Analysis of Planar Light Fields from Homogeneous Convex Curved Surfaces Under Distant Illumination

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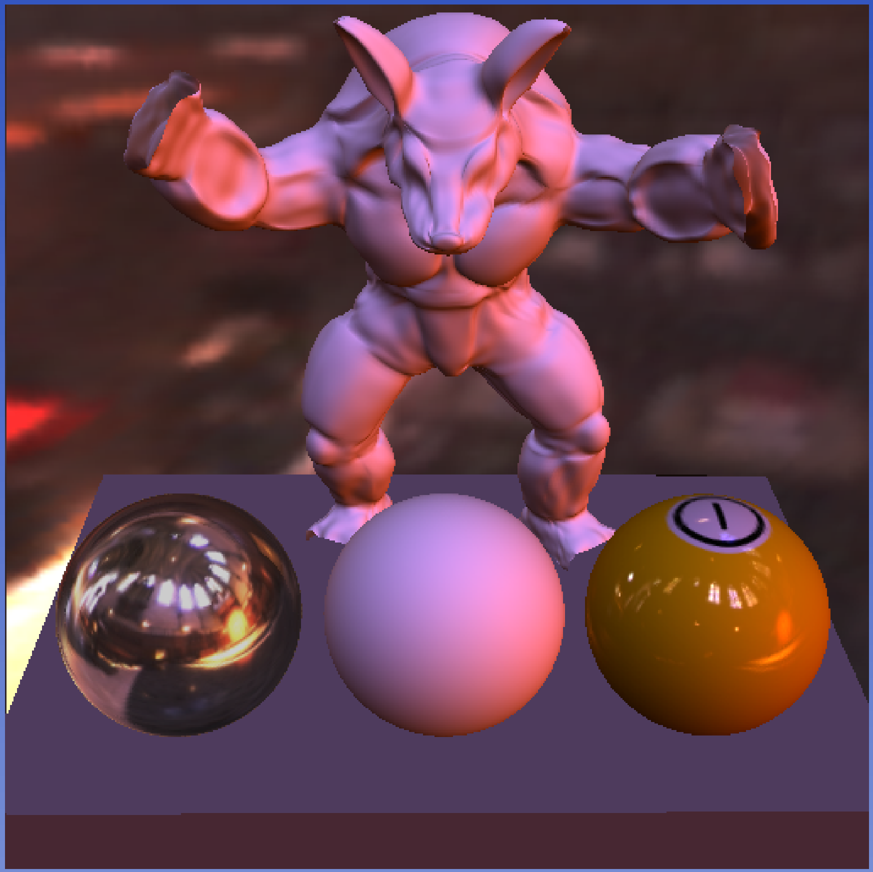
http://graphics.stanford.edu/papers/planarlf/

Motivation

Forward Rendering (Computer Graphics)

- Complex Lighting (Environment Maps)

