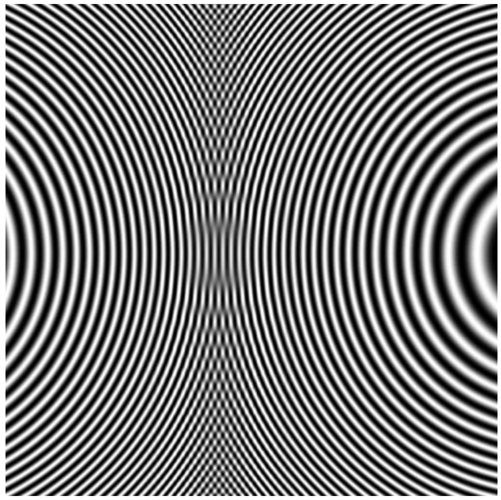
Sampling,
Aliasing, Antialiasing



Sampling

Sampling process

Aliasing

Nyquist frequency

Reconstruction process

The sampling theorem

Antialiasing

Signal Processing Review

Convolution of a "Spike"

Signal/Image

0	0	1	0	0	0	…

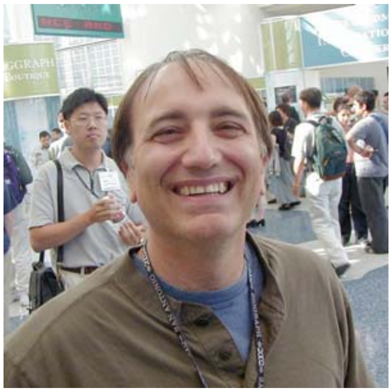
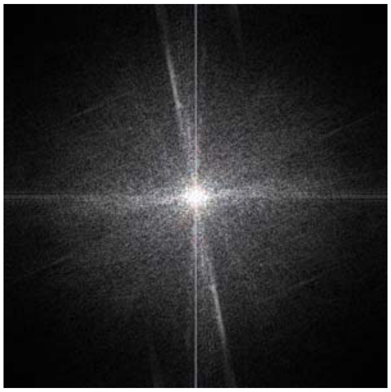
Filter

1	2	1

Result: copy of the filter centered at the spike

0	1	2	1	0	0	…

Pat’s Frequencies

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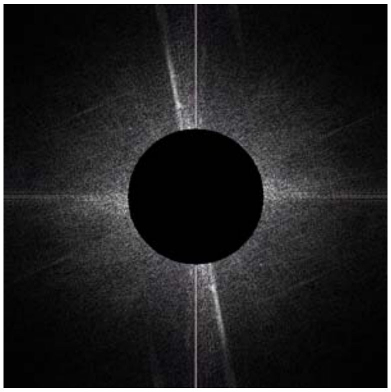
Pat’s Frequencies

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Convolution

***Convolution Theorem*: Multiplication in the frequency domain is equivalent to convolution in the space domain.**

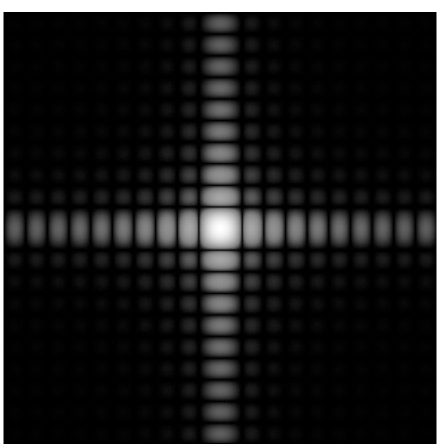
$$f \otimes g \leftrightarrow F \times G$$

***Symmetric Theorem*: Multiplication in the space domain is equivalent to convolution in the frequency domain.**

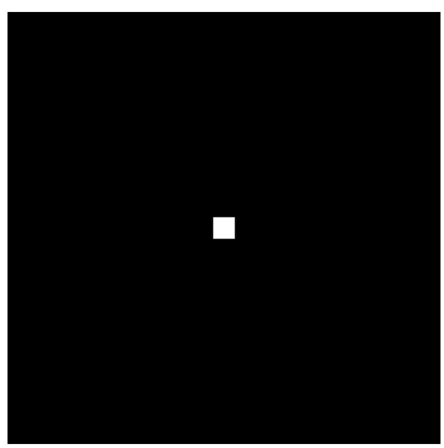
$$f \times g \leftrightarrow F \otimes G$$

