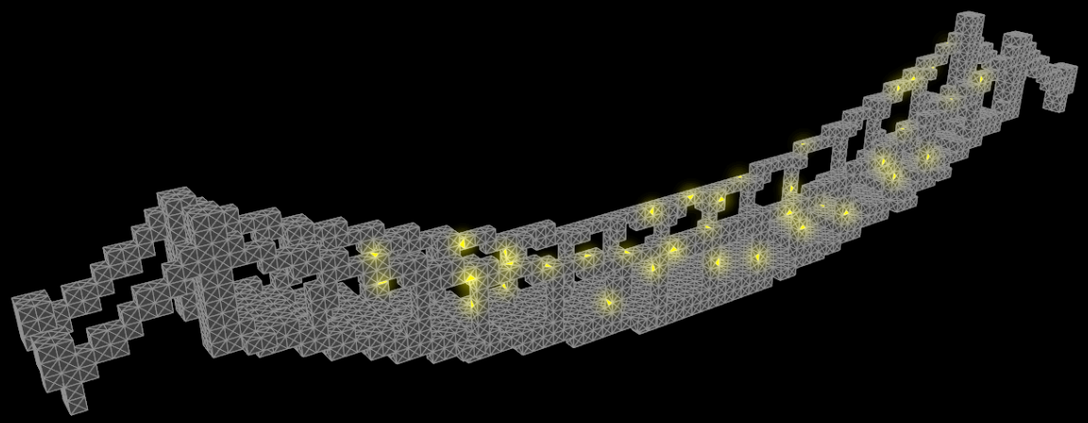
for Efficient Integration of Subspace Deformations

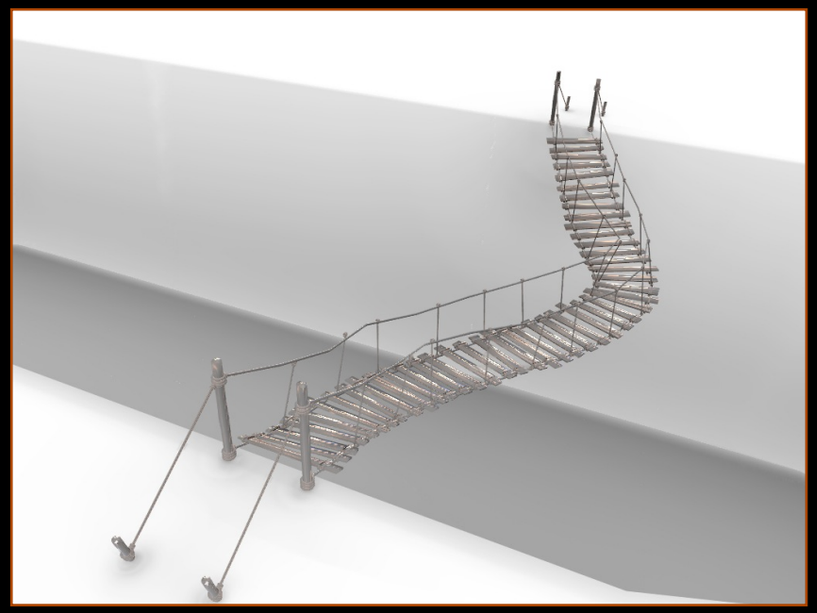
(with Steven An and Ted Kim)

- Fast, non-linear subspace forces
- Complex geometry
- Non-linear hyperelastic materials
- Scalable training pre-process:
- INPUT: Subspace model, training poses
- OUTPUT: Fast $O(r^2)$ subspace force model