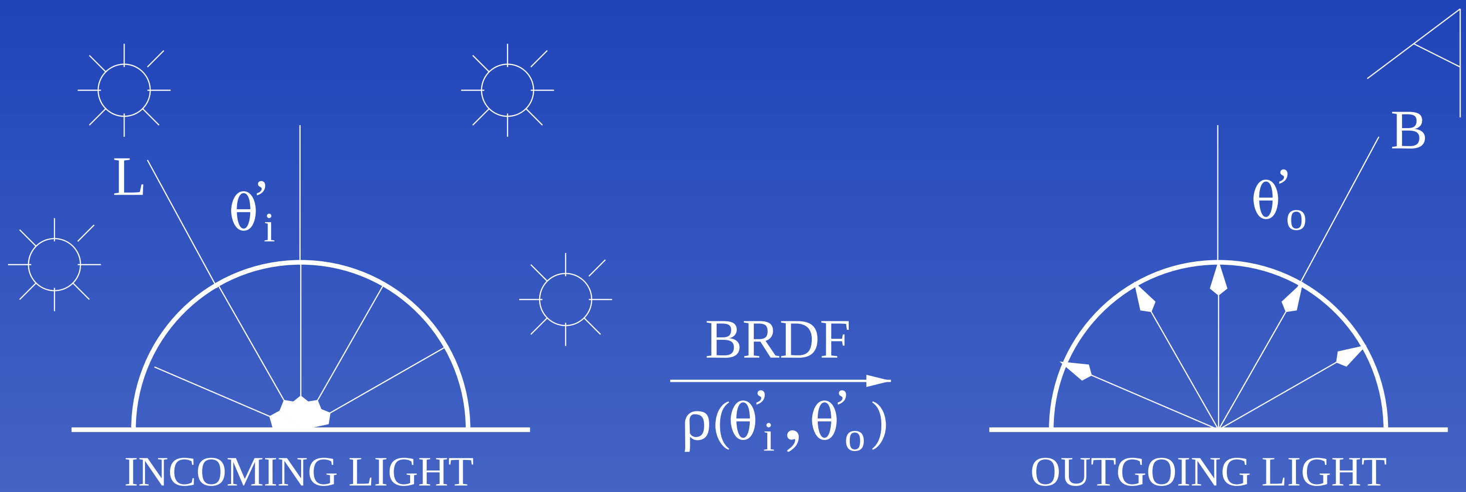


Motivation

Inverse Rendering (Computer Vision and Graphics)

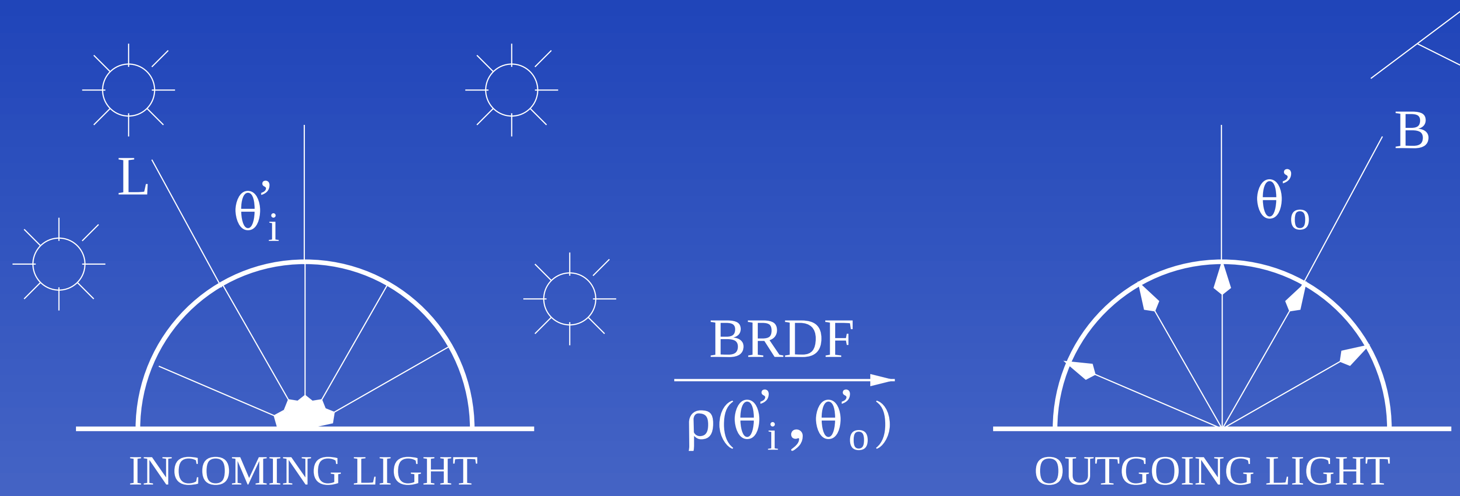
- *Estimate* BRDF, Lighting, *both* BRDF and Lighting
  - ⋆ Theoretically Possible?
  - ⋆ Practically Feasible?