Perfect Low-Pass = Sinc Convolution

Spatial Domain

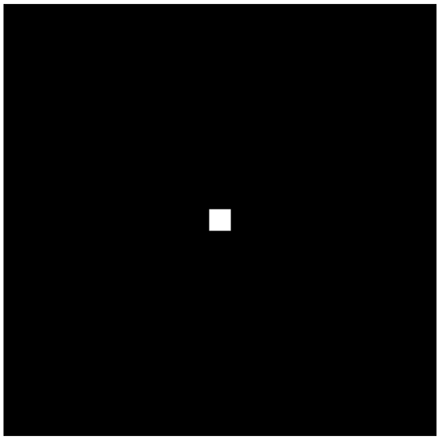


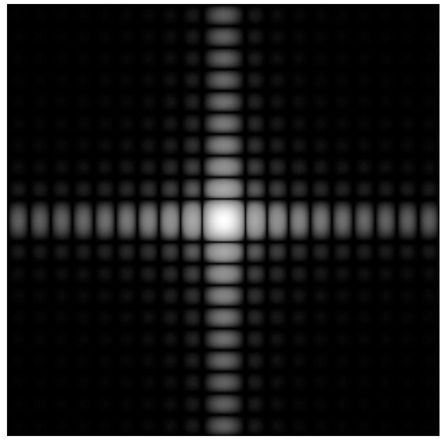
Frequency Domain



Box Convolution = Sinc-Pass Filter

Spatial Domain | **Frequency Domain**





Imagers = Signal Sampling

Imagers convert a continuous image to a discrete image.

Each sensor integrates light over its active area.

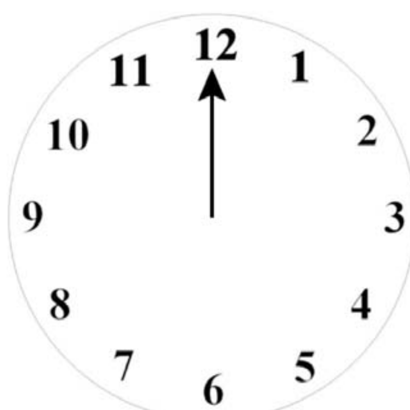
$$R(i,j) = \int_{A_{i,j}} E(x,y) P_{i,j}(x,y)\, dx\, dy$$

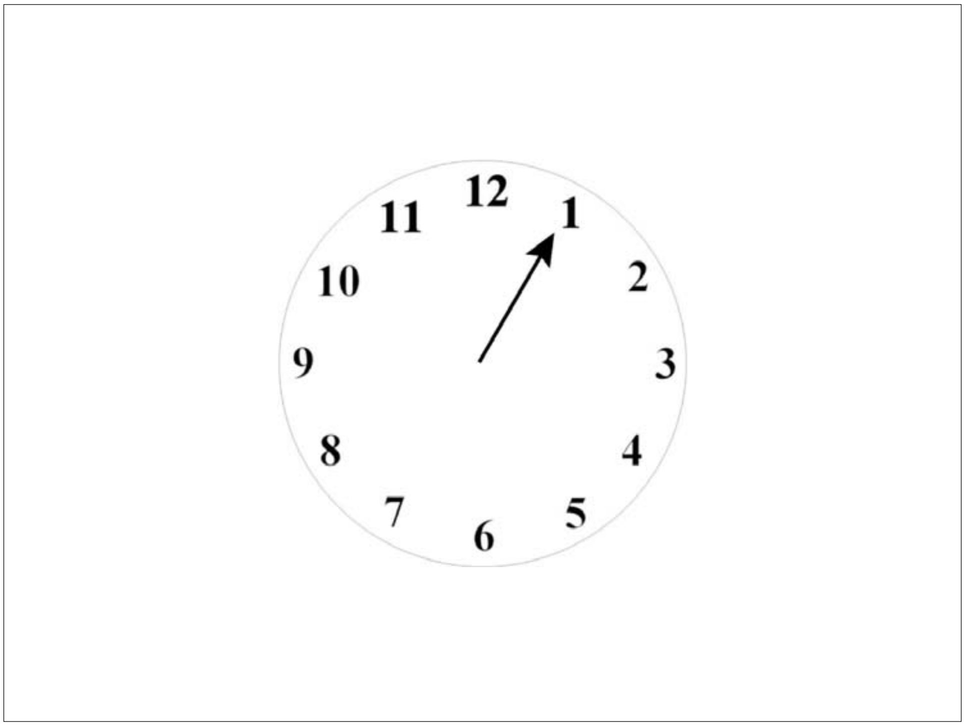
Examples:

- **Retina: photoreceptors**
- **Digital camera: CCD or CMOS array**
- **Vidicon: phosphors**

How should this be done in computer graphics?