Thin-Shell Cubature

- Subspace dynamics

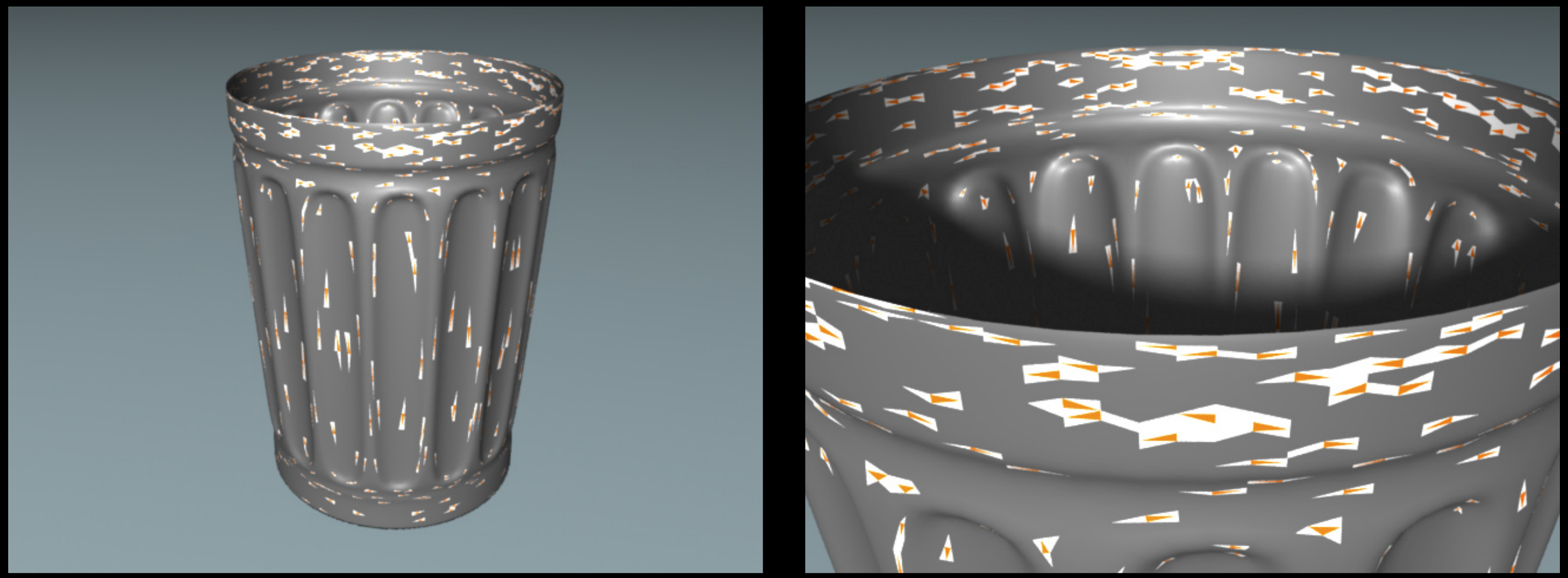
$$\ddot{\mathbf{q}} + \widetilde{\mathbf{D}}\dot{\mathbf{q}} + \widetilde{\mathbf{f}}_{int}(\mathbf{q}) = \widetilde{\mathbf{f}}_{ext}$$

- Subspace forces

$$\widetilde{\mathbf{f}}_{int}(\mathbf{q}) = \mathbf{U}^T \mathbf{f}_{int}(\mathbf{U}\mathbf{q}) = \sum_{i=1}^{N_\Delta} A_i \, \mathbf{g}_i(\mathbf{q})$$