Reflection Equation



$$B(\mathbf{x}, \theta'_o) \;=\; \int_{-\pi/2}^{\pi/2} L(\mathbf{x}, \theta'_i)\rho(\theta'_i, \theta'_o)\cos(\theta'_i)\, d\theta'_i$$

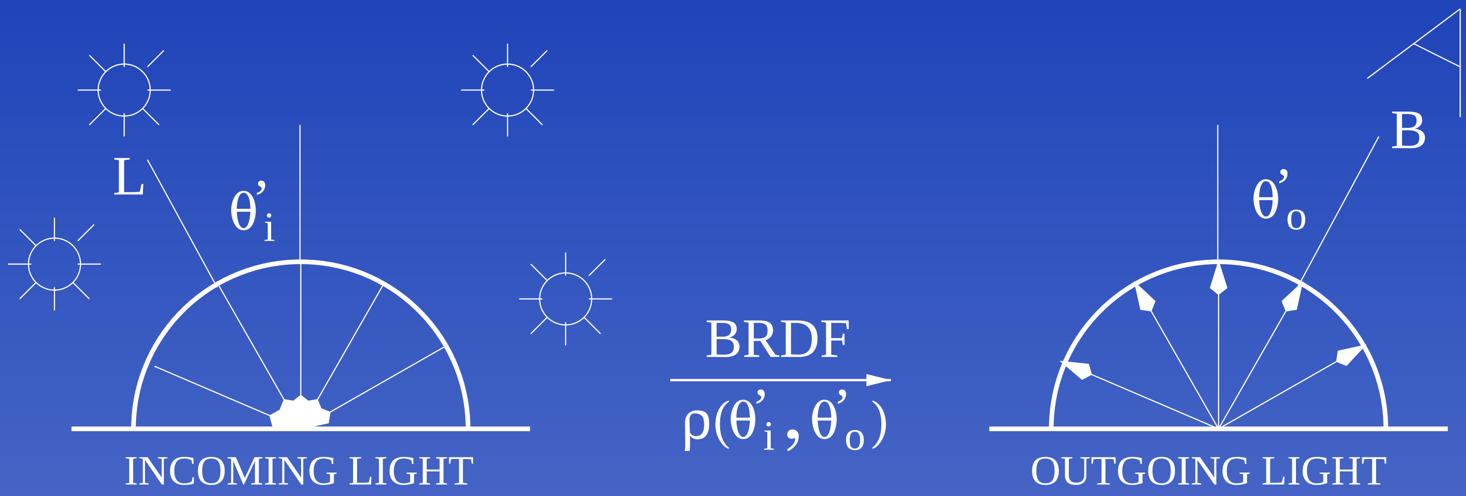
Reflection Equation



$$B(\mathbf{x}, \theta'_o) \;=\; \int_{-\pi/2}^{\pi/2} L(\mathbf{x}, \theta'_i)\rho(\theta'_i, \theta'_o)\cos(\theta'_i)\, d\theta'_i$$

$$\hat{\rho}(\theta'_i, \theta'_o) \;=\; \rho(\theta'_i, \theta'_o)\cos(\theta'_i)$$

Reflection Equation



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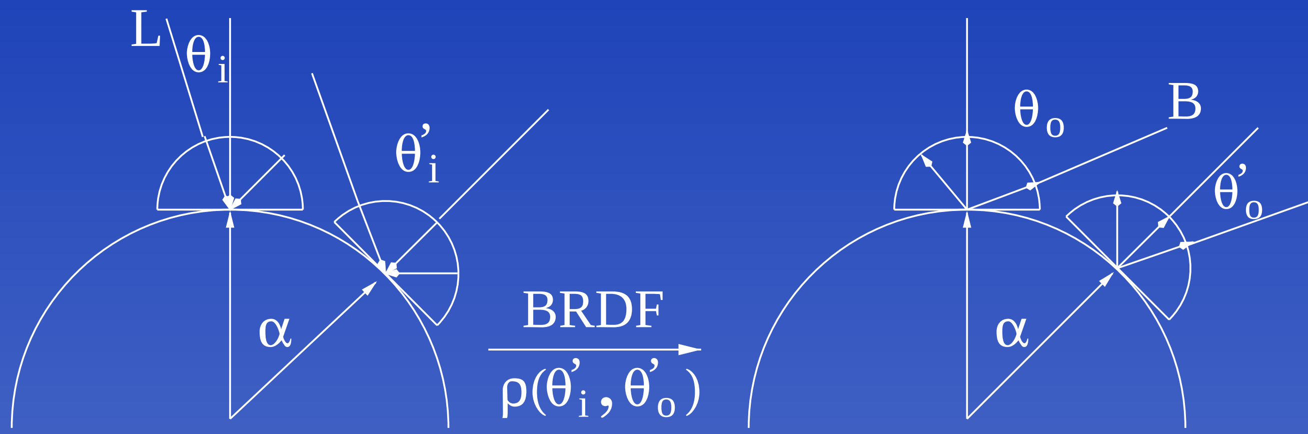
$$B(\mathbf{x}, \theta'_o) = \int_{-\pi/2}^{\pi/2} L(\mathbf{x}, \theta'_i)\hat{\rho}(\theta'_i, \theta'_o)\, d\theta'_i$$