Aliasing

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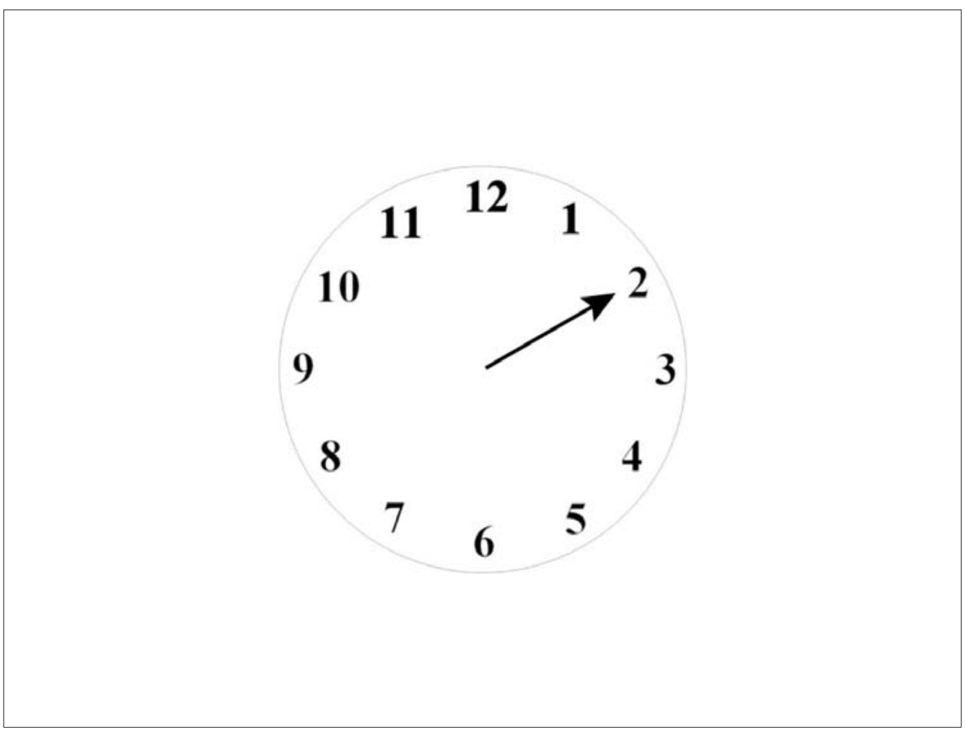
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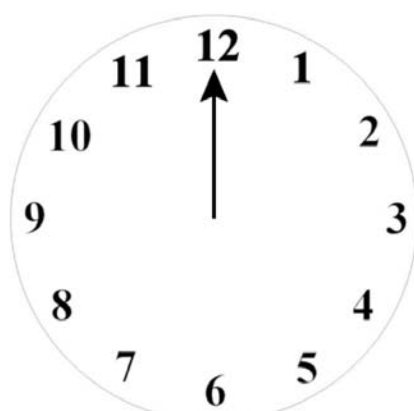
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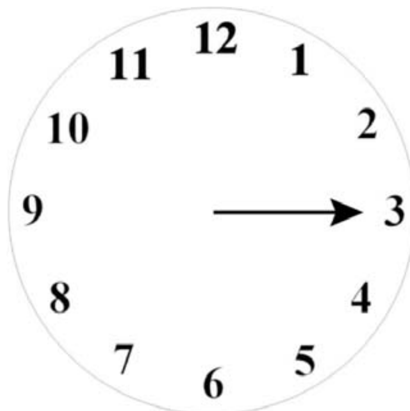
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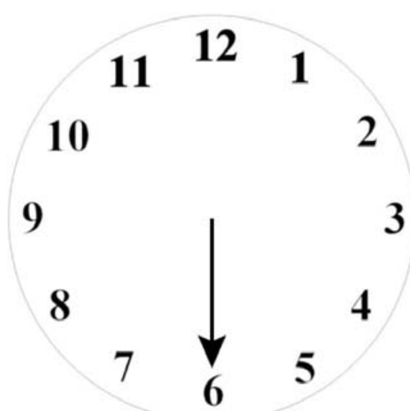