$$\approx \sum_{i \in \mathcal{C}} w_i \, \mathbf{g}_i(\mathbf{q})$$

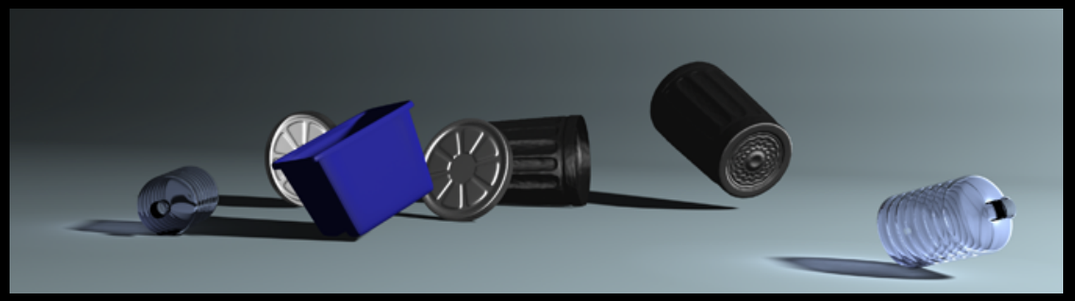
Thin-Shell Cubature



Trash can (200 modes) with 800-feature cubature scheme

Results

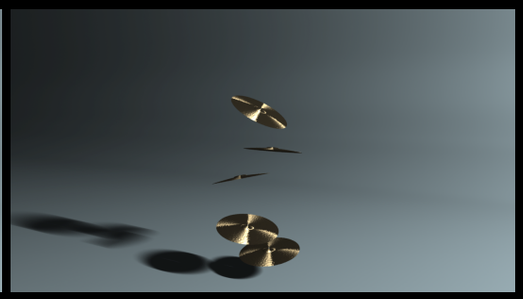
Model Statistics

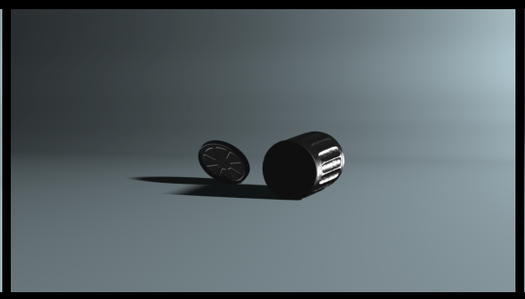
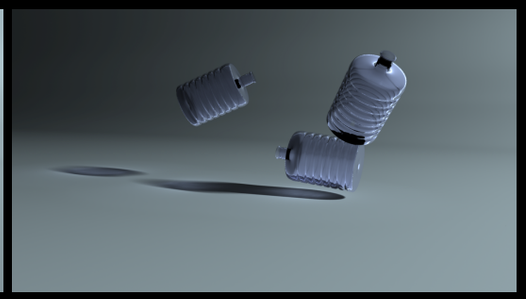


Model	L (m)	tri	vtx	N	modes	freq (kHz)	material	ν	Y (GPa)	h (mm)	α	β (10^{-9})	n_{cuba}	Error$_{cuba}$	kL	Δt (s)
Trash Can	0.75	77536	38833	116499	200	0.071 – 4.43	Steel	0.30	190	2	0.5	75	800	10.3%	0.98 – 61	1/44100
Trash Lid	0.55	34312	17286	51858	200	0.112 – 6.79	Steel	0.30	190	2	0.5	75	800	11.5%	1.1 – 68	1/44100
Water Bottle	0.46	28658	14418	43254	300	0.116 – 3.59	Polycarb.	0.37	2.4	2.25	0.5	400	900	10.7%	0.98 – 48	1/44100
Recycling Bin	0.61	109568	54945	164835	300	0.062 – 2.21	Polycarb.	0.37	2.4	5	4.0	300	1200	15.7%	0.70 – 30	1/44100
Cymbal	0.50	61952	31104	93312	500	0.061 – 9.94	Bronze	0.33	124	0.7	1.0	6.25	1500	10.7%	0.57 – 92	1/88200

Model	Modes r	Modal Analysis	Cubature Precomp.	Timestep Cost	Simulation Cost (per second of audio)	FFAT Precomp. (average time/mode)	FFAT Eval (all modes, $M=4$)	FFAT Storage (floats, $M=1$)
Trash Can	200	569 s	2.49 hr	16.1 ms	714 s	109.2 min	0.151 ms	56 MB
Trash Lid	200	170 s	1.87 hr	14.6 ms	642 s	85.5 min	0.151 ms	113 MB
Water Bottle	300	314 s	4.31 hr	23.6 ms	1026 s	25.6 min	0.227 ms	54 MB
Recycling Bin	300	2332 s	9.65 hr	27.8 ms	1224 s	48.0 min	0.227 ms	25 MB
Cymbal	500	1155 s	3.88 hr	44.3 ms	3900 s	318 min	0.378 ms	512 MB

Table 2: Representative Timings: *All timings are for a single 2.66GHz Xeon X5355 processor core, except "Cubature Precomp" which used 8 cores.*