Approach: Reflection is Convolution



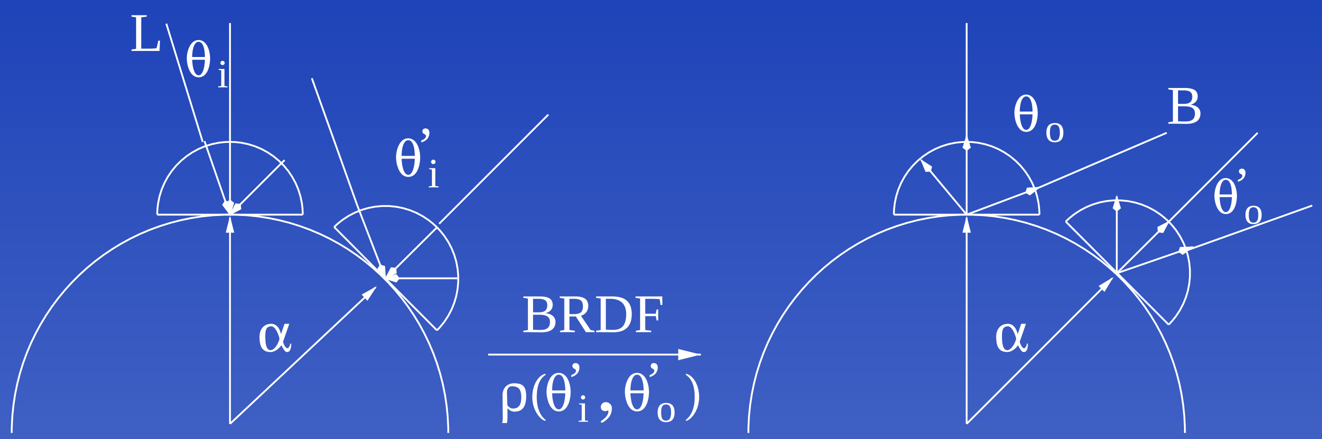
$$B(\alpha, \theta'_o) \;=\; \int_{-\pi/2}^{\pi/2} L(\alpha, \theta'_i)\hat{\rho}(\theta'_i, \theta'_o)\, d\theta'_i$$

Approach: Reflection is Convolution

$$B(\alpha, \theta_o') \;=\; \int_{-\pi/2}^{\pi/2} L(\alpha, \theta_i') \hat{\rho}(\theta_i', \theta_o') \, d\theta_i'$$

$$L(\alpha, \theta_i') = L(\theta_i) \;=\; L(\alpha + \theta_i')$$

Approach: Reflection is Convolution



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CONVOLUTION : $B = L \otimes \hat{\rho}$

Related Work

Graphics: Prefiltering Environment Maps

- Qualitative Observation that Reflection is Convolution
- Miller & Hoffman 84, Greene 86
- Cabral Max Springmeyer 87, Cabral Olano Nemec 99

Vision, Perception

- D'Zmura 91: Reflection as Operator in Frequency Space
- Basri & Jacobs: Lambertian Reflection as Convolution

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Our Contribution: Formal Analysis in General 2D case

- Key insights extend to 3D (more recent work)

Fourier Analysis

$$L(\theta_i) = \sum_p L_p e^{Ip\theta_i}$$

$$\hat{\rho}(\theta_i', \theta_o') = \sum_p \sum_q \hat{\rho}_{p,q} e^{Ip\theta_i'} e^{Iq\theta_o'}$$

$$B(\alpha, \theta_o') = \sum_p \sum_q B_{p,q} e^{Ip\alpha} e^{Iq\theta_o'}$$

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Note: Can fix output direction: $\boxed{B_p(\theta_o') = 2\pi L_p \hat{\rho}_{-p}(\theta_o')}$