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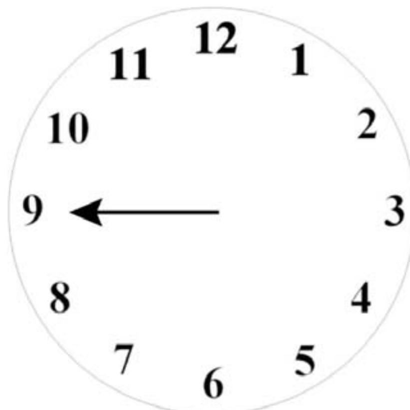
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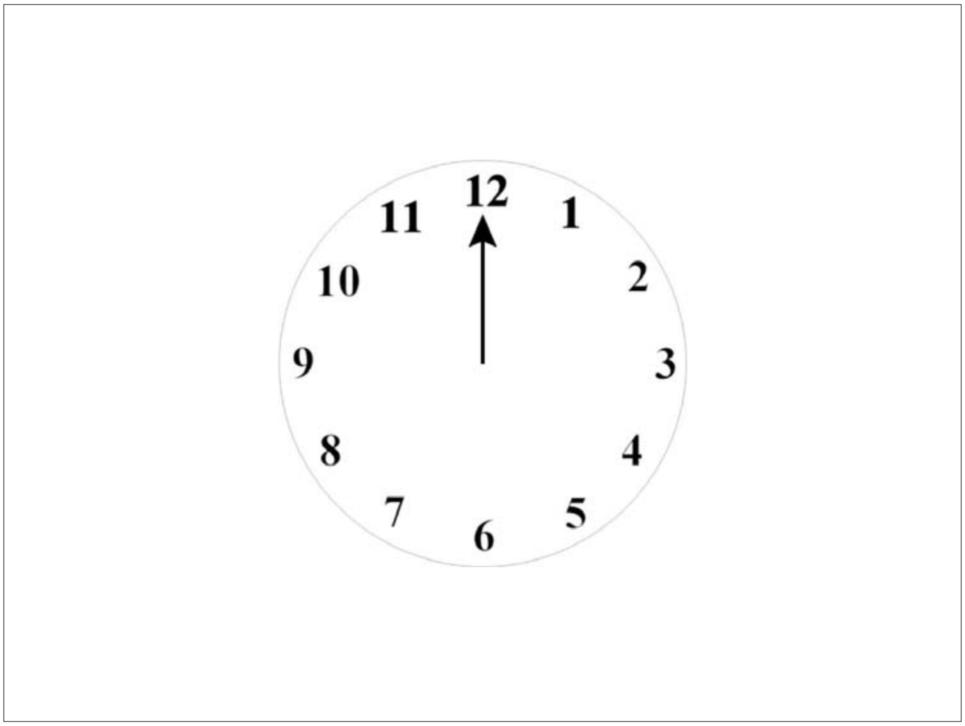
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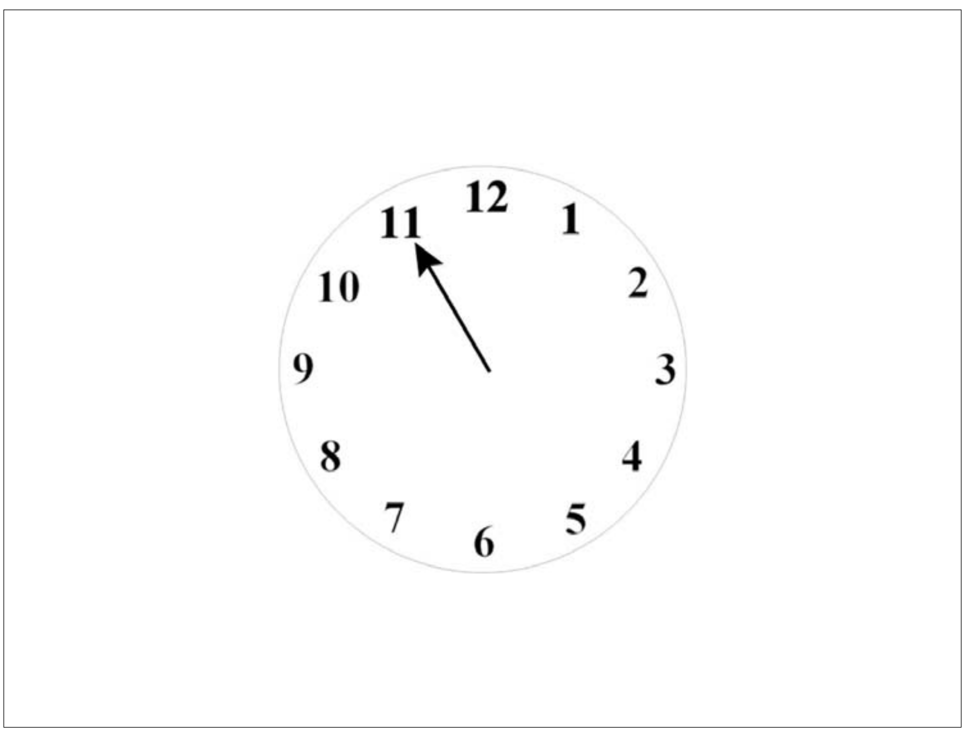
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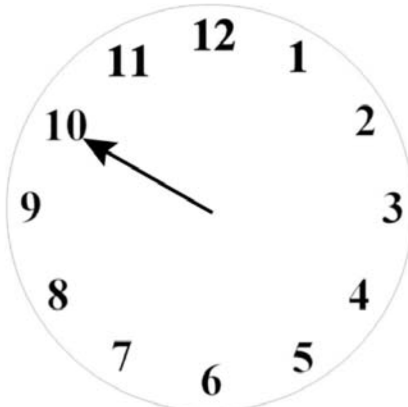
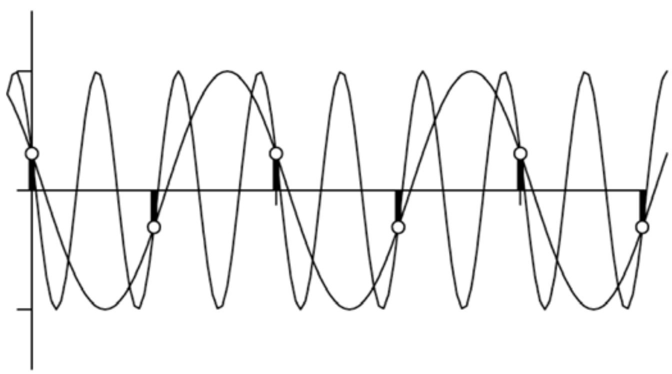
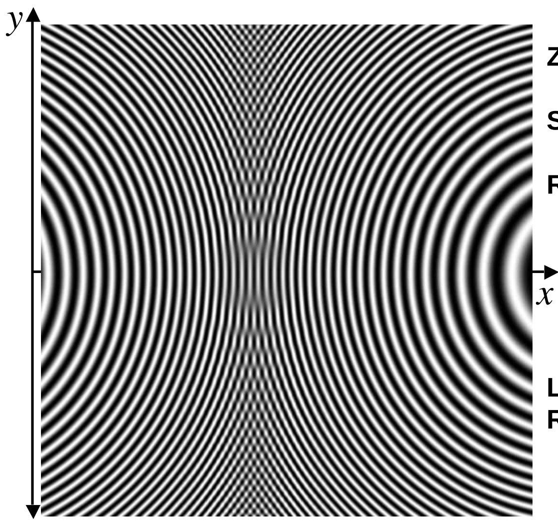