Harmonic Shells

A Practical Nonlinear Sound Model for Near-Rigid Thin Shells

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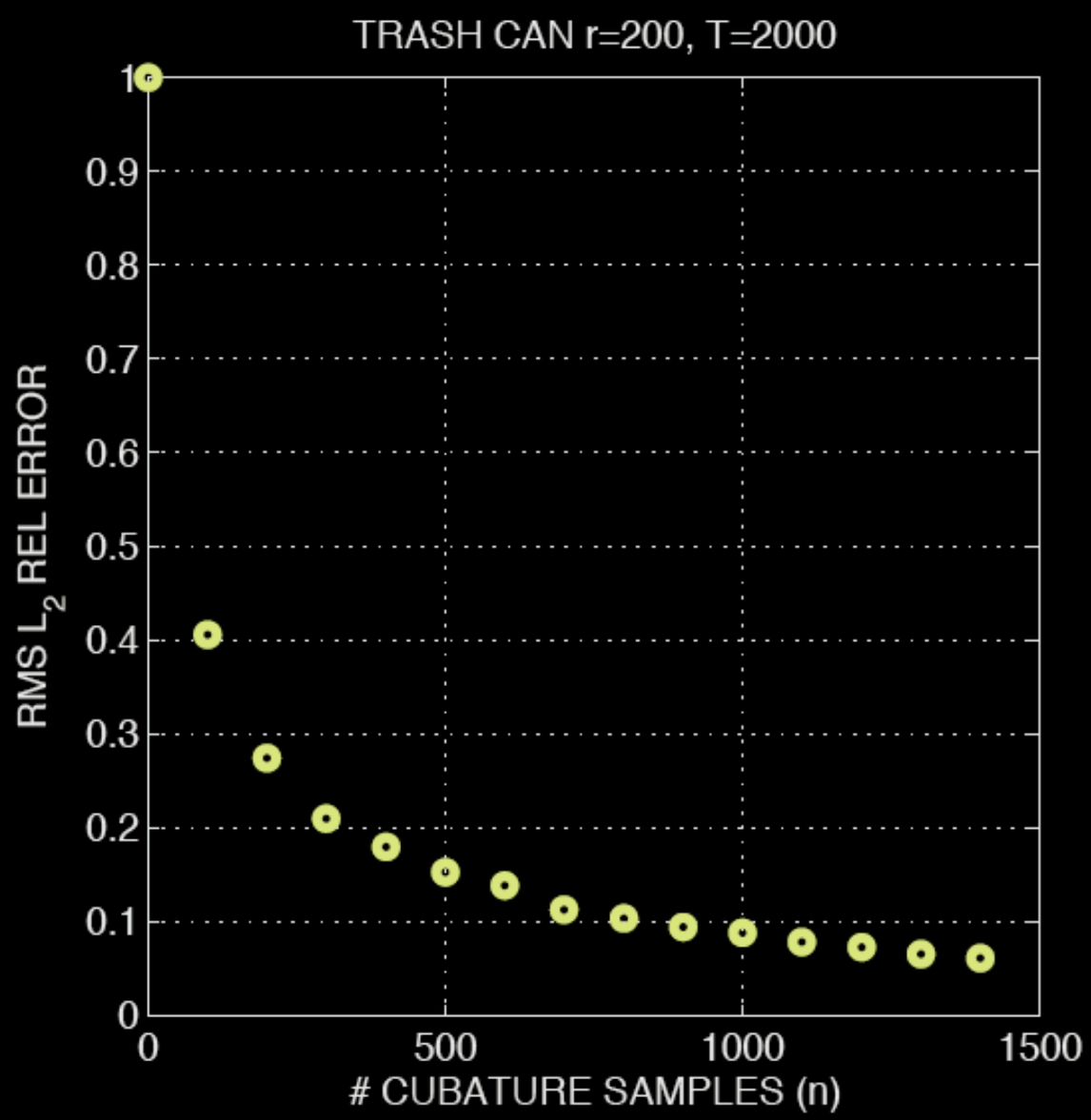
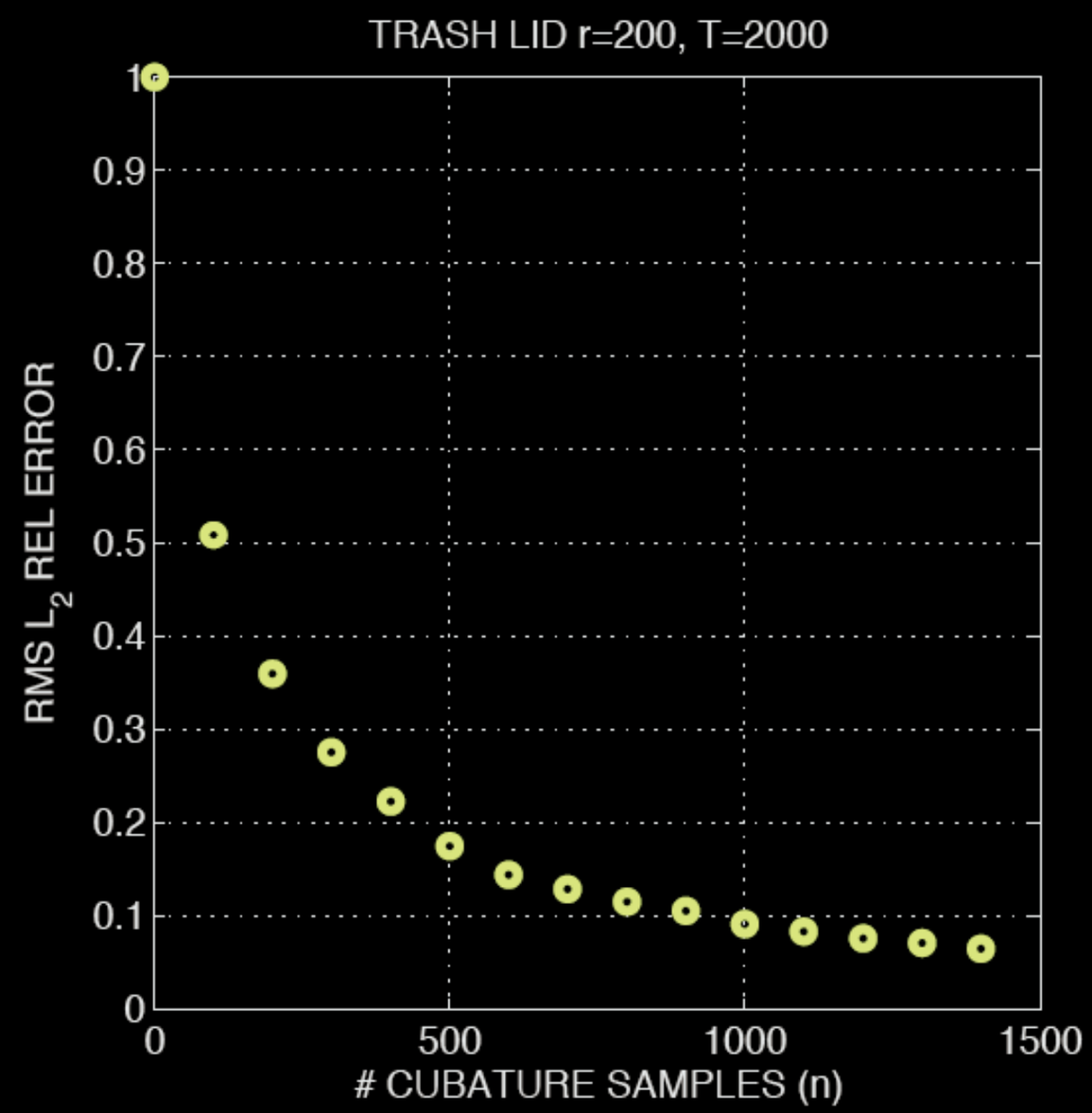
SIGGRAPHASIA2009