Insights

Reflected Light Field is *Convolution* of Lighting, BRDF

Convolution Theorem $\Rightarrow$ Product of Fourier Coefficients

Signal Processing: Filter Lighting using BRDF Filter

$$\begin{aligned} \text{Lighting} &\leftrightarrow \text{Input Signal} \\ \text{BRDF} &\leftrightarrow \text{Filter} \end{aligned}$$

Inverse Rendering is *Deconvolution*

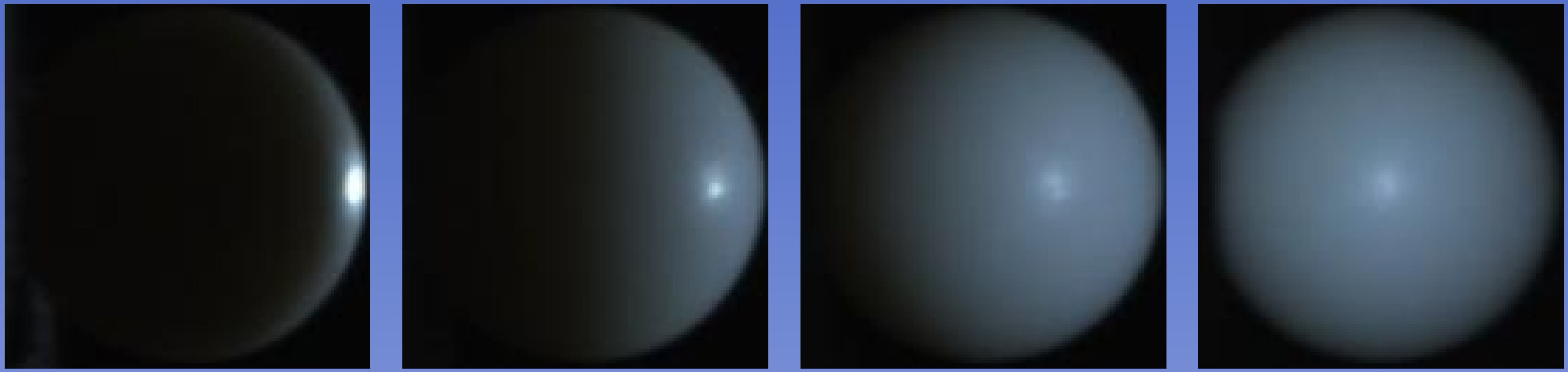
Example: Directional Source at $\theta_i = 0$

$$L(\theta_i) \;=\; \delta(\theta_i) \qquad L_p = \frac{1}{2\pi}$$

$$\boxed{B_{p,q} = \hat{\rho}_{-p,q}}$$

Reflected Light Field corresponds directly to BRDF

- *Impulse Response* of BRDF filter

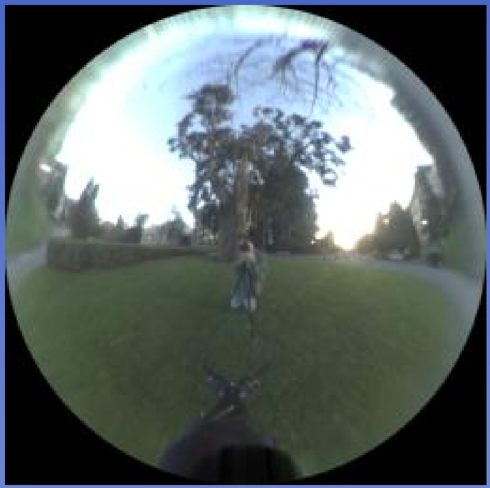


Example: Mirror BRDF

$$\hat{\rho}(\theta_i', \theta_o') = \delta(\theta_i' + \theta_o') \qquad \hat{\rho}_{p,q} = \frac{\delta_{p,q}}{2\pi}$$