"Aliases"



Aliases [θ and (2π- θ)] are indistinguishable after sampling!

Sampling a “Zone Plate”



Zone plate: $\sin x^2 + y^2$

Sampled at 128x128

Reconstructed to 512x512
Using a 30-wide
Kaiser windowed sinc

Left rings: part of signal
Right rings: prealiasing

Nyquist Frequency

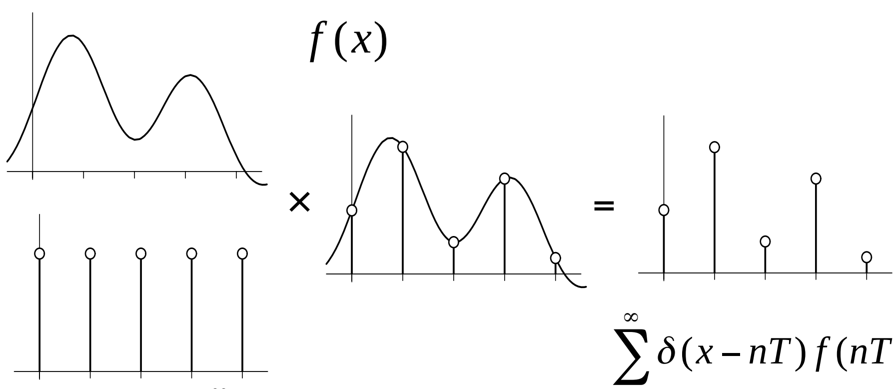
Definition: The Nyquist Frequency is 1/2 the sampling frequency

A periodic signal with a frequency above the Nyquist frequency cannot be differentiated from a periodic signal below the Nyquist frequency

The various indistinguishable signals are called aliases

Sampling

Sampling: Spatial Domain



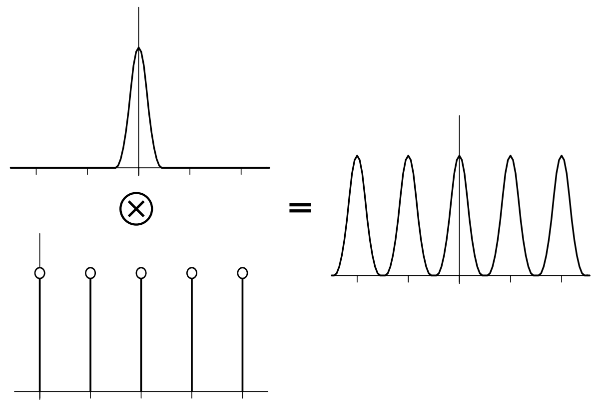
$$f(x)$$

$$\sum_{n=-\infty}^{\infty} \delta(x-nT) f(nT)$$

$$\mathrm{III}(x) = \sum_{n=-\infty}^{n=\infty} \delta(x-nT)$$

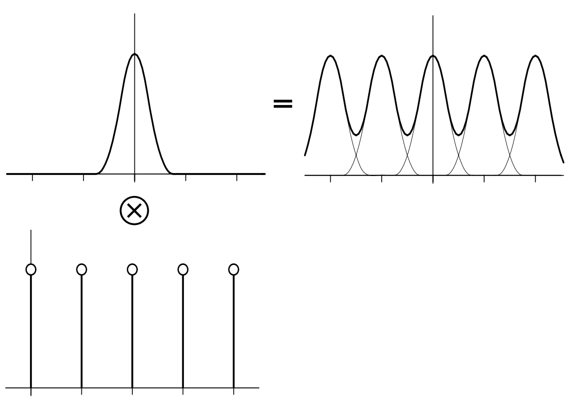
Sampling: Frequency Domain

$F(\omega)$



$$III_{1/T}(\omega) = \sum_{n=-\infty}^{\infty} \delta(\omega - n/T)$$

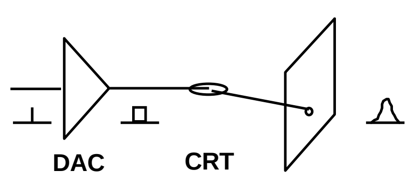
Undersampling: Aliasing



Reconstructing

Displays = Signal Reconstruction

All physical displays recreate a continuous image from a discrete sampled image by using a finite sized source of light for each pixel.