Thin-shell Cubature

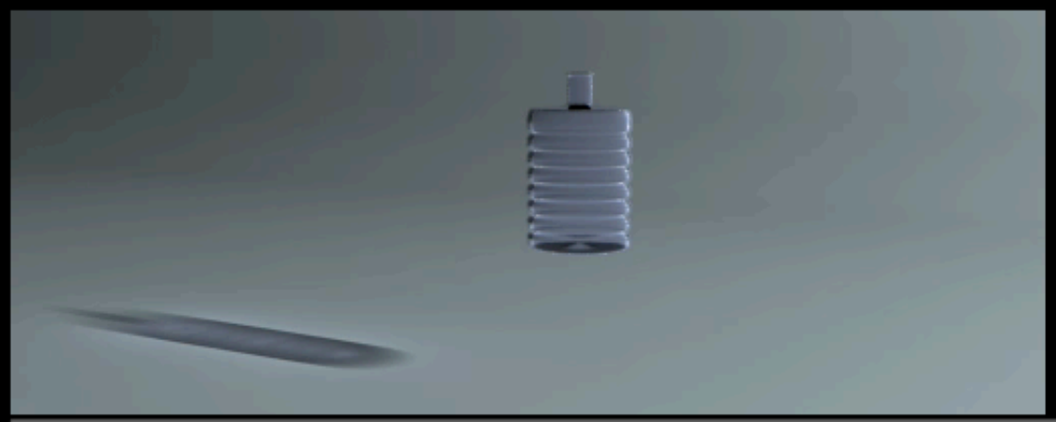




Cubature Error Comparison

0% Error	6% Error	10% Error	15% Error
167ms / timestep	**28ms / timestep**	**16ms / timestep**	**10ms / timestep**
	6.1x Speed-up	**10x Speed-up**	**17x Speed-up**
Traditional subspace integration [Krysl et al. 2001]		(shown previously)	

Comparison to unreduced dynamics



Unreduced Dynamics	Reduced Dynamics
(Explicit Newmark)	(11% cubature error)
~90 hrs / second of simulated sound	17 min / second of simulated sound
(This 5 sec clip took ~19 days to compute.)	10 sec clip 441,000 time steps in 2.8 hours

Comparison to Other Methods

1. **Nonlinear dynamics + Transfer**
   i.e., “Harmonic Shells”

2. **Linear dynamics + Transfer**
   e.g., [James et al. 2006]

3. **Linear dynamics + Monopole**
   e.g., [O'Brien et al. 2002; Boneel et al. 2008]

4. **Nonlinear dynamics + Monopole**