$$\boxed{B_{p,q} = \delta_{p,q} L_{-p}}$$

Reflected Light Field corresponds directly to Lighting



Gazing Sphere

Example: Lambertian BRDF

Transfer function is *Clamped Cosine*

No output dependence, drop index q

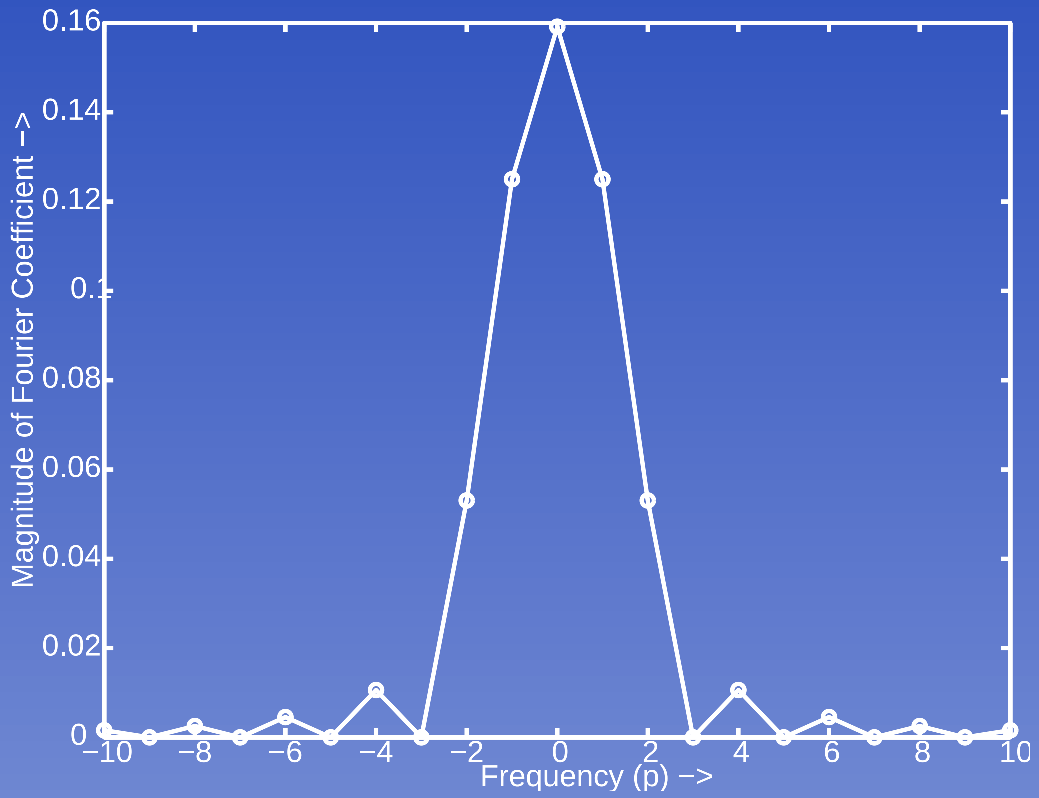
$$B_p = 2\pi L_p \hat{\rho}_{-p}$$

Lambertian BRDF is *Low-Pass* filter

Incident Reflected

Properties: Lambertian BRDF Filter

$$\hat{\rho}_{2p} = \frac{(-1)^{(p+1)}}{2\pi\left(4p^2-1\right)}$$



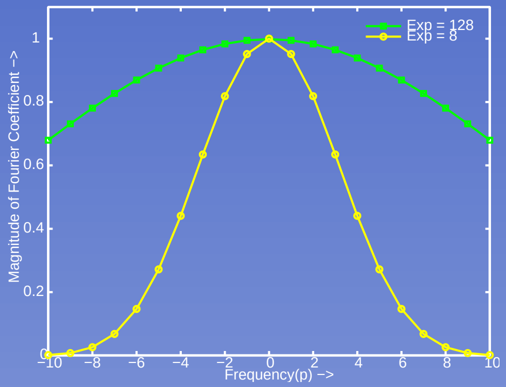
Good approximation using only terms with $p \leq 2$

Phong, Microfacet BRDFs

Rough surfaces blur highlights

Microfacet BRDF is Gaussian

- Hence, Fourier Spectrum also Gaussian
- Similar results for Phong (analytic formulae in paper)



Inverse Rendering

BRDF \ Lighting	Known	Unknown
Known	X	Miller & Hoffman 84 D'Zmura 91 Marschner&Greenberg97
Unknown	Sato et al. 97, Yu et al. 99, Dana et al. 99 Debevec et al. 00, Marschner et al. 00, ...	? Sato et al. 99 (shadows)

Often estimate *Textured* BRDFs (3rd axis of table)

Inverse Rendering

General Complex Illumination?