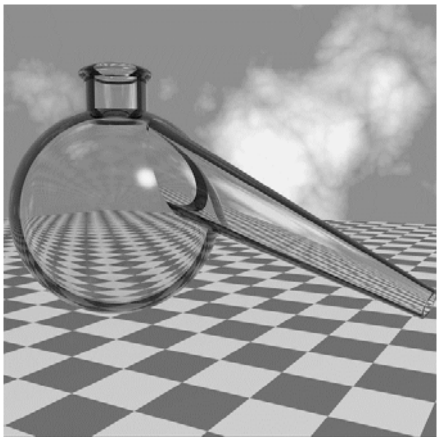


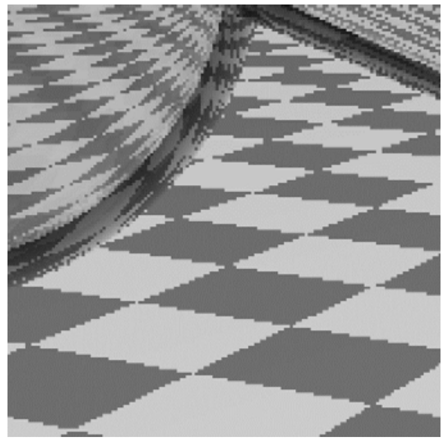
Examples:

- LCDs: Finite aperture
- DACs: sample and hold
- Cathode ray tube: phosphor spot and grid

Jaggies

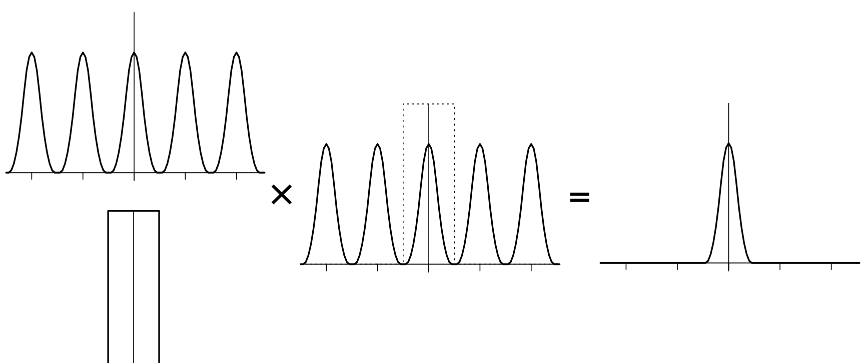
Retort image by Don Mitchell





Staircase pattern or jaggies

Perfect Reconstruction: Freq. Domain

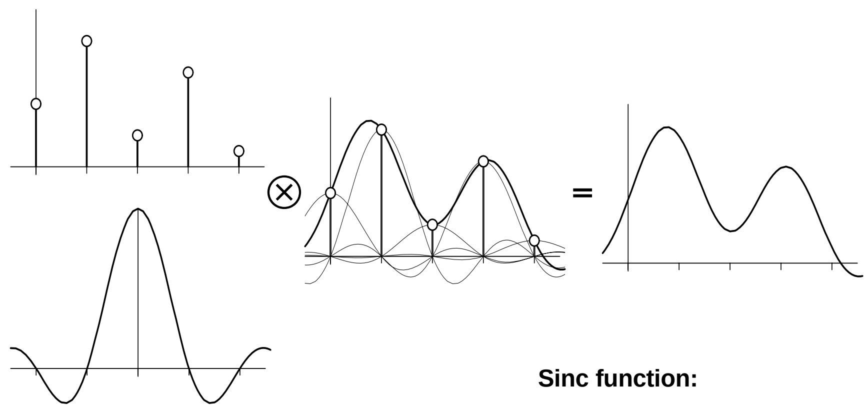


$$\prod\nolimits_{1/T}(x)$$

Rect function of width T:

$$\prod\nolimits_T(x) = \begin{cases} 1 & |x| \le \frac{T}{2} \\ 0 & |x| > \frac{T}{2} \end{cases}$$

Perfect Reconstruction: Spatial Domain

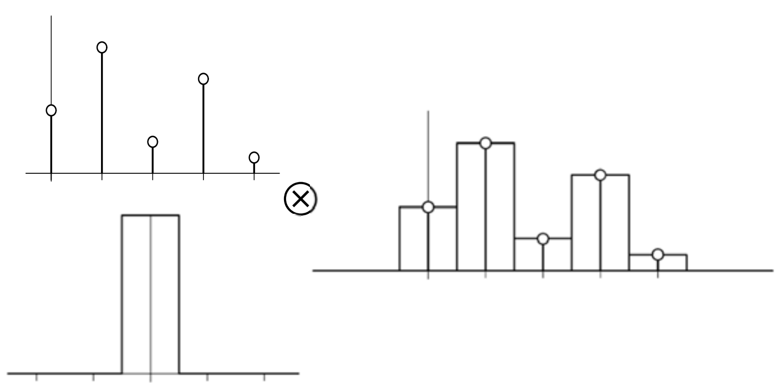


$sinc\ (x/T)$

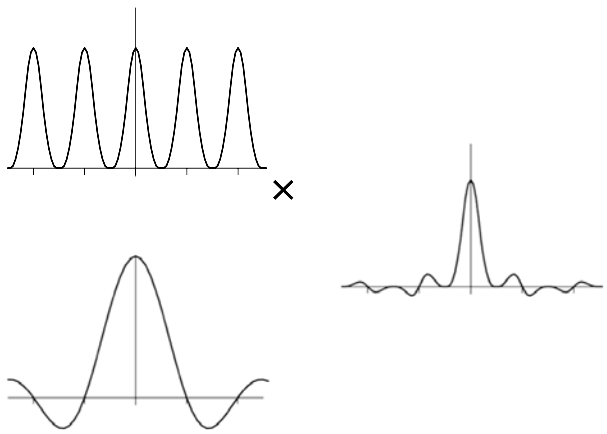
Sinc function:

$$sinc(x) = \frac{\sin \pi x}{\pi x}$$

Reconstruction: Spatial Domain



Reconstruction: Freq. Domain



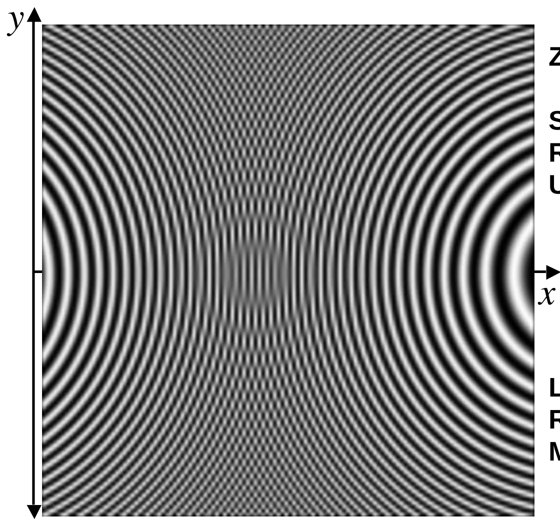
Reconstruction

Ideally, use a perfect low-pass filter - the sinc function - to bandlimit the sampled signal and thus remove all copies of the spectra introduced by sampling

Unfortunately,

- The sinc has infinite extent and we must use simpler filters with finite extents. Physical processes in particular do not reconstruct with sincs
- The sinc may introduce ringing which are perceptually objectionable

Sampling a "Zone Plate"



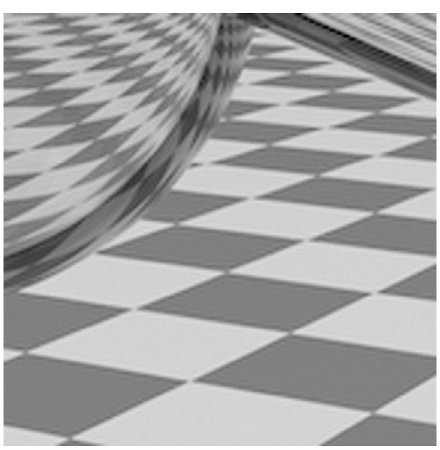
Zone plate: $\sin x^2 + y^2$

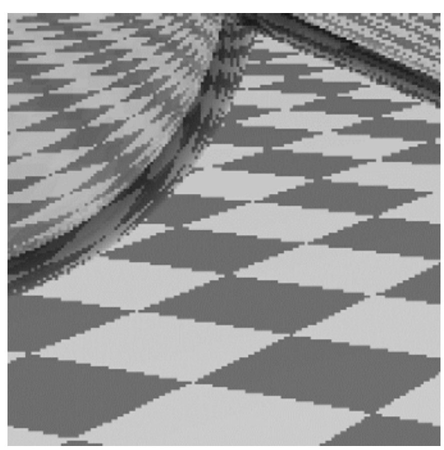
Sampled at 128x128
Reconstructed to 512x512
Using optimal cubic