- Most inverse-BRDF methods use point source
- Outdoor methods: Sato&Ikeuchi94, Yu&Malik98

Well-Posedness, Conditioning?

- Well Posed if unique solution
- Well Conditioned if robust to noisy data

Factorization of BRDF,Lighting (find both)?

- Sato et al. 99 use shadows

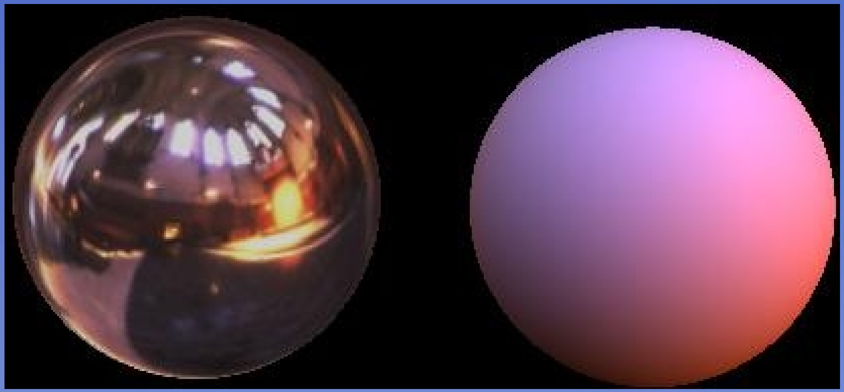
Inverse Lighting

$$L_p = \frac{1}{2\pi}\frac{B_{p,q}}{\hat{\rho}_{-p,q}}$$

Well posed unless $\hat{\rho}_{-p,q}$ vanishes for all q for some p.

Well conditioned when Fourier spectrum decays slowly.

- Need high frequencies in BRDF (sharp specularities)
- Ill-conditioned for diffuse BRDFs (low-pass filter)



Mirror Lambertian

BRDF estimation

$$\hat{\rho}_{p,q} = \frac{1}{2\pi}\frac{B_{-p,q}}{L_{-p}}$$

Well Posed if all terms in Fourier expansion L_{-p} nonzero.
Well Conditioned when Fourier expansion decays slowly.

- Need high frequencies in lighting (sharp features)
- Ill-conditioned for soft lighting (low-frequency)

Directional Source   Area Source (same BRDF)

Light Field Factorization

Up to a global scale, Light Field can be factored

- Can *simultaneously* estimate Lighting, BRDF

Number of Knowns (B) $>$ Number of Unknowns (L , ρ)

- $(B \rightarrow 2D) > (L \rightarrow 1D + \rho \rightarrow 1/2(2D))$

Explicit Formula in paper

3D

Fourier Series $\longrightarrow$ Spherical Harmonics $Y_{lm}(\theta, \phi)$

$\longrightarrow$ Representation Matrices of $SO(3)$ $D^l_{mm'}(\alpha, \beta)$

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L(\theta_i, \phi_i) &= \sum_{l,m} L_{lm} Y_{lm}(\theta_i, \phi_i) \\
\hat{\rho}(\theta_i', \phi_i', \theta_o', \phi_o') &= \sum_{l,p,q} \hat{\rho}_{lq,pq} Y_{lq}^*(\theta_i', \phi_i') Y_{pq}(\theta_o', \phi_o') \\
B(\alpha, \beta, \theta_o', \phi_o') &= \sum_{l,m,p,q} B_{lmpq} \left(D^l_{mq}(\alpha, \beta) Y_{pq}(\theta_o', \phi_o') \right)
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$$

2D: $B_{pq} = 2\pi L_p \hat{\rho}_{-p,q}$

3D: $B_{lmpq} = L_{lm} \hat{\rho}_{lq,pq}$

Implications

Lambertian BRDF

- 2D: Only first 2 Fourier coefficients important
- 3D: First 2 orders of spherical harmonics → 99% energy
  - ⋆ *Only the first 9 coefficients are important*
- Similar results independently derived by Basri & Jacobs
- Formally, recovery of radiance from irradiance ill-posed
  - ⋆ See *On the relationship between Radiance and Irradiance: Determining the illumination from images of a convex Lambertian object (submitted)*

Phong & Microfacet BRDFs

- Gaussian Filters. Results similar to 2D

Practical Issues (in 3D)

Frequency spectra from Incomplete Irregular Data

Concavities: Self-Shadowing and Interreflection

Textures: Spatially Varying BRDFs

Practical Issues (in 3D)

Frequency spectra from Incomplete Irregular Data

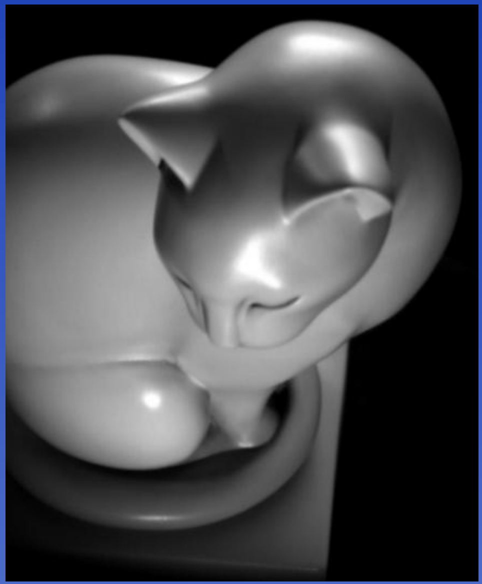
Concavities: Self-Shadowing and Interreflection

Textures: Spatially Varying BRDFs

Issues can be addressed; can derive practical algorithms

- Use Dual Angular and Frequency-space Representations
- Associativity of Convolution
- See *A Signal Processing Framework for Inverse Rendering (submitted)*

Experiment: Cat Sculpture



3 photographs of cat sculpture of known geometry

Microfacet BRDF under complex unknown lighting

Lighting also estimated

Then use recovered BRDF for new view, new lighting

Results: Cat Sculpture

Images below show new view, new lighting

REAL PHOTOGRAPH

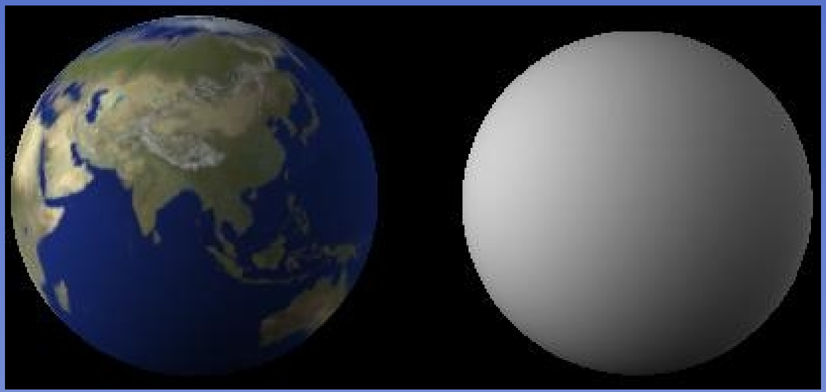
RENDERED IMAGE

Numerical values verified to within 5%

Implications for Perception

Assume Lambertian BRDF, no shadows

- Perception: Separate Reflectance, Illumination
- Low frequency $\leftrightarrow$ lighting, High frequency $\leftrightarrow$ texture
- Theory formally: lighting $\rightarrow$ only low-frequency effects
- Find high-frequency texture independent of lighting
- But ambiguity regarding low-frequency texture, lighting



Conclusion

Reflection as convolution

Fourier analysis gives many insights

Extends to 3D and results in practical inverse algorithms

Signal-Processing: A useful paradigm for Forward and Inverse Rendering