Left rings: part of signal
Right rings: prealiasing
Middle rings: postaliasing

Antialiasing

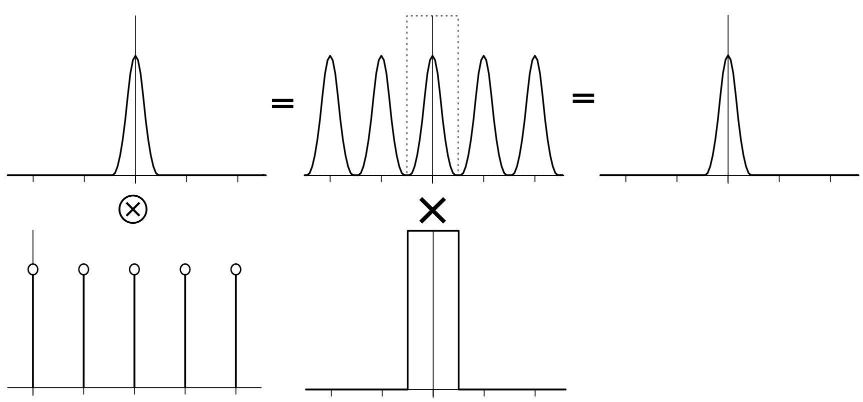
Retort image by Don Mitchell





The Sampling Theorem

Sampling and Reconstruction



Sampling Theorem

This result if known as the Sampling Theorem and is due to Claude Shannon who first discovered it in 1949

> A signal can be reconstructed from its samples without loss of information, if the original signal has no frequencies above 1/2 the Sampling frequency

For a given bandlimited function, the rate at which it must be sampled is called the ***Nyquist Frequency***

Sampling in Computer Graphics

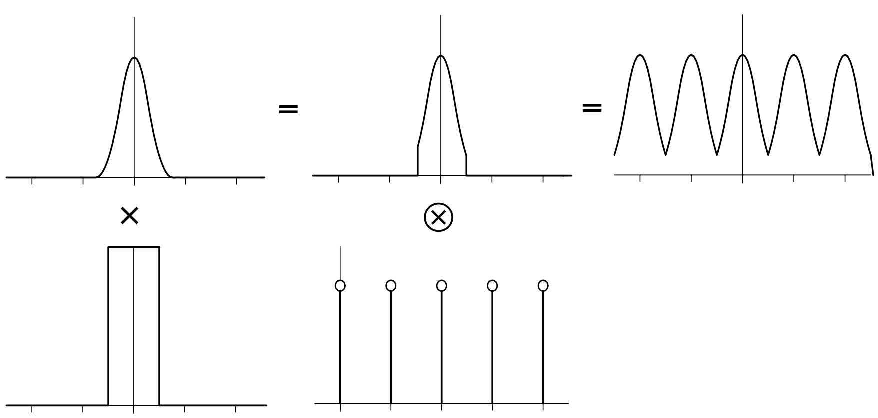
Artifacts due to sampling - Aliasing

- Jaggies
- Moire
- Flickering small objects
- Sparkling highlights
- Temporal strobing

Preventing these artifacts - Antialiasing

Antialiasing

Antialiasing by Prefiltering

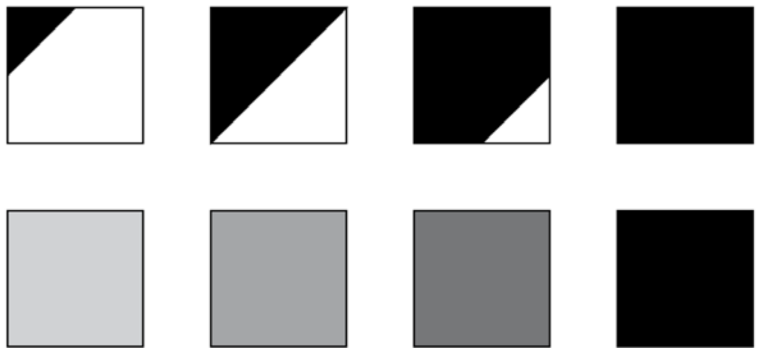


Frequency Space

Prefiltering by Computing Coverage

Pixel area = Box filter

Equivalent to area of pixel covered by the polygon



25% 50% 75% 100%

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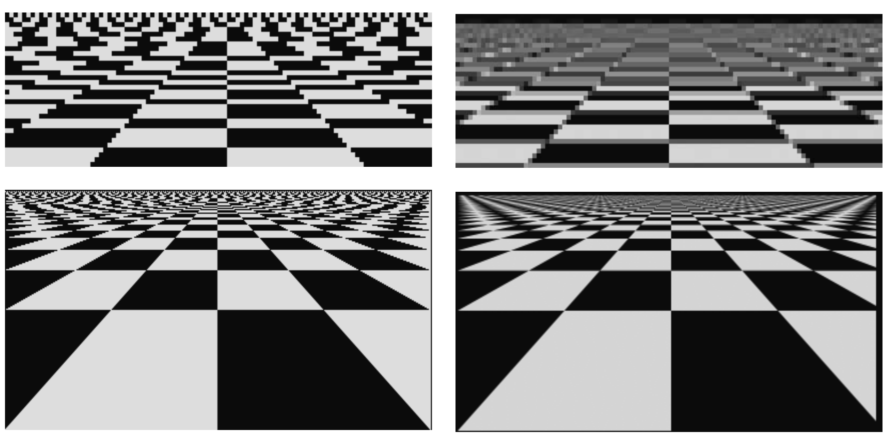
Point vs. Area Sampled



Point Exact Area

Checkerboard sequence by Tom Duff